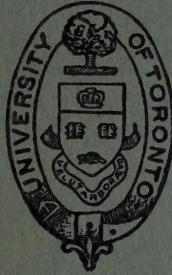


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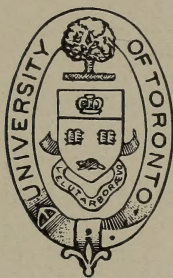
University of Toronto
FACULTY OF APPLIED SCIENCE AND ENGINEERING
SCHOOL OF ENGINEERING RESEARCH



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1927

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THE NEW AERODYNAMIC LABORATORY OF THE UNIVERSITY OF TORONTO*

J. H. PARKIN, M.E., F.R.A.E.S.

SUMMARY

The present paper contains descriptions of the new 4 ft. R.A.E. type wind channel recently installed at the University of Toronto, and of the auxiliary equipment including a modified Chattock micromanometer and alignment instruments, to which reference will be made in certain subsequent papers. This information will assist those interested in judging of the value of the results obtained. In addition, the results of air flow and power consumption studies of the wind channel at various stages during its construction are given, together with similar data pertaining to the original 4 ft. N.P.L. type wind channel at Toronto.

INTRODUCTION

It is considered advisable, for several reasons, to present at this time a paper containing a brief description of the wind channel and equipment of the new aerodynamic laboratory of the University of Toronto. In the first place, the nature of the equipment of the laboratory should be known by those interested, to assist in estimating the accuracy of the results of the investigations made and reported. Also, it has become customary to publish information relative to the characteristics of each wind channel, in order that comparison may be made with other channels, and to permit of the results of tests being co-ordinated with similar results from other laboratories. A certain amount of information of this kind pertaining to the Toronto laboratory, is contained in the present paper.

Then again, having the channel and general auxiliary equipment described in one paper enables the descriptions to be omitted from future reports, with consequent gain in conciseness of the latter. Thus references have been made in several research papers already prepared to apparatus being described in this report.

*Printed also in The Engineering Journal, Aug. (1927) p. 390.

Further, the results of air flow studies and power consumption tests made at different periods during the construction of the new channel are considered to be of sufficient interest to justify their presentation in the form of a research paper. As this information, to be of maximum value, should be accompanied by complete descriptions of the channel, it is presented as an appendix to this paper.

HISTORICAL

In 1917 the Board of Governors of the University of Toronto authorized the installation of an Aerodynamic Laboratory in the Department of Mechanical Engineering, and voted a special grant for its equipment. The laboratory was set up the following year in the Hydraulic Laboratory of the Mechanical Building. In 1923 conditions in the Mechanical Building became such as to force the removal of the channel. The Air Board of Canada, realizing the importance of such a laboratory to the development of aeronautics in Canada, and the necessity of having available facilities for experimental and research work in aerodynamics for the Government, offered a grant toward the erection of a suitable building to house the wind channel. The offer was made through the National Research Council and was gratefully accepted by the University. The building was erected, equipment installed and the laboratory formally opened in February 1924.

THE ORIGINAL CHANNEL

The original wind channel of the laboratory was of the N.P.L. type, 4 ft. square at the working section with graded latticed distributor $6' 3\frac{1}{2}'' \times 6' 3\frac{1}{2}'' \times 20' 0''$ inside. Speeds as high as 48 f.p.s. were obtained with the 5' 6" diameter propeller, chain driven from a 20 h.p. variable speed motor. Subsequently a latticed intake was fitted to this channel, together with a second honeycomb at the entrance to the cone, and a new propeller, enabling speeds of about 55 f.p.s. to be attained with improved steadiness and uniformity in the air stream. This channel and equipment are fully described in Aero Research Paper No. 1, September 1920.

THE NEW BUILDING

The new building is of brick and steel construction (see Fig. 1) with flat roof and concrete floor, and provided with ample window area. Its size, determined on a basis of that found satisfactory in an investigation conducted at the Royal Aircraft Establishment and described

in R. & M. No. 574*, is $23' 3'' \times 60' 0'' \times 20' 3''$ high inside. The walls are smooth and the building free from obstructions other than those necessary to support the equipment.

THE NEW CHANNEL

Before erecting the channel in the new building, the question of the best type of channel to be used in order to obtain maximum range and efficiency, was carefully considered, and, bearing in mind the relative merits of the different existing types of wind channel, together with the limitations imposed by local conditions, it was decided to use the R.A.E. type.

The R.A.E. (Royal Aircraft Establishment) channel is of the closed type with an experimental portion of square section, and free return of the air. It differs from the N.P.L. type channel in that the short expanding cone, small diameter propeller and latticed distributor of box form, are replaced by a long regenerative cone, large diameter propeller and cellular wall distributor. The channel has been described in a number of publications. (Note particularly R. & M.'s Nos. 574 and 847).

The general arrangement of the new Toronto channel and building, as shown in Fig. 2, is very similar to that of the No. 2 7 ft. channel of the Royal Aircraft Establishment. The channel is $44' 7''$ long overall, made up of the tube 4 ft. square in cross section and about 24 ft. long and the regenerative cone, roughly 20 ft. long and $9' 10''$ in diameter at the large end.

The latticed Toronto intake (refer Aero Research Paper No. 3, Sept. 1920) which was developed and applied to the original channel to correct certain irregularities of air flow arising from the presence of obstructions in the building† was abandoned, as unnecessary under the new conditions, and a faired bellmouth entrance of usual form employed.

CONSTRUCTION OF CHANNEL

The $4' \times 4' \times 24'$ long tube of the original channel, with its steel framework and supports, was used in the new channel, with only minor alterations.

The design of the regenerative cone was based on that found so

*The model building there tested was $5.7 \times 5.2 \times 14.3$ diameters, and the building housing the No. 2 7 ft. channel at the R.A.E. is of these proportions.

†The Toronto Intake is used in the 4 ft. N.P.L. channel of the Daniel Guggenheim School of Aeronautics, New York University, to correct similar troubles and "has proved extremely satisfactory". Aviation, May 1, 1926, p. 293.

satisfactory in the model studies made at the R.A.E., to which reference has already been made. It was found in these tests that by the use of a suitable regenerative cone the power required to produce a given air speed could be reduced 33%, as compared with that required with the usual short cone, without sacrificing either steadiness or uniformity of velocity distribution in the air stream.

In the cone there is a very easy transition in form and area of cross section from the 4 ft. square section of the tube to a circular section 9' 10" in diameter at the outlet or propeller end in a length of 19' 3". This is accomplished by replacing the straight sides of the square section by circular arcs of progressively smaller and smaller radius until at about 8 ft. from the outlet end the section becomes circular, of diameter slightly greater than the diagonal of the 4 ft. square. The corners of the square tube are carried through straight in the cone for a distance of about 6 ft. before beginning to flare out and merge into the circular.

The principal objection to cones of double curvature, such as the one described, is that of construction, which, with rigid materials, is troublesome and costly. This difficulty has been satisfactorily overcome at Toronto by using heavy canvas for the covering of the cone.

The construction of the cone will be apparent from Fig. 5. Nine transverse wooden frames, enclosing openings, corresponding to the different cross sections of the cone, are supported at the level of the channel axis on horizontal beams carried on steel supports. The frames are tied together by numerous longitudinal strips of wood, the frames and strips together constituting a latticed form over which heavy canvas is stretched under great tension, and fastened with tape and tacks. The canvas is given several coats of cellulose acetate dope. To permit walking inside the cone a narrow cat walk is planked along the bottom of the cone.

It was found when the propellor rotated in this cone that the canvas vibrated somewhat between the wooden strips as the blades passed, and it was feared that this might result eventually in the canvas pulling away; consequently, the portion of the cone between the last two transverse frames is sheeted with galvanized iron bolted to the strips.

This form of construction is relatively inexpensive and has proved most satisfactory in every way.

The construction of the intake is generally the same as that of the outlet cone, except that the original unfaired bellmouth, sheeted with wood, has been incorporated in the present faired intake. The whole is covered with doped canvas.

Subsequently, to prevent any possible infiltration of air through the wooden walls of the channel, the latter were also covered with canvas and the canvas doped.

A cellular honeycomb of sheet metal with 3" square cells 30" long, as used in the original channel, is placed about 42" from the entrance of the channel.

Longitudinal swaying of the channel is prevented by diagonal tie rods between the steel frames.

POSITION OF CHANNEL IN BUILDING

Care was taken in arranging the laboratory to place the channel as nearly in the centre of the building cross section as possible (see Fig. 3) in order to secure an even return flow of air through the building. The channel axis is equidistant from the side walls and practically midway between floor and roof, having regard to the presence of the roof beams. The centre line is 10' 3" above the floor, and an average of 10' 0" below the slightly sloping 12" I beams supporting the roof. Placing the channel at this height necessitated setting the steel frames used in the original channel on 9" concrete pedestals.

The intake end is 9' 6" from the end wall of the building, and the end of the regenerative cone practically 6' 0" from the other end wall.

OPERATING PLATFORMS AND SUPPORT OF INSTRUMENTS

Due to the height at which the channel is placed, an elevated platform is built below the channel at the experimental section, to enable the N.P.L. balance to be conveniently operated. Further, a small platform, reached by steps, is built at the level of the channel floor, outside the door of the experimental section to render access to the interior of the channel convenient in setting up and adjusting models and also to enable the roof balances, now being made, to be manipulated.

The N.P.L. balance is set on a heavy concrete pedestal and a small table is supported in a convenient position on an iron standard set in concrete to provide a rigid support for the manometers.

The platforms, stairs and table have all been built to offer a minimum resistance to the return flow of the air past them. (See Fig. 3).

CELLULAR WALL DISTRIBUTOR

The cellular wall distributor is described in Appendix I to this paper.

PROPELLER AND DRIVE

The propeller is four bladed, 9.5 ft. diameter by 2.75 ft. pitch, and rotates $2\frac{1}{2}$ " from the outlet end of the cone. This leaves a clearance of $1\frac{1}{2}$ " between the tips of the blades and the metal sheeting of the pro-

PELLER RACE. The tests at the R.A.E., already mentioned, and others at Langley Memorial Laboratory (N.A.C.A. Report No. 98) showed it to be disadvantageous to place the propeller far from the end of the cone.

The propeller is direct connected to the same motor as was used in the original channel, an 18-20 h.p. variable speed shunt motor. The shaft, just back of the propeller, is carried in a ball thrust bearing.

The motor and propeller are supported, quite independently of the channel, on a steel frame bolted to the end wall of the building and resting on a concrete pedestal. In this position the motor helps to deflect the air and is itself cooled.

SPEED CONTROL

The connections are arranged so that the motor can be run on either 110 or 220 volts, thus giving a wide speed range. The speed range of the channel is from 15 f.p.s. to 88 f.p.s.

Speed is at present controlled manually, using a drum type controller in the armature circuit, and coarse and fine rheostats in the field circuit.

There is now being installed in the Mechanical Building a constant voltage unit, controlled by a Tyrrell type regulator, which will deliver current at constant voltage to a special circuit. The wind channel motor will be connected to this circuit, rendering speed control in the channel much less troublesome.

AUXILIARY APPARATUS

(a)

MICROMANOMETER

A Chattock tilting micromanometer (See A.R.P. No. 2 p. 45 and Fig. 11) was used in the original channel, in conjunction with the side plate for indicating air speeds in the channel. This gauge possessed several distinctly valuable features, and while it was found generally satisfactory, it was open to two criticisms, namely, great fatigue and eyestrain in long continued use of the microscope and the excessive fragility of the glassware, necessitating extreme care in handling and manipulating the gauge. Further, with the increased speeds obtainable in the new channel, a gauge of greater range was required, since structural difficulties limit the range of the tilting gauge to pressures of about one inch of water.

Mr. Douglas describes in R. & M. No. 657, (1919), a modified form of Chattock manometer developed at the R.A.E. which overcomes the disadvantages and limitations of the tilting gauge, while retaining its desirable features. In this gauge, instead of tilting the glass U tube, one arm is reared to balance the pressure to be measured.

A gauge has been designed and several constructed at Toronto based on the R.A.E. design, but embodying certain features of sufficient interest to justify presenting a brief description of the gauge here.

The new gauge differs in three principal particulars from the Chattock tilting gauge, namely:—

1. The tilting principle is replaced by vertical movement.
2. The simplicity of the glassware.
3. The microscope is replaced by a projector.

The Toronto design, see Figs. 6 and 7, consists of a brass base carried on three levelling screws and fitted with a spirit level. Screwed to this base at one side is a C frame carrying a micrometer screw and large diameter graduated micrometer disk. The micrometer screw rotates in cylindrical bearings, and the end play, which necessarily must be zero, is eliminated by holding the screw axially between two steel balls, the upper ball being pressed down by a flat steel spring, seen in the figures.

A large diameter glass bulb, forming one arm of the U tube is carried on an arm, which may be raised or lowered by means of the micrometer screw, the amount of movement being shown by an indicating point registering on a graduated scale on the back of the C frame (See Fig. 7) and by the micrometer disk graduated in thousandths of an inch.

At the other side of the base is a long arm which may be raised or lowered slightly by means of an adjusting screw. The arm carries the glass bubble tube, forming the other arm of the U tube, and at either end a lamp and shield, and an optical projector.

The bubble tube corresponds to the middle one of the three tubes of the tilting gauges and, as in the latter, consists of two concentric vertical glass tubes. The inner tube, of about $\frac{1}{4}$ " bore, is ground to a knife edge at the top and at the bottom connects through a long rubber tube and two stop cocks to the bulb on the elevating screw, thus completing the U tube, which is filled with distilled water. The water rises to the knife edge of the inner tube, forming there a bubble or meniscus in the paraffin oil* with which the outer tube is filled.

First setting the micrometer head on zero and having filled the system, being careful to eliminate all air bubbles, the arm carrying the bubble tube is raised or lowered until the bubble rises to its position of maximum sensitivity. The lamp and optical system are then adjusted until a clear image of the bubble is thrown on the ground glass screens, the top of the image of the bubble being tangent to the cross hairs of the projector. Pressures may then be measured by raising the larger

*Standard Oil Co., medicinal paraffin oil, "Nujol".

bulb by means of the screw until the pressure is just balanced by the applied pressure, as is shown by the bubble remaining tangent to the hair line. The pressure is then read, in inches of water, directly from the scale and micrometer head.

Provision is made in the gauge for vertical adjustment of the concentrated filament lamp, and also of the asbestos lined aluminum shield, in order to bring the light and aperture in the shield into the best positions for proper illumination of the bubble.

The image of the bubble is thrown on two ground glass screens at right angles to enable the operators at both lift and drag arms of the balance to conveniently observe the air speed.

Of the two glass stop cocks provided, that forming part of the bubble tube safeguards the bubble when the gauge is being moved and is not ordinarily used, the other, conveniently clamped at the front of the stand, is used to close the gauge in ordinary routine operation.

The stand is employed to raise the gauge so that the rubber tubing may hang more or less freely and not change its shape appreciably for different elevations of the movable bulb. Two of the seats in which the three levelling screws of the base rest are in the form of V grooves, at right angles, the third being a conical seat. This enables variations in centre distance between the screws to be accommodated while at the same time preventing the gauge from slipping off the stand.

The advantages of this gauge are:

1. Its great sensitivity. The gauge will indicate a change of pressure of 0.0001 inch of water.
2. Its great range. The gauge shown has a range of 3 inches of water, but gauges of this type can be readily made with greater ranges.
3. Simplicity of the glassware.
4. Convenience of observation and absence of eyestrain.
5. Extreme accuracy in making the gauge is confined to the two parts, the screw and nut. The accuracy of the gauge depends solely on the accuracy of these parts.
6. The gauge is an absolute standard. No calibration is necessary provided the screw is accurate.

Gauges of this design have been used at Toronto in routine operation of the channel for some three years and have proved most satisfactory. After a long continued run, on returning the gauge to zero, the check is invariably perfect.

(b)

ALIGNING APPARATUS

In aligning the chords of aerofoils parallel to the air stream, in the horizontal plane, *i.e.* when testing aerofoils in the usual manner on an end spindle, an optical alignment device of great sensitivity and convenience is employed, replacing the apparatus previously used and described in Aero Research Paper No. 1. The device is similar to that employed in the atmospheric channel at the Langley Memorial Laboratory, as described in N.A.C.A. Technical Note No. 35, (1921), but with certain modifications rendered necessary in applying it to the R.A.E. type channel.

The apparatus consists of two main elements (See Fig. 8) a projector, throwing a narrow beam of light perpendicular to the air stream through a window in the channel wall, and a mirror clamped to the aerofoil (See upper instrument in Fig. 9) which reflects the beam of light back on a target carried by the projector. The aerofoil is adjusted until the reflected beam registers on a point on the target directly above the aperture through which the beam passed from the projector. The aerofoil chord is then parallel to the air stream.

The lens, slit, lamp, and target comprising the projector are carried on a frame, arranged to have a transverse motion on a base, which in turn slides on ways machined on the upper flange of a horizontal I beam, bolted to two of the steel frames of the channel. This permits the projector to be brought up quite close to the glass of the window and to be moved along the channel to accommodate different sizes and positions of aerofoil. The projector is at the level of the centre of the channel. Both slit and lamp carrier are adjustable for focusing purposes, and the 100 watt concentrated filament lamp is enclosed in a casing, not shown in the figure. The vertical target plate is carried in front of the lens, and as close to the glass of the window as possible. There is a narrow vertical slot in the target plate through which the beam of light is projected, and directly above the slot a small hole. The latter is illuminated by the beam of light, rendering it visible, through a window in the channel floor, to the operator at the balance.

The mirror may be turned about a spindle fitted to a light steel straight edge. The latter is arranged with a spirit level and clamp, by means of which it may be attached to the aerofoil, tangent to the chord, and horizontal.

In aligning an aerofoil, the procedure is as follows: The spindle carrying the aerofoil is first gripped in the chuck of the balance with the aerofoil in approximate alignment, and the straight edge carrying the mirror clamped to the aerofoil and levelled.

The projector is moved along the ways until the beam of light strikes the mirror. The latter is adjusted on its spindle to return the beam to the target at the proper level. The vertical arm of the balance is then rotated until the reflected beam strikes the target, bisecting the small illuminated hole. The operation of aligning an aerofoil may thus be carried out by a single operator. The device is initially calibrated by testing an aerofoil in the usual way in normal and reversed positions for lift and drag.

The device is exceedingly convenient and accurate. The reflected beam can be easily located on the target to within $1/64''$ which, on a length of 48" means that the aerofoil can be lined up to within one minute of the desired direction.

For aligning complete models or aerofoils in normal flight position, the lower instrument in Fig. 9 has been found very convenient. This instrument is a level, consisting of a light steel straight edge, provided with adjustable spring clamps, and to which is fastened at one end a protractor. The vernier indicator of the protractor, fitted with a lens, is attached to a spirit level. The desired angle of incidence is set accurately on the protractor, with the aid of the vernier and lens, and the instrument attached to the model with the straight edge tangent to the chord. The model is then adjusted until the spirit level is level. The straight edge, and therefore, the chord, are then at the required angle to the horizontal.

(c)

AERODYNAMIC BALANCE

The balance of the channel is of the N.P.L. type and has been fully described elsewhere. In addition to the improvements incorporated in the balance as originally made, described in Aero Research Paper No. 1, additions have since been made to the balance to improve its range and convenience of operation.

1. The weight arms have been fitted with steel tie rods to limit deflection under the greater weights to be carried at the higher air speeds possible in the new channel.
2. The vertical arm of the balance has been fitted with a large gear and pinion to enable settings of incidence to be more easily made.
3. A small microscope permits the protractor to be read with greater precision.
4. A turntable has been inserted in the channel floor surrounding the vertical arm of the balance to enable biplane and similar

tests to be conveniently made. The under side of the turntable is graduated in degrees and provided with a vernier to enable accurate settings of angle being made. A series of holes, arranged in a spiral, are tapped into the turntable to take a standard fitted with a chuck for holding a second spindle supporting an aerofoil or other model.

At the present time there are under construction two roof balances for the channel patterned after those used in the British 7×14 channel. These balances are to be carried on steel beams above the channel. The forward balance is arranged for measuring lift and rolling moments, while the rear balance will measure lifts and drags. The two will thus enable complete determinations of lift, drag, centre of pressure, pitching and rolling moments to be made, with one setting of the model, and yawing moments with a second setting. An advantage of the balances will be the absence of any wires or other obstructions upstream from the models.

AERODYNAMIC CHARACTERISTICS OF THE CHANNEL

The aerodynamic characteristics of the present Toronto channel are plotted on Plate No. 1. Input power to the propeller and propeller speed* are plotted on average speed over the central 30" square of the channel cross section. The maximum observed speed was 87.8 f.p.s., practically one mile per minute, at a propeller speed of 710 r.p.m. and a power input of 24.03 horsepower. Side plate pressures are also plotted on this diagram.

For purposes of comparison the characteristics of the original N.P.L. type channel at Toronto, under two sets of conditions, have also been plotted on Plate No. 1. In one case the channel was equipped with a simple bellmouth intake, honeycomb and four bladed propeller, chain driven, of 5' 6" dia. and 3' 0" pitch, in the other case the channel was fitted with the Toronto intake, and a more efficient propeller of N.P.L. design, 5' 6" dia. and 2' 3" pitch (See Fig. 1a Aero Res. Paper No. 1).

A comparison of these curves is instructive. The very high power consumption of the original channel in the first case was due principally to two causes, namely, large exit losses due to high velocity at exit from the cone, and an inefficient propeller. In the second case, while the addition of the latticed intake increased the power consumption slightly,

*Propeller speeds were read on an aircraft tachometer ordinarily used to roughly indicate the propeller speeds. Precise readings were not possible with this instrument and the plotted points are consequently in some cases rather far from the line.

this was more than offset by the greater efficiency of the new propeller. The very large saving in power effected by the new design of channel employing the regenerative cone and large diameter propeller is well indicated by the power curves.

A further comparison of the channels is afforded in the following table:—

COMPARISON OF TORONTO CHANNELS

Channel	Propeller		$\frac{V}{N D}$	Power Factor
	(All propellers have four blades)			
	Diameter	Pitch		
Original N.P.L. type....	5.5	3.00	.00984	10.83
Original N. P. L. type with Toronto intake..	5.5	2.25	.01042	6.50
Present R.A.E. type.....	9.5	2.75	.01298	2.19

In brief, the original channel used five times, and as modified used three times the power of the present channel for the same air speed.

The variation of static pressure along the channel axis at different air speeds is shown on Plate No. 2. The pressures were determined from pressure differences between the static pressure openings of a standard Pitot Tube and the side plate of the channel, measured on the micromanometer previously described in this paper.

APPENDIX I.

CELLULAR WALL

It was anticipated that placing the channel in the centre of the building cross section would result in an evenly distributed flow of air in the building, and consequently a uniform distribution of velocity in the channel. However, velocity traverses made in the channel, see A Plate, No. 5, disclosed the fact that the variation in velocity in the channel was excessive. In an effort to determine the cause of the variation an extensive study of the air flow in the building was made, using fine silk threads and smoke, the results of which are shown in Plate No. 4. It will be seen that the flow had a decided rotary motion imparted to it by the propeller, that there was much eddying in the building, and that the platforms and apparatus under the channel displaced the air currents to a considerable extent. The bulk of the air entered the bellmouth at the top and east side, resulting in the uneven distribution of velocity found in the channel.

A further study of the rate of the return flow, using vane anemometers, confirmed the results of the smoke investigation and showed (See Plates No. 3 A and B) that the highest velocity of flow was along the boundaries of the building cross section, walls, roof, and floor, and that very little return flow took place in the neighborhood of the channel.

In tests with models made at the Royal Aircraft Establishment, and described in R. & M. 574, Dec. 1918, it was found that placing a cellular wall across the wind channel building improved considerably the distribution and steadiness of flow in the channel, particularly in a small building. It was further found that the ratio of hole area to total wall area should be about one-third, and that the best position of the wall is from one-half to one propeller diameter from the large end of the cone.

Hence, to correct the previously mentioned defects in the air flow, both within and without the channel, a cellular wall was erected across the building (See Fig. 4). The wall was placed 8' 3" from the outlet end of the cone. This position conformed to the recommendation quoted in the foregoing, and was also convenient due to one of the steel supporting frames being located at this section.

The wall was composed of the latticed gratings used in the distributor of the original N.P.L. type channel (See Aero Research Paper No. 1), the area of the wall being only slightly less than the total area of the distributor. The gratings are made with openings $1\frac{1}{4}$ " square and $1\frac{1}{4}$ " long, to have a directive effect on the air flow, and the various gratings have different percentages, varying from 12 to 51% of the whole area, occupied by the openings. The total area of the openings in all of the gratings composing the wall is 22% of the cross sectional area of the building.

As will be seen in Fig. 4, in arranging the wall, an effort was made to secure uniform distribution of return flow throughout the building by placing the gratings with minimum open area around the boundaries of the building cross section, and those with maximum hole area next the channel. The arrangement of the gratings in the wall was, as far as possible, symmetrical about the channel.

The distribution of velocity in the channel still proved unsatisfactory (See B Plate No. 5) and a further anemometer investigation of the flow through the wall and around the intake showed (See Plates No. 3 A and B) that most of the return flow was taking place along the west side of the building, the flow along the east side, particularly near the floor, being quite small.

Consequently, the hole area on the West side of the wall was somewhat reduced, and that on the East side, and especially near the floor, was

increased. Further, as apparently the presence of the platforms, pedestals, instruments, etc., below the channel seriously restricted the flow below the channel, in an attempt to establish more symmetrical flow, a slotted grating, 6' 6" square, with a slot area equal to about 50% of the whole area, was placed above the channel between roof of building and top of channel, at the working section*. These changes effected a fairly uniform distribution of return flow of air into the bellmouth (See Plates Nos. 3 A and B), and the resulting distribution of velocity over the working part of the channel proved to be satisfactory. (See C Plate No. 5).

It was found, from power measurements made and described in Appendix II, that the presence of the cellular wall actually reduced slightly the power necessary to produce a given air speed in the channel, apparently due to the elimination of power loss in eddies in the building.

APPENDIX II.

POWER CONSUMPTION TESTS

At various stages during the erection of the channel power consumption tests were made on the channel. The power input to the motor was measured and the accurately determined losses in the motor deducted to yield the power input to the propeller. Velocity traverses were made at the experimental section of the channel with a standard Pitot tube (See Aero Research Paper No. 2 Fig. 8, right hand tube) and a slanting or Krell manometer, described in the same paper, p. 43 and Fig. 9, readings being taken every six inches, and in many of the latter tests every three inches. The reference velocity was held constant by means of a side plate and Chattock manometer, later a modified Chattock manometer, as described in the foregoing paper. From the traverses the average velocity over the central or working part of the air stream 30" $\times$ 30" was determined.

In all, tests were made under seven sets of conditions, as follows:—

- Test A—Bare channel, without honeycomb or cellular wall and with canvas untreated.
- Test B—Canvas of the regenerative cone treated with five coats of cellulose acetate dope.
- Test C—Canvas of the regenerative cone treated with six coats of dope, that of intake with one coat of dope.

*Compare with R. & M. 847, (1922), in which it is stated that with the No. 2 7 ft. channel at the R.A.E. it was found necessary to blank off 100 sq. ft. of area of wall above the channel.

Test D—Honeycomb built in the channel.

Test E—Propeller race sheeted with galvanized iron.

Test F—Cellular wall erected.

Test G—Channel tube covered with canvas and latter doped.

The results of these tests are tabulated in Table No. 3 and plotted on logarithmic axes on plate No. 6.

The graphs show clearly the effects of the different changes made in the channel during its erection. Doping the canvas reduces appreciably the power necessary to produce a given air speed. The introduction of the honeycomb, as would be expected, greatly increases the power, while each of the subsequent changes, namely, sheeting the propeller race, covering the tube with canvas, and erecting the cellular wall, results in slight, though definite, reduction in power.

The reduction in power effected by preventing infiltration of air through the channel walls agrees with similar results secured with the No. 2 7 ft. R.A.E. channel. (See R. & M. 847, (1922), p. 4).

That the cellular wall would reduce the expenditure of power necessary for a given air speed in the channel was rather unexpected, but apparently the increased resistance to flow, at the low air speeds, is more than offset by the reduction in the power lost in eddies in the building.

Theoretically, the power required for a wind channel varies as the cube of the air speed. The slope of the lines on Plate No. 6 is 1.473, *i.e.*, the power varies as the 1.473 power of the dynamic, or impact, pressure, or as the 2.946 power of the velocity.

In the case of the experiments at the R.A.E. on model channels, the "power factor" is used as a means of comparing the consumption of power of different channels, where the power factor is the input power to the propeller to produce a velocity of 100 f.p.s. over 1 sq. ft. of working section of the channel, *i.e.*,

$$\text{power factor} = \frac{\text{input horse power}}{A V^3} 10^{-6}$$

By extrapolation on Plate No. 6 the power to produce 100 f.p.s. air velocity in the Toronto channel would be 35 h.p., corresponding to a power factor of 2.19. The best arrangement of model, propeller, etc., in the R.A.E. tests yielded a factor of 1.9, and the factor for the No. 2 7 ft. R.A.E. channel, of which the design was based on the model tests, was 1.9.

POWER CONSUMPTION TESTS

TABLE 1

Original 4 ft. N.P.L. Type Wind Channel

Power to Propeller Watts	Horse Power	Propeller Speed	Air Speed f.p.s.
505	0.677	270	14.0
550	0.737	307	16.2
1,084	1.450	372	19.6
1,292	1.732	400	21.5
2,235	3.000	473	25.4
2,932	3.925	510	27.6
3,810	5.11	560	30.4
4,845	6.49	613	32.9
6,250	8.38	663	36.3
8,822	11.83	735	40.9
12,042	16.16	815	46.1
13,500	18.10	843	47.5

TABLE 2

Modified 4 ft. N.P.L. Type Wind Channel

(Toronto Intake and N.P.L. Propeller)

219	0.294	340	18.5
579	0.776	390	22.2
1,223	1.641	455	26.0
1,666	2.232	510	29.1
2,671	3.578	575	33.2
3,720	4.985	645	36.9
4,197	5.63	670	38.4
6,110	8.19	755	43.1
8,275	11.09	825	48.1
11,980	16.09	905	53.1

POWER CONSUMPTION TESTS

TABLE 3

Toronto 4 ft. R.A.E. Type Wind Channel

Power to Propeller Watts	Horse Power	Propeller Speed	Impact Pressure Inches Water	Air Speed f.p.s.
<i>Test A—Bare Channel</i>				
567	0.76	205	0.216	30.78
777	1.04	270	0.287	35.45
1,203	1.61	300	0.369	40.20
1,849	2.48	340	0.501	46.90
2,677	3.59	390	0.631	52.60
<i>Test B—Canvas of Regenerative cone doped (5 coats)</i>				
366	0.491	204	0.185	28.48
554	0.744	225	0.240	32.45
844	1.132	260	0.347	39.00
1,202	1.61	295	0.396	41.65
1,901	2.55	340	0.537	48.50
2,773	3.72	390	0.707	55.60
<i>Test C—Canvas doped—Exit cone 6 coats, Intake 1 coat</i>				
216	0.289	175	0.129	23.78
527	0.707	225	0.241	32.5
791	1.06	260	0.318	37.3
1,165	1.56	295	0.408	42.3
1,299	1.74	305	0.427	43.3
1,809	2.43	340	0.527	48.1
2,507	3.36	375	0.664	54.0
<i>Test D—Honeycomb built in</i>				
361	0.484	208	0.115	22.42
549	0.736	230	0.153	25.90
847	1.135	263	0.201	29.75
1,204	1.610	295	0.259	33.70
1,345	1.805	305	0.295	35.95
1,908	2.560	335	0.359	39.70
1,863	2.495	335	0.365	40.00
2,736	3.670	380	0.475	45.70
<i>Test E—Propeller race sheeted</i>				
547	0.734	225	0.153	25.9
832	1.116	260	0.206	30.0
1,197	1.605	290	0.268	34.25
1,333	1.790	305	0.288	35.45
1,894	2.540	330	0.365	40.00
2,054	2.752	345	0.378	40.65
2,508	3.365	365	0.437	43.8
3,279	4.390	440	0.528	48.1
4,259	5.725	450	0.650	53.4
5,165	6.920	475	0.717	56.0
6,170	8.275	500	0.812	59.6

Power to Propeller		Propeller	Velocity Head	Air Speed
Watts	Horse Power	Speed	Inches Water	f.p.s.
6,885	9.24	525	0.886	62.3
7,795	10.44	545	0.950	64.5
14,302	19.20	655	1.420	78.8
15,763	21.13	700	1.528	81.8
19,010	25.50	735	1.740	87.2
19,330	25.90	750	1.735	87.1

Test F—Cellular Wall erected

441	0.590	215	0.141	24.85
682	0.913	250	0.191	28.90
1,023	1.370	275	0.242	32.55
1,189	1.591	285	0.269	34.30
1,603	2.150	320	0.328	37.85
1,697	2.275	325	0.342	38.7
2,038	2.725	335	0.391	41.4
2,781	3.730	375	0.482	45.9
3,626	4.865	400	0.578	50.3
4,278	5.73	440	0.641	53.0
6,625	8.89	500	0.874	61.9
8,205	11.00	550	1.028	67.1
8,860	11.88	550	1.025	67.0
11,475	15.39	620	1.289	75.0
15,197	20.19	675	1.546	82.2
16,435	22.05	700	1.673	85.5
17,385	23.3	700	1.736	78.1
17,385	23.3	700	1.746	87.4
18,460	24.9	710	1.785	88.0
18,460	24.9	710	1.772	88.0

Test G—Channel tube covered with doped canvas

336	0.450	200	0.118	22.7
463	0.621	220	0.156	26.1
770	1.032	250	0.205	30.0
1,147	1.537	280	0.268	34.3
1,295	1.737	300	0.292	35.7
1,849	2.476	325	0.366	40.0
1,897	2.545	335	0.385	41.0
2,370	3.177	350	0.447	44.2
3,116	4.178	390	0.535	48.4
4,064	5.450	425	0.652	53.5
4,878	6.540	455	0.729	56.4
6,940	9.310	520	0.942	64.3
7,150	9.580	525	0.942	64.3
7,405	9.925	525	0.967	65.0
7,855	10.530	525	0.981	65.5
8,062	10.810	540	1.028	67.1
8,931	11.960	550	1.036	69.3
13,382	17.920	650	1.459	79.9
14,791	19.810	670	1.564	82.7
17,923	24.030	710	1.764	87.8



FIG. 1.

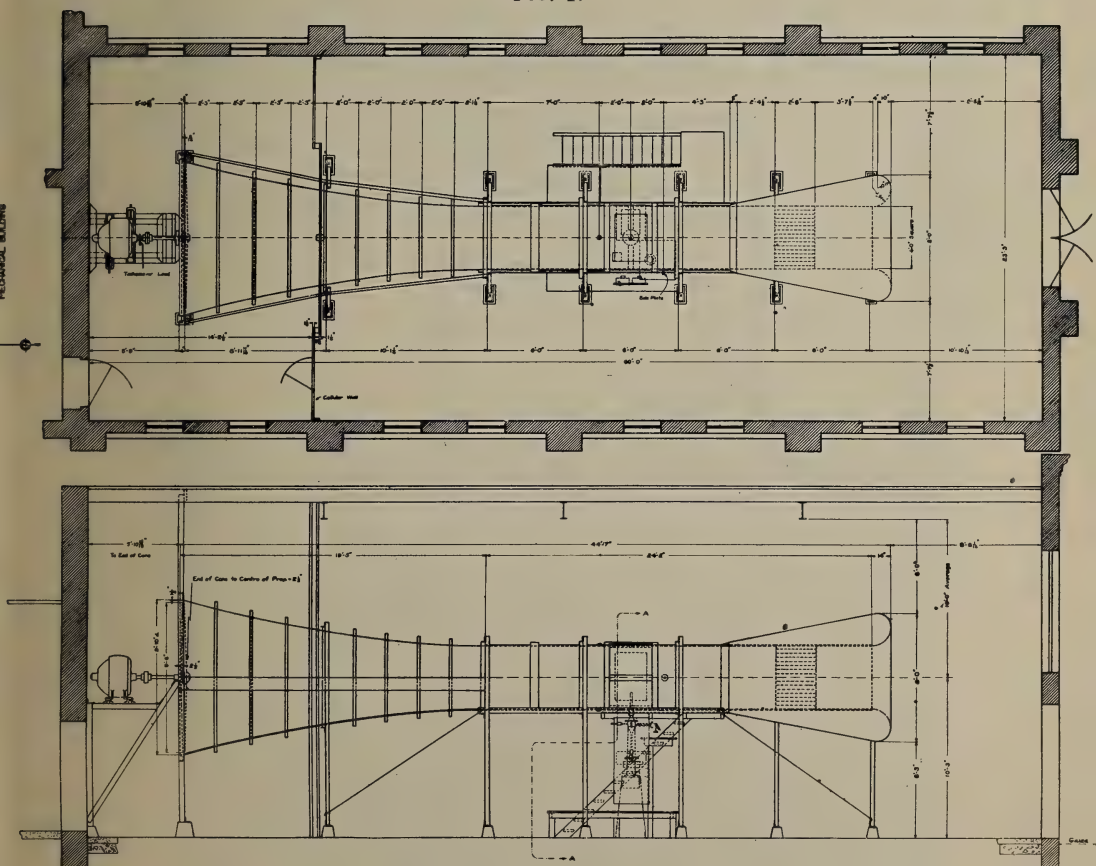


FIG. 2.

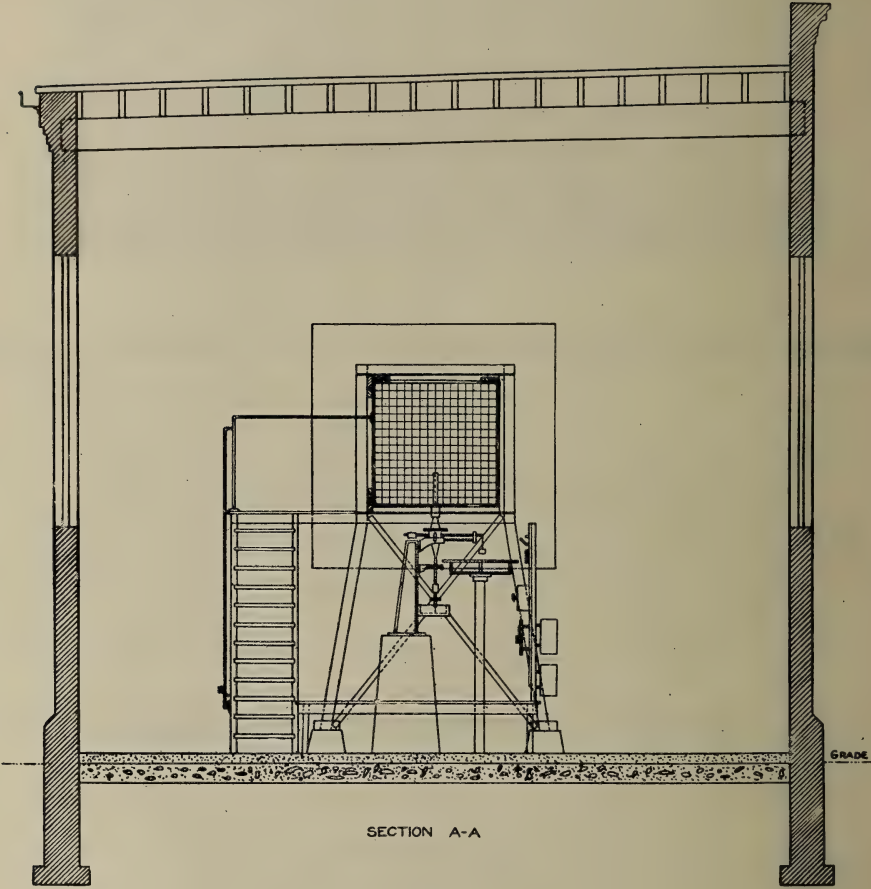


FIG. 3.

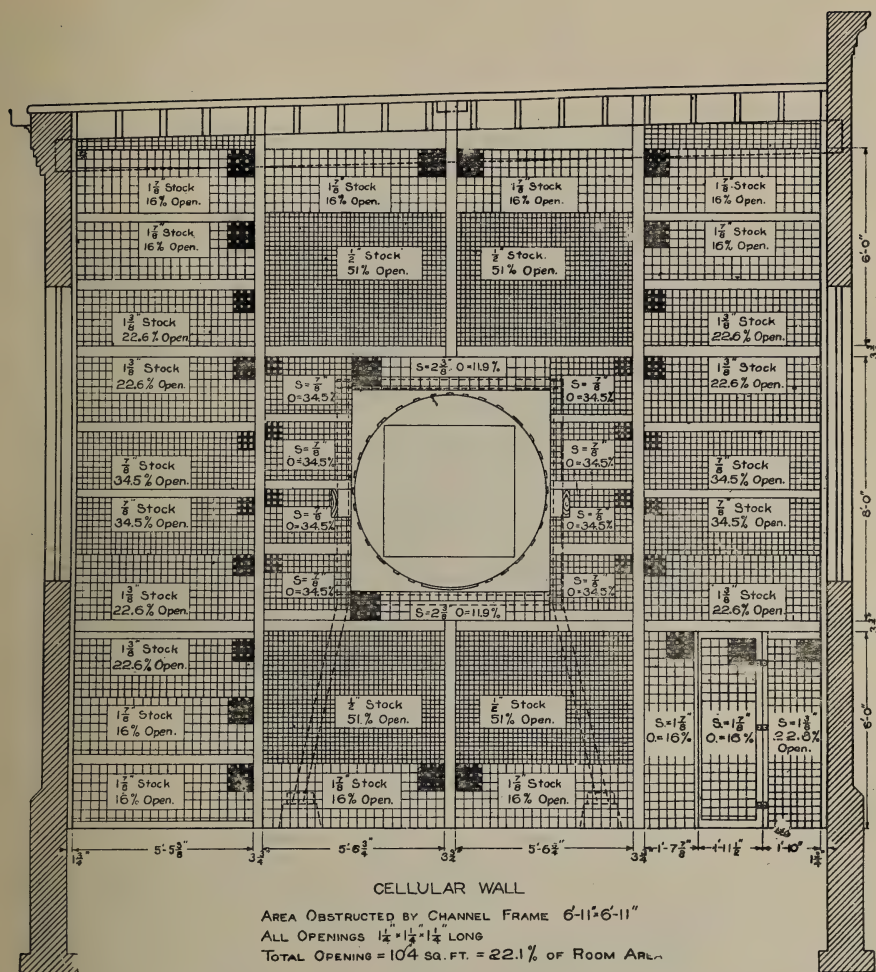


FIG. 4.



FIG. 5.

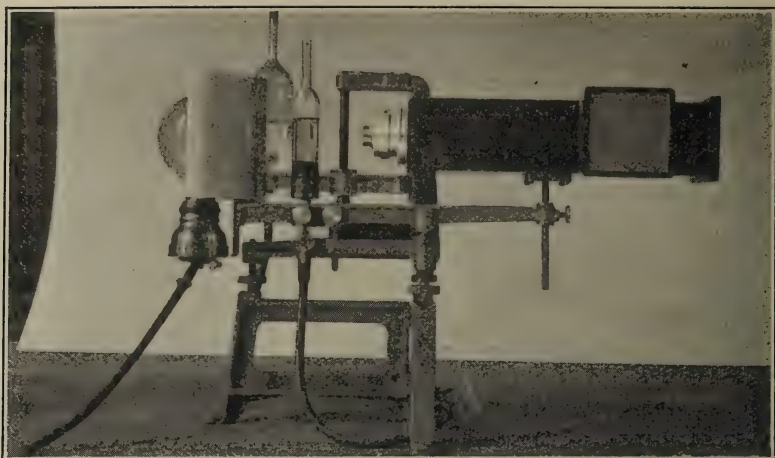


FIG. 6.

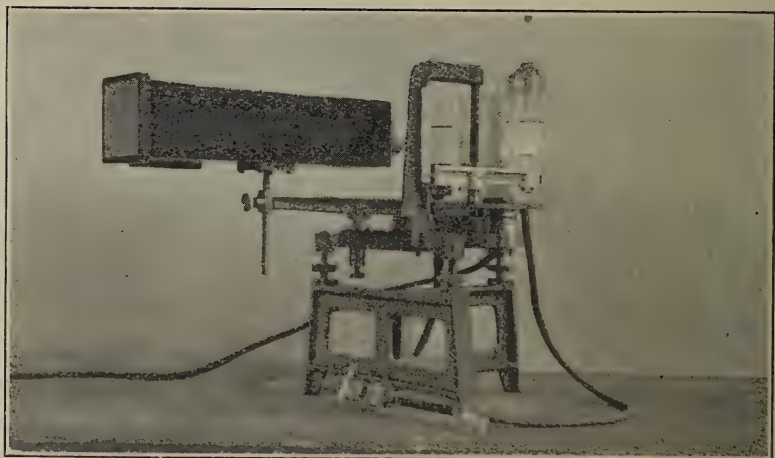


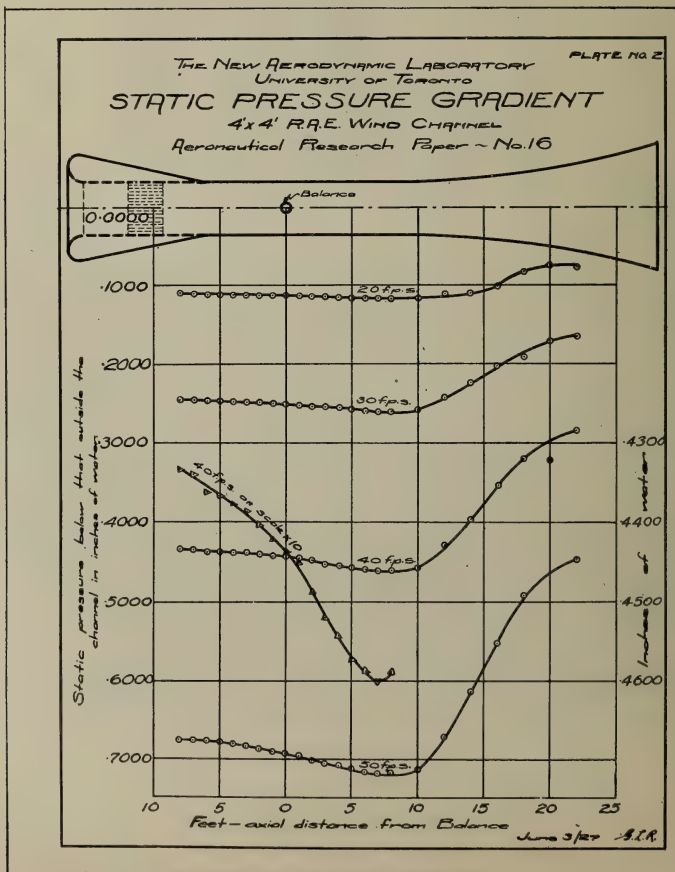
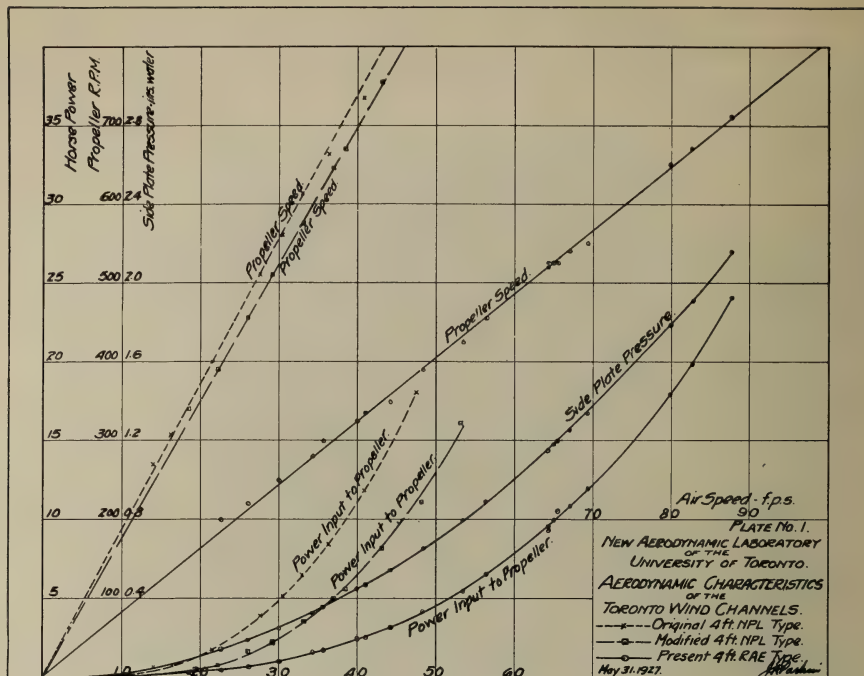
FIG. 7.

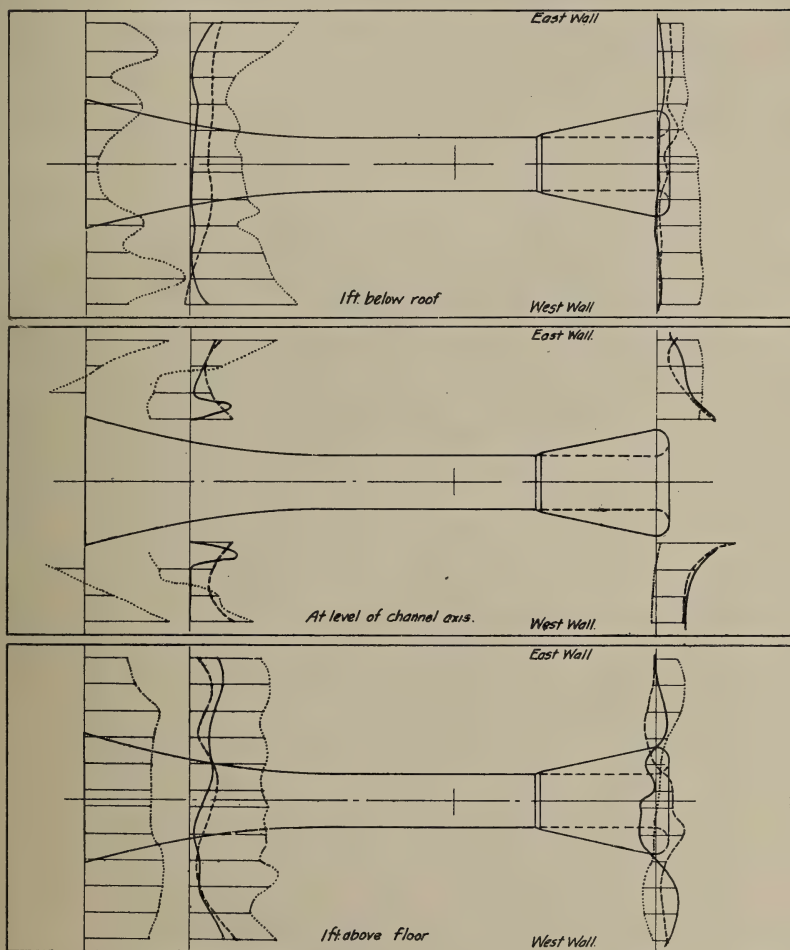


FIG. 8.



FIG. 9.





PLANS

THE NEW AERODYNAMIC LABORATORY
OF THE
UNIVERSITY OF TORONTO.

AIR SPEEDS IN THE BUILDING

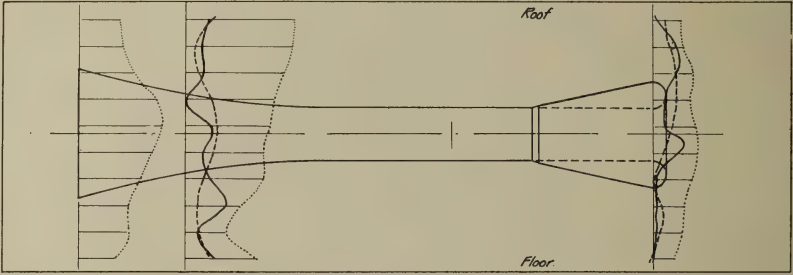
Air Speed 0 5 10 f.p.s.

JUNE 14, 1927

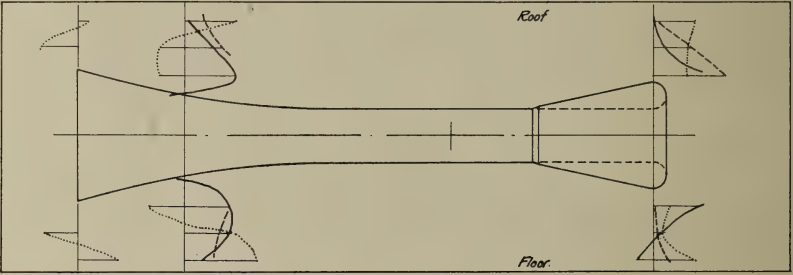
J. H. P.

..... Before erection of cellular wall.
----- With symmetrical wall.
———— With unsymmetrical wall.

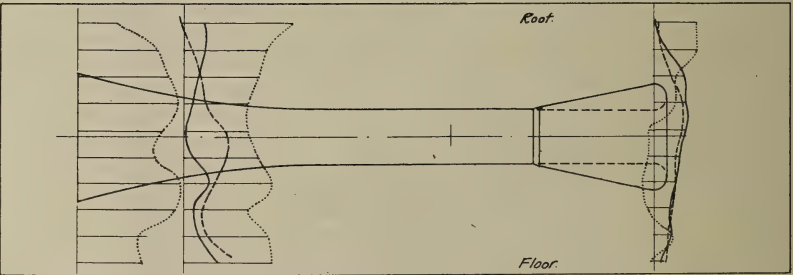
PLATE No. 3A.



1 ft. from East Wall.



At channel axis.



1 ft. from West Wall.

ELEVATIONS.

THE NEW AERODYNAMIC LABORATORY
OF THE
UNIVERSITY OF TORONTO.

AIR SPEEDS IN THE BUILDING.

Air Speed. 0 5 10 f.p.s.

June 14, 1927.

Handwritten signature

PLATE No. 3 B.

..... Before erection of cellular wall.
----- With symmetrical wall.
———— With unsymmetrical wall.

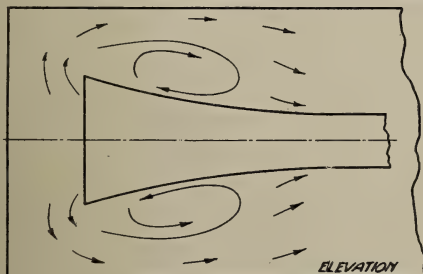
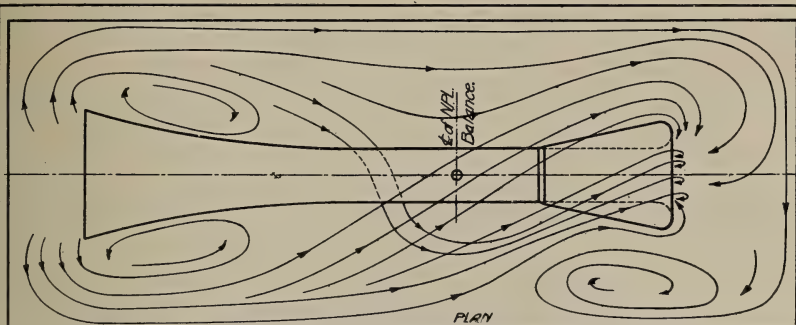
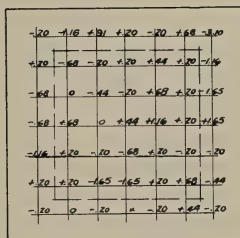
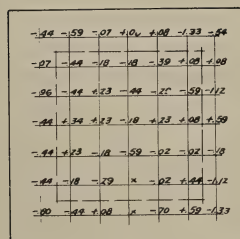


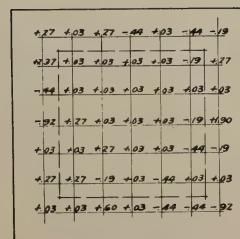
PLATE No 4:
 THE NEW AERODYNAMIC LABORATORY
 OF THE
 UNIVERSITY OF TORONTO
 GENERAL AIR FLOW
 IN THE
 BUILDING
 BEFORE THE ERECTION OF CELLULAR WALL.
 AERONAUTICAL RESEARCH PAPER 11916
 May 31/27 G.J.K.



A Before erection of cellular wall.

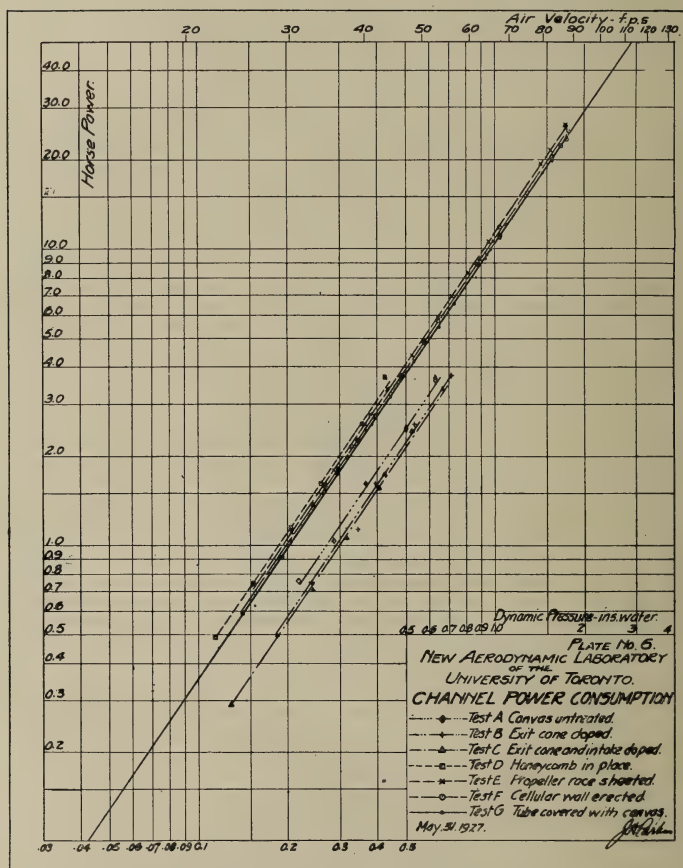


B With symmetrical wall



C With unsymmetrical wall

THE
 NEW AERODYNAMIC LABORATORY
 OF THE
 UNIVERSITY OF TORONTO
 VELOCITY DISTRIBUTION
 IN THE
 WIND CHANNEL
 Percentage Variation from Mean.



RESEARCH ON CHANNEL WALL INTERFERENCE

By J. H. PARKIN, B.A.Sc., M.E., F.R.Ae.S., M.S.A.E., M.A.S.M.E.,
J. E. B. SHORTT, B.A.Sc. and C. G. HEARD, B.A.Sc.

Summary

An investigation of channel wall interference with the object of verifying experimentally the accuracy of the results given by the theory developed by Prandtl and extended by Glauert. Tests were made, with cylinders and aerofoils, using the "mirror" method, for some ten different channel sizes. The results show:

1. The presence of the channel walls results in an increase in the drag of a symmetrical body, such as a cylinder.
2. Agreement with the theoretical corrections for incidence and drag for channels of reasonable size.
3. The extent and nature of the wall interference effects on the aerodynamic characteristics of aerofoils in much restricted streams, beyond the limits of applicability of the theory.

General Introduction

The wind channel is one of the principal sources of numerical information for the development of aerodynamics and the design of aircraft. In the wind channel, a reduced scale model of the complete aircraft or aircraft component is supported in a moving current of air, and the forces and moments exerted upon it are measured on an aerodynamic balance. The conditions of the wind channel test differ materially from those of the full sized aircraft in free flight, and hence the results obtained from the model test are subject to various corrections before becoming strictly applicable to full scale free flight conditions.

The modifying influences which must be taken into consideration when using wind channel* data may be listed as follows:

1. Those due to the model
 - (a) Size
 - (b) Nature of surface
2. Those due to support of model

*Atmospheric pressure channels only are here considered.

3. Those due to the wind channel
 - (a) Type (open or closed)
 - (b) Cross section. I. Shape
II. Size.
4. Those due to the air current
 - (a) Speed
 - (b) Turbulence or flow texture.

Some of these, notably those due to model, model support and air speed, have been much investigated, and considerable experimental information accumulated. On others, but little experimental work has as yet been done, and knowledge is limited. This particular research was concerned with the corrections necessary due to the wind channel, on which experimental information is lacking.

The work was performed in the Aerodynamic Laboratory of the Department of Mechanical Engineering, University of Toronto, under the auspices of the School of Engineering Research. A grant was made by The National Research Council for the purchase of the models and other special apparatus, including the erection of the cellular wall across the building, used in the research.

After the design and construction of the special holding devices and necessary apparatus during the summer of 1924, preliminary tests were made on cylinders during August and September 1924. The bulk of the work on cylinders, and all of that on aerofoils, was done from May to September 1925. Pressure of other work prevented the preparation of the results until the present, August 1926.

Channel wall interference

In wind channels the air stream is of limited extent, and restrictions are hereby imposed on the air flow past the model. Hence, in order to enable the results of wind channel tests to be applied to aircraft in an air space infinitely extended in all directions, the nature and magnitude of the influence exerted by the boundaries of the stream on the air flow, and resulting forces on the model, must be known.

Modern aerofoil theory has provided a basis for estimating the effect of the channel walls on the air flow past aerofoils, and the present study was made primarily with the object of securing further experimental confirmation of the effects predicted by theory.

There are two types of wind channel in general use. In one the air flow takes place through a closed tube and the air stream is thus limited by fixed walls parallel to the direction of flow. The air stream is of square, rectangular or circular cross section. In the other type, the air

flow takes the form of a free stream or jet of circular section without fixed boundaries.

The theory of the channel wall interference for an aerofoil in a circular section channel has been developed by Prandtl (Gottingen Nachrichten 1918-19, N.A.C.A. Report 116, N.A.C.A. Tech. Note. No. 10) and the theory extended by Glauert (R. and M. 723 and 867) to the cases of channels of square and rectangular sections.

In a channel with open working section, the presence of the boundaries deflects the air stream downward at the model, so that for a given lift the measured angle of incidence and drag are both higher than for an unlimited stream; while in a channel of closed working section, the stream is deflected upward and the incidence and drag are both measured too low.

The present research was confined to an investigation of the wall interference effects in wind channels of the closed tube type, of square and rectangular cross sections.

PART I

GENERAL THEORY OF AN AEROFOIL IN AN UNLIMITED STREAM.

(R. AND M. 723, ENGINEERING (LOND.) JAN. 4, 1924)

According to the aerofoil theory, first suggested by Lanchester and developed in recent years by L. Prandtl and others at Gottingen, an aerofoil derives its lift from a circulation around the aerofoil which is superimposed on the motion of translation, resulting in an increase in velocity over the upper surface of the aerofoil, accompanied by a decrease in pressure, and a decrease in velocity over the lower surface accompanied by a corresponding increase in pressure.

Flow conditions close to the aerofoil are modified by the skin friction between the fluid and surfaces, and the two streams re-uniting at the trailing edge have, in general, different velocities, and are separated by an unstable sheet. The action of any small viscosity in the fluid will cause the sheet to develop into a thin region of vortex motion springing from the aerofoil in the immediate neighborhood of the trailing edge. The extent of the region depends on the form and attitude of the aerofoil. The drag, D_o^* , due to these causes is termed the "profile drag" and for aerofoils of good shape may approach the skin friction drag. It is the

*A list of symbols is given at the end of the report.

only drag occurring for the wing of infinite span around which the flow is two dimensional.

For the actual wing the flow is three dimensional. The lift of a wing of finite span varies from a maximum at mid span to zero at the tips, as a result of which a sheet of trailing vortices springs from the trailing edge of the wing, and being unstable, this sheet rolls up into two trailing vortices whose distance apart is slightly less than the wing span. Under their mutual reaction these two vortices will move slowly downwards and so will be inclined at a small angle to the wing. The effect of these trailing vortices is to give rise to a vertical velocity, w , at the aerofoil, termed the "induced velocity".

The induced velocity reduces the angle of incidence α by an amount $\frac{w}{V}$ so that the lift force L_0 for the effective angle of incidence α_0 is inclined backwards at an angle $\frac{w}{V}$ to the normal to the air stream direction, and possesses a drag component D_1 termed the "induced drag". The total drag of the aerofoil is the sum of the profile drag and the induced drag.

Assuming an elliptic distribution of lift across the span the induced velocity is constant across the span and equal to

$$w = \frac{L}{2\pi\rho V s^2}$$

The relations between the angles of incidence of the finite and infinite span wings are then

$$\alpha = \alpha_0 + \frac{w}{V} = \alpha_0 + \frac{K_L S}{2\pi s^2} = \alpha_0 + \frac{2K_L}{\pi A}$$

The induced drag of the finite wing is given by

$$D_1 = \frac{w}{V} L \text{ or } K_{D_1} = \frac{w}{V} K_L \text{ and } K_{D_1} = \frac{K_L^2 S}{2\pi s^2} = \frac{2K_L^2}{\pi A}$$

and the relationship between the drags of finite and infinite wings are then

$$K_D = K_{D_0} + K_{D_1}$$

Extension of the theory to the case of an aerofoil in a limited stream

The limited extent of the airstream in a wind channel places restrictions on the flow about the model and modifies the above results. The boundaries of the stream then give rise to induced velocities with corresponding deviations of the air stream.

Prandtl states that the boundary condition for a channel with a closed working section is zero normal velocity, and for the jet type channel with open working section, constant pressure. To secure these conditions for a channel of circular section Prandtl combines with the

aerofoil, another obtained by reflexion according to reciprocal radii. For a monoplane with assumed elliptic distribution of lift in the centre of the stream, it is shown that the induced velocity due to the stream boundary at a distance, x , from the middle of the stream is

$$w^1 = \frac{L}{4\pi R^2 \rho V} \left[1 + \frac{3}{4} \left(\frac{2s}{R^2} x \right)^2 + \frac{5}{8} \left(\frac{2s}{R^2} x \right)^4 + \frac{35}{128} \left(\frac{2s}{R^2} x \right)^6 + \dots \right]$$

At the centre of the stream where $x=0$ the deviation of the air stream due to this induced velocity will be

$$\frac{w^1}{V} = \Delta \alpha = \frac{1}{4} \frac{S}{C} K_L$$

The change in drag calculated from the above velocity is

$$D^1 = \frac{L^2}{4\pi R^2 \rho V^2} \left[1 + \frac{3}{16} \left(\frac{s}{R} \right)^4 + \frac{5}{64} \left(\frac{s}{R} \right)^8 + \dots \right]$$

The first term in this equation is found to be the same for a uniform distribution of lift. Further, since with $\frac{s}{R}$ as great as 0.6 the second term is less than 3% of the first, all terms except the first may be neglected. Prandtl has also found that the expression is the same for any small wing system in the centre of a circular stream. Hence, in general, for circular channels, the induced drag due to the channel wall is

$$D^1 = \frac{L^2}{4 C_\rho V^2} \text{ or } K_{D^1} = \frac{K_L^2 S}{4 \pi R^2} = \frac{1}{4} \frac{S}{C} K_L^2 = \frac{K_L^2 s^2}{\pi A R^2}$$

This is the extent to which the drag coefficient, as determined in a channel, will differ from that in an unlimited stream. For an open working section or jet K_{D1} is to be subtracted from the measured coefficient, and for a closed channel or tube is to be added to the measured coefficient to obtain the coefficient applicable to free air conditions.

The induced drag for the unlimited stream being

$$K_{D1} = \frac{K_L^2}{2\pi s^2} = \frac{2 K_L^2}{\pi A}$$

the total induced drag in a stream of radius R is

$$K_{D1} + K_{D^1} = \frac{2 K_L^2}{\pi A} \left(1 \pm \frac{1}{2} \frac{s^2}{R^2} \right)$$

where the plus sign applies to the open channel and the minus sign to the closed channel.

Wall interference in square and rectangular cross section channels

The theory of channel wall interference has been extended to the cases of square and rectangular channels by H. Glauert (R. & M. 723 and 867).

In order to satisfy the boundary conditions for a closed type channel there must be introduced a doubly infinite series of images correspond-

ing to the "mirror" images to be seen if the channel walls were optical mirrors. The images are the same as the aerofoil but alternate ones, commencing with the first, are inverted with respect to the aerofoil.

The case is first considered of an air stream having boundaries perpendicular to the span, but none above or below. There is only one series of images, all of which are identical with the aerofoil. The interference for this case, assuming elliptic distribution of lift and a line vortex system, is shown to be

$$\Delta \alpha = \frac{\pi S}{12 b^2} K_L = 0.252 \frac{S}{b^2} K_L$$

$$\Delta K_D = K_L \Delta \alpha = 0.252 \frac{S}{b^2} K_L^2$$

where b is the breadth of the channel.

In obtaining this expression, as before, all terms other than the first are regarded as of negligible importance for values $\frac{2s}{b}$ as great as 0.6. The equation applies strictly at the centre of the channel, but the increase in interference from the centre to tips is also negligible. The same expression applies for uniform distribution of lift.

For boundaries above and below the wing only, that is a channel of height, h , but of infinite breadth, the images are of alternate sign, and the resulting expressions for the interference are:

$$\Delta \alpha = \frac{\pi S}{24 h^2} K_L = 0.131 \frac{S}{h^2} K_L$$

$$\Delta K_D = K_L \Delta \alpha = 0.131 \frac{S}{h^2} K_L^2$$

A comparison of the above expressions indicates that the interference is due mainly to the side walls of a rectangular channel, and not to the boundaries above and below the wing.

For a rectangular channel of breadth, b , and height, h , there is a doubly infinite series of images, and the interference is given by

$$\Delta \alpha = \delta \frac{S}{C} K_L \text{ in radians}$$

$$\Delta \alpha = 57.3 \delta \frac{S}{C} K_L \text{ in degrees}$$

where C —cross sectional area hb of the rectangular channel

$$\delta = 2\pi\lambda \left[\frac{1}{2!} + \sum_{i=1}^{\infty} \frac{p}{1 + e^{2i p \lambda \pi}} \right]$$

$$\lambda = \frac{h}{b}$$

Numerical values of the coefficient δ for different values of the ratio $\lambda = \frac{h}{b}$ are plotted in Plate 3.

This curve appears to indicate that the interference of the walls on an aerofoil is a minimum for a rectangular channel having a breadth $\sqrt{2}$ times the height. There are also the curious results that δ is unaltered when λ is replaced by $\frac{1}{2\lambda}$ so that the interference is the same for a channel of breadth b and height h as for one of breadth $\sqrt{2}h$ and height $\frac{b}{\sqrt{2}}$.

The equation is applicable for aerofoils such that the span does not exceed 0.6 times the channel dimension parallel to the span. If this figure is exceeded the interference will be greater than that given by the equation.

The formula can also be used for biplanes, since to a first approximation both wings can be regarded as at the centre of the channel, and the strength of the images is therefore doubled, which is taken account of automatically in the formula in the symbol S .

It should be noted that the interference according to the above equations depends on the area of the wing, and not on the chord or span independently. The percentage interference will, however, be less the smaller the aspect ratio, since the induced drag increases with decrease in aspect ratio.

PART II

Method of test

Experimental confirmation of the accuracy of the wall interference corrections according to the Prandtl theory is difficult (Glauert—Int. Air Congress, 1923, p. 254) and has usually been made by testing the the same aerofoil in channels of different sizes (R. & M. 889 and 898) or testing geometrically similar aerofoils of different sizes in the same channel.

For a closed working section channel the boundary condition of zero normal velocity is secured analytically by the introduction of a system of images of the test aerofoil, obtained by regarding the channel walls as optical mirrors. The test aerofoil is thus the centre of a system of mirror images of itself, as shown in Figs. 1 and 2, Plate 1. In the method of test employed in this research, which was suggested by Colonel E. W. Stedman, R.C.A.F., models are introduced in the channel to act as the first images adjacent to the test model, as indicated in Plate 2. The model under test is then in effect in a wind channel of dimension equal to one-half the distance between two corresponding image or mirror

models (Figs. 5 and 7, Plate 2). The method has been termed the "mirror" method and is analagous in principle to the reflection method previously used in determining the ground influence on aerofoils (N.A.C.A. Tech. Note No. 67, 1921).

The mirror method possesses several points of superiority compared with methods so far used for studying channel wall interference:

1. The measurements are made throughout on one and the same model. This effectually eliminates any effects due to slight differences in geometrical similarity of the models, roughness or nature of surfaces, etc., that may occur if models of different sizes are used.
2. The nature of the general air flow in the channel is the same for all tests, hence variations in results due to differences in turbulence, velocity distribution, etc., which may be present when the same model is tested in different channels are obviated.
3. The method in effect provides an elastic channel whose cross sectional dimensions can be varied at will, within wide limits.

On the other hand:

1. A number of exactly similar models are required.
2. Only the first "mirror" images are used. These in turn have images outside the channel walls, and the influence of the actual channel walls on the air flow about the "mirror" models may affect the results.

Wind channel employed

The tests were carried out in the new R.A.E. type 4 ft. wind channel of the Aerodynamic Laboratory of the Department of Mechanical Engineering, University of Toronto.* The aerodynamic forces were measured on an N.P.L. type balance employing the usual vertical steel spindle method of support of the models. Air speeds were controlled manually and indicated by side plate and modified Chattock micromanometer.

Models

Preliminary tests were made on cylinders for the following reasons:

1. Simple geometrical shape and low cost.
2. Permit the use of mirror models at the ends of the test model, since the mirror model may surround the balance standard

*For a description of the channel and apparatus see Aero. Research Paper No. 16.

which is not possible with aerofoils of usual form. This permits the study of the effect of variation of channel width as well as height.

3. Absence of cross wind force simplifies measurements somewhat and also provides data on wall interference for symmetrical models on which only drag is exerted.

The cylinders employed were of selected drawn No. 18 gauge brass tubing, 18" long and of two outside diameters, 1" and 2". Brass plugs were fitted in the ends of the cylinders to provide plain ends normal to the axis and also to take the 5/16" diameter steel spindles.

The aerofoil section employed was that known as the Airscrew 4 (R. & M. 322, Bairstow—Applied Aerodynamics p. 304, Aero Research Paper No. 8. Toronto) and was selected for the following reasons:

1. It is a comparatively simple section with flat under-surface and good efficiency.
2. It is a moderately thick, $t/c=0.127$, high lift section. The higher the lift the greater the wall interference effects, hence the advantage in using a high lift section.
3. Deflection difficulties are lessened with a rigid thick section.

The aerofoil models were made by W. H. Nichols, of Waltham, Mass., on a special wing generating machine, (N.A.C.A. Tech. Note No. 172) which produces aerofoils guaranteed accurate within 0.002 inch on any ordinate. The models were of duralumin.

Two sets of aerofoil models were tested, one of 2" chord and 12" span, the other of 3" chord and 18" span, in order to check the effect of ratio of model to channel size and of vl effect, and also to insure obtaining general conclusions.

The larger models were supported on very slightly tapered 5/16" dia. steel end spindles, the smaller ones on spindles of somewhat greater taper. All spindles fitted in accurately reamed and threaded holes in the aerofoils.

The mirror models were supported on similar end spindles held in by set screws in the dummy balance guards (or end mirror cylinders). The later in turn were held in sockets fitted to sliders. The sliders were arranged with tapered clamping strips enabling them to be clamped in any position on steel rails. The steel rails were sunk in the floor and roof of the channel, extending perpendicularly to the air stream from side to side of the channel directly in line with the balance axis. This enabled the mirror models to be accurately located with reference to the test model. The general arrangement of the apparatus will be evident from the photographs.

PART III.

Tests on cylinders

There were two series of tests made on the cylindrical models.

SERIES I. Resistance measurements

Measurements of drag were made on cylinders under the following conditions:

1. For channels of various heights, from a maximum equal to 4 feet to the minimum possible with the apparatus 0.15 to 0.166 feet, the channel width being constant at 4 feet. In these tests two mirror cylinders only were used, their arrangement being as shown in Figs. 5 and 6, Plate 2. Figs. 1 and 2 show the special arrangement employed for a channel height equal to 4 feet. The diagrams indicate the method used in determining spindle and guard corrections. These tests were made on both 1" and 2" diameter cylinders.
2. For channels of various heights and various widths. The arrangement of the cylinders is shown in Figs. 8, 9 and 10 and photograph 1, eight mirror cylinders being used. The mirror cylinder surrounding the guard tube of the balance was provided with an end plug having a hole somewhat larger than the spindle to provide clearance. The presence of the balance standard and guard tube within this cylinder limited the maximum channel width that could be tested in this way, as it was thought to be undesirable to permit these to project beyond the end of the mirror cylinder. The range of channel widths tested was thus from 1.5 ft. (equal to cylinder length) to 1.812 ft. using the set up shown in Figs. 8 and 9, with the set up of Figs. 5 and 6 a special or limiting case for a width of 4 ft. The range of heights tested was as above. These tests were made on the 2" diameter cylinders only.

The spacing of the mirror cylinders was made as accurately as possible using micrometers. The final check on the accuracy of the setting was the absence of any cross wind force as shown by the balance. This check was very sensitive, a very small error in spacing causing the test cylinder to move to one side or the other, to the limit of its travel.

The drag of the test cylinder was measured for each set up at nominal air speeds of 20, 30, 40 and 50 feet per second. The projected or frontal area of the 2" cylinders, test and mirror in the extreme case (see photo. 1) totals 2 sq. ft. out of a total channel cross section of 16 sq. ft. The nominal air

speed upstream from the cylinders is hence increased at the cylinders in proportion of 16:14. The actual air speed past the test cylinder is, however, probably quite different to either of these speeds. Hence, it was considered advisable to use the nominal channel air speed in calculating the coefficients, regarding any increase in speed due to presence of model as part of the channel effect.

SERIES II. *Pressure distribution measurements.*

In seeking an explanation of the peculiar variation in drag discovered in the Series 1 tests, an investigation of the distribution of pressure around the 2" x 18" cylinder was made for typical channel heights and widths.

The test cylinder was fitted with air tight end plugs, the lower one carrying the spindle and also a small diameter pressure connection, quite close to the spindle, from which a fine rubber tube passed to a micromanometer. The pressure on the surface of the cylinder was communicated to the interior by means of a 0.02 inch diameter hole carefully drilled radially through the cylinder wall. Two such holes were drilled, one midway between the two ends of the cylinder, and the other 1.26 inches from the upper end. The hole not in use was plugged with plasticene.

The pressure measured was the difference between that at a given point on the surface of the cylinder and that at the side plate in the channel wall. It was found that the difference between the pressure at the side plate and the static pressure in the centre of the channel in the position occupied by the cylinder was so small that for the purposes of this test the pressure at the side plate could be regarded as the static pressure at the cylinder. The pressure difference measured was thus that between the actual pressure and the normal static pressure at the same point on the cylinder surface.

The pressure difference was measured on a Krell slanting manometer previously calibrated against a Chattock micromanometer.

The cylinder was supported on an end spindle on the N.P.L. balance in the usual way, and the balance protractor used to enable the cylinder to be rotated through the proper angle to bring the pressure opening into any desired position. The pressures were thus measured at angular intervals of 10° for the full 360° .

The air speed was 40 f.p.s. throughout.

Results

SERIES I. The results of the drag measurements on cylinders have been presented in the non-dimensional coefficient

$$K_D = \frac{D}{\rho S V^2}$$

where K_D is the drag coefficient

D is the measured drag in pounds corrected for spindle interference.

ρ is the kinematic viscosity of air.

S is the projected area of the cylinder, *i.e.*, diameter $\times$ length in sq. ft.

V is the air speed in ft. per. sec.

The coefficients are tabulated in Tables 1 and 2 and plotted in Plates 4—9.

SERIES II. The pressures observed in the pressure distribution tests on the cylinders have also been expressed non-dimensionally in the terms of normal impact pressure, the observed pressures being divided by $\frac{1}{2}\rho V^2$. Due to the very large number of the pressure observations and also to the fact that the individual readings are of relatively minor importance, these results have not been tabulated, but are presented graphically in Plates 10—14.

Discussion of results

SERIES I. The cylinder drag coefficients are plotted on Plates 4—9, logarithmic scales being used because of the wide range covered and the closeness of the readings at lower end of range. Plate 4 shows the variation of drag coefficient with channel height for different speeds for the 1" $\times$ 18" cylinder. On Plates 5—8 are plotted drag coefficients for the 2" $\times$ 18" cylinders, each plate carrying the results for one speed. In the lower part of each of these plates are drawn curves showing the variation of the drag coefficient with channel height for a number of channel widths. These curves are drawn from the plotted observations. In the upper part of each plate are plotted the drag coefficients as observed, or where there was no observation, taken from the curves in the lower part of the plate, on a channel width base. The resulting curves show the variation of the coefficient with channel width for different constant channel heights.

An examination of the curves of variation of drag coefficient K_D discloses a number of interesting features:

1. The drag increases with decrease in channel height and channel width. The increase is relatively less for the 1" than for the 2" cylinders. The increase in drag for a channel of height

0.2 feet as compared with that for a channel of height 4 feet, is about 25% for the 1" cylinder, while for the 2" cylinder for channel widths as small as 1.666 ft., the corresponding increase is about 100%.

Plotting K_D on ratio of channel height to cylinder diameter, as in Plate 9, indicates, however, that the increases in both cases are of the same general nature. The reason that the curves for the 1" and 2" cylinders do not exactly coincide in this plotting is probably due to the difference in the ratio of length to diameter of the two cylinders and resulting end effects.

2. There is a region of instability, or critical point, encountered as the channel height is decreased where the drag abruptly decreases, and beyond which conditions are unstable for a greater or less range, afterwards becoming stable again and the drag increasing. In the case of the 1" cylinders the drag drops at the critical point, a channel height of 0.2 ft., to less than that for the 4 ft. channel. The readings in the unstable region are generally steadier than for the larger cylinders, and with further reduction the drag tends to increase. It was impossible to make tests with the 1" cylinders at channel heights as small relatively as for the 2" cylinders, owing to the size of the balance guard tube, but the curve through the critical region shows the same upward tendency as for the larger cylinder, which is borne out by the plotting on Plate 9. For the 2" cylinders the curves of K_D variation beyond the unstable region appear to be of the same curvature as that before the break, but displaced downward. The displacement appears to become less as the channel width decreases, and for the minimum width the curves before and after the unstable region appear to be part of the same curve. It is therefore surmised that if the channel width could be reduced sufficiently to prevent air flow past the cylinder ends that the instability would disappear completely.

For channel widths from 4.0 to 1.666 ft. (2" cylinders) the unstable region is very narrow, a reduction of a small fraction of an inch being sufficient to cause a reduction of about 20% in drag, and the instability is confined to this narrow region. For channel widths less than 1.666 ft. the instability occurs for larger and larger channel heights, until for the minimum channel width the unstable region extends from a height of 0.38 to about 0.2 ft. As the extent of the region

increases the unsteadiness and reduction in drag within it also increases, until with the minimum channel width the drag in this region is much less than for the 4 ft. channel (compare 1" cylinder). The unstable region is narrower and more clearly defined the greater the channel width.

The increase in the unstable region corresponds to the reduction in the critical angle for aerofoils with reduction in channel size. (See later).

Within the unstable region readings are exceedingly difficult to secure, and several different drags may be indicated for the same conditions, as shown by the points plotted in this region on the graphs. Beyond the unstable region, *i.e.* for the region between a channel height of 0.2 and one of 0.166 (for 2" cylinders) conditions are fairly stable, and most of the curves can be plotted here with reasonable definiteness. The maximum drag occurs for the minimum channel width.

3. The variation of drag with channel width, as shown by the curves for the 2" cylinder, is of much the same character as that with channel height, but rather less proportionately until a width of about 2 ft. is reached. Between 2 and 1.5 ft. the increase in drag is very rapid, averaging about 40%.
4. The curves enable the increase in measured drag of a cylinder in a channel of small dimensions to be estimated. Thus, a 2" $\times$ 18" cylinder in a 2 $\times$ 2 channel would have about 10% greater drag than in a 4 $\times$ 4 channel, and about 15% greater drag in a 1 $\times$ 2 channel.
5. The curves plotted in the upper part of Plate 9 show the general nature of the scale effect on cylinder drag. The drag is seen to increase with speed or VD . This is in agreement with the general curve of scale effect for cylinders, given in R. & M. 102, 1913-14.

There is some slight indication that the scale effect is reduced the smaller the channel height and width, since the curves for a channel width of 1.5 ft. on Plate 5—8 practically coincide, as do those for a channel height of 0.40 ft. But the differences are so small as to be within the limits of experimental error.

The critical point is influenced by speed to some extent. Thus, an examination of Plates 4—8 shows that the critical point occurs at a smaller channel height the higher the speed. The unstable region is reduced by increase in speed. This is in conformity with the observed scale effect on the critical angles for aerofoils, which are increased with speed.

In R. & M. 106, 1913-14, the measured drag per foot run on a 2" cylinder, with cylindrical end guards, at 30 f.p.s. is 0.191 pds. equivalent to 0.287 for the 18" length.

This condition corresponds to that in the present research for a channel 4 ft. high and 1.5 ft. wide for which the observed drag was 0.308 pds.

In R. & M. 102, a curve is given for the resistance of smooth wires. For a value of $\log \frac{vl}{\nu}$ equal to 4.5 (corresponding to the 2" cylinder at 30 f.p.s.) the value of $\frac{F}{\rho V^2 L^2}$ shown is about 0.59, from which the resistance per diameter length is $F = 0.035$ pds. and for the 18" length the drag is 0.315 pds.

The difference in these results is probably due to the different flow texture or turbulence of the air stream in the different channels. (See N.A.C.A. Report 231 and Tech. Note No. 191).

The critical point in the curves of K_D on channel height for both 1" and 2" cylinders for the larger channel widths, occur at a height of about 0.2 ft. The corresponding absolute dimension of the air gap between test and mirror cylinders is 0.4 inches for the 2" and 1.4 inches for the 1" cylinder. Again plotting K_D on ratio of channel height to cylinder diameter (Plate 9) the critical point is seen to occur at quite different values of the ratio. There is not sufficient information to determine what factors control the critical region. These tests indicate that:

1. It is lessened *i.e.* occurs for smaller channel heights the higher the speed.
2. It is increased the narrower the channel (Plates 5—8). It is also considered probable that the critical region depends on:
3. Absolute dimension of the gap between models.
4. Shape of the gap, ratio of length to width.
5. The curvature of the boundary surfaces of the gap.
6. End effects, *i.e.* ratio of length to diameter of boundary models.

SERIES II.

Pressure distribution curves have been plotted in Plates 10-14. In the case of Plates 10—13 each plate shows the pressure distribution for one channel width. The figures plotted are the averages of the observed pressures at corresponding points on each side of the cylinder. The averages were used owing to the difficulty of initially setting the pressure opening exactly upstream. The average figures are plotted for the semi cylinder only.

The central group of plottings on each plate is that for the central hole, *i.e.* the pressure distributions in the median plane of the cylinder, 9" from either end.

The left group of plottings on each plate show the pressures as taken at the end hole, that is the distribution of pressure around the cylinder 1.26" from its end, with the end spindle removed.

The right set of curves on each plate shows the corresponding pressures, but with the 5/16" dia. end spindle in place. A comparison of these two latter groups of curves shows the effect of the end spindle.

There are, in general, four curves in each of the above groups, each showing the distribution of pressure for a certain channel height. One curve shows the distribution for the full channel height of 4 ft., or that is without mirror cylinders. The second curve shows the pressures for a channel height of about 0.333 ft. The third curve shows the distribution around the cylinder for a channel height as close as practicable to the critical point or break in the curves of Plates 4—8. In the fourth curve the pressure distribution is shown for the minimum channel height of 0.166 ft., equal to the cylinder diameter, where test and mirror cylinders are side by side and practically touching.

An examination of the curves showing the distribution of pressure around the median plane of the cylinder affords an explanation of the variation of drag with channel height. The full curve (central group Plate 14) for channel height equal to 4 ft. is similar to that of Fig. 2, R. & M. 106, 1913-14, although the higher air speed of the present tests causes the distribution to be somewhat different.

As the channel height is decreased the curves show that the air banks up more and more on the front of the cylinders, the region of positive pressure becoming larger and larger, while at the same time the air jets between the test and mirror cylinders with higher and higher velocity the pressure on the cylinder surface being correspondingly reduced. The point of minimum pressure, corresponding to the maximum jet velocity, moves farther and farther back in conjunction with the increase in frontal area of positive pressure. The higher the jet velocity, the lower the pressure in the dead or eddy region behind the cylinder. These effects all conduce to increase the cylinder drag as the channel height becomes less.

Beyond the critical point the air flow about the cylinders changes. There is a sudden decrease in the jet velocity between the cylinders, with the result that the suction on the sides and back of the cylinder is reduced. This is particularly noticeable for the central hole plottings on Plates 12, 13 and 14; the corresponding plottings on Plates 10 and 11 are in the transition region. The positive pressure area on the front of the cylinder

continues to increase, but at a slower rate than the reduction of suction on the back, with the net result that the drag of the cylinder is considerably reduced. The three cylinders (two mirror and one test) are here acting partially as one body, in that the air is banking up in front on all three, but there is some leak through between the cylinders, which serves to partially break the suction at the rear.

With further decrease in the channel height, down to the minimum where the cylinders are side by side, the air flow between the cylinders is more and more restricted until it finally practically ceases, the positive pressure exists over nearly the whole 180° of cylinder face, and at the same time the suction at the rear becomes larger, since it is now due to the dead region in the rear of three cylinders, practically unrelieved by any leakage between the cylinders. The three cylinders now function as a single body, the air piling up in front and leaving a single large dead region of low pressure in the rear. As a consequence, the drag of the test cylinder is here a maximum.

Comparing these curves of pressure distribution for the median plane, for different channel widths, it is seen that the effect of decreasing the channel width is to decrease somewhat the region of positive pressure on the face of the cylinder and to increase the negative pressure on the back, the latter effect predominating so that there is a slight net increase in drag, as shown by the drag measurements.

The pressure distribution near the end of the cylinder is of the same general character as that at the centre, but the positive pressure region is smaller, while the negative pressure region and pressures are increased. This is doubtless due to the combination of normal air flow perpendicular to the cylinder axis with flow parallel to the axis set up by the passage of the air around the cylinder end. In other words, the air jets from the end as well as the two sides, and thus sets up a greater suction in the neighborhood of the ends at the rear.

While for the central hole the suction at the back of the cylinder is always least for the channel height of 4 ft., this is not the case for the end hole. The flow around the end of the cylinder is apparently much influenced by the proximity of the mirror cylinders (channel height). In most cases the minimum suction on the back of the cylinder at the ends, occurs for a channel height of less than 4 ft. due, it would seem, to end leakage. Nor does the maximum suction on the back occur for the minimum channel height of 0.166 ft.

The presence of the 5/16" diameter end spindle is seen to have very little influence on the distribution of pressure around the cylinder near the ends.

PART IV.

Tests on aerofoils

In making the aerofoil studies it was not possible to arrange a guard of the aerofoil section around the standard of the balance and spindle, representing the end image *i.e.* the image in the floor of the channel). Hence, only the effect of variation in channel height was studied, the channel width remaining constant at 4 ft.

Three aerofoils were therefore used, that under test and two mirror aerofoils (See Figs. 5—7, Plate 2 and Photos 2—4). The $1\frac{1}{4}$ " guard tube around balance and spindle was duplicated, as shown, for the mirror aerofoils, and in determining spindle resistances dummy spindles and guards were arranged, supported from the rail in the roof of the channel.

Tests were made, with the $3" \times 18"$ aerofoils, for ten channel heights, 4.00, 1.75, 1.50, 1.0, 0.5, 0.33, 0.25, 0.166, 0.125 and 0.104, with constant breadth of 4.0 ft. For the $2" \times 12"$ aerofoils tests were not made for the heights of 0.33 and 0.125 ft. The minimum height of 0.104 ft. is the smallest possible spacing, the $1\frac{1}{4}"$ diameter guard tubes being then in contact.

For each channel height the lift and drag* were measured at a series of incidences, -8, -4, 0, +4, 8, 12 and 16° . In certain cases measurements were made at intermediate angles, where necessary to locate critical angles definitely.

Further, to study speed effects, each test was made at four air speeds, 20, 30, 40 and 50 feet per second.

Results

The observations, corrected for spindle interference, are tabulated as the usual non-dimensional lift and drag coefficient in Tables 3—6, and have been plotted on curve sheets as follows:

*The aerofoil here used is of the same section as that used previously for a research on thick aerofoils, (See Aero. Research Paper No. 8, Sept. 1921). The models in this earlier research were of wood and were tested in the old 4 ft. N.P.L. type wind channel at Toronto. The models in the present research are of metal, and tested in the new R.A.E. type channel, some four years later by entirely different observers. Yet the agreement between the coefficients given in the earlier report and those here given for similar conditions, 40 f.p.s. and 4×4 channel, will be found, on comparison, to be surprisingly good. This affords some indication of the accuracy of the models used and the reliability of the observations.

- Plates 15 to 22. Lift and drag coefficients on incidence as base, one plate for each speed and model size. Because of the fact that the points were, in many cases, very close together, the actual observation points have not been plotted on these plates.
- Plates 23 to 26. Lift coefficient on drag coefficient (for the four larger channel sizes only) for different speeds and channel sizes.
- Plates 27 to 28. Lift and drag coefficients on channel height as base for different incidences.
- Plates 29 to 30. Lift and drag coefficient on v_l base for different channel sizes.

The figures for the low speeds, 20 f.p.s., and particularly for the smaller models, are erratic due to the small absolute magnitude of the forces, and in some cases have not been plotted. At all speeds and for both series of models the figures in the neighborhood of the critical angle and at negative angles are also uncertain, due to the unstable nature of the flow.

Discussion of results

Plates 15 to 22. An examination of these curves of lift and drag coefficient on incidence as base discloses a number of points of interest:

- (a) As indicated by theory the incidence corresponding to a certain lift decreases as the channel height is reduced. The presence of the channel walls results in an induced velocity which is equivalent to an "upwash". The effect is seen to be small until a channel height of about 1 ft. is reached.
- (b) There is a marked effect on the critical angle. Both angle and maximum lift are lowered with reduction in channel height until a height between 1.0 and 0.5 ft. is reached, beyond which, with further reduction in channel size, the reduction in angle is slight and the maximum lift exhibits a rapid increase until instability of flow is set up.

These results confirm the statement of McKinnon Wood (Aero Journal, Dec. 1922) that wall interference reduces the stalling angle.

It will be observed that for the very small channel heights the curves break off abruptly, sometimes at very low incidences. This was occasioned by the great instability of flow occurring beyond these angles preventing the securing of further readings. The instability was so great as to cause the test aerofoil to vibrate to such an extent that, in certain cases, it came

in contact with the adjacent mirror aerofoils, necessitating discontinuing the test. This instability varies with the speed occurring at a lower angle the greater the speed. Thus, for the 3×18 aerofoil and smallest channel instability occurs at -4° at 50 f.p.s., and $+10^\circ$ at 20 f.p.s., and for the 2×12 models at 0° and $+8^\circ$ respectively for the same speeds. A similar effect is referred to in N.A.C.A. Tech. Note No. 67. p. 3.

As the channel height is decreased the induced velocity due to the channel walls (virtual upwash) increases, but at a slower rate than that of theory, until a peak or critical size is reached, beyond which there is an abrupt decrease. A break-down or change in the type of flow apparently occurs at this point, as with the cylinders, as the space between the models becomes very restricted, with the result that the lift is greatly decreased and the drag much increased. It would appear to be analagous to the burbling action, in fact it really means that the burbling point is brought so low that the flow over the whole test range, in certain cases, is of the type occurring beyond the critical angle.

With the 3×18 model at 20, 30 and 40 f.p.s. and for the minimum channel height, it was found that the flow was wholly of the kind existing beyond the critical, and it will be noted that the curves corresponding to these conditions differ materially from the others. The lifts are very low and the drags correspondingly high.

In certain cases, at high speeds, for small channel height and at negative incidences, the balance became very sluggish, giving the impression of some very viscous fluid surrounding the aerofoils.

- (c) The influence on the form of the lift curve at negative lifts is peculiar. With reduction in channel size the lift curve here develops a decided sag. This, it will be seen later, is partly a speed effect.
- (d) According to theory the angle correction is proportional to the lift, and hence the lift curves should cross at zero lift. In no case does this occur for the experimental results. The curves, it will be observed, cross at a lift coefficient of about 0.05.

Plates 23 to 26. These plottings were made to indicate the wall effects on the drag. It will be observed that the general effect is as predicted by theory, namely, the presence of the channel walls results in the drag

being measured too low. The effect is seen to be small for the channel dimensions plotted.

Plates 27 to 28. The lift and drag coefficients at several incidences and for the different speeds, have been here plotted on the channel dimension as base, using logarithmic scales.

This plotting shows clearly the general effect of the channel walls on the aerofoil characteristics.

With reduction in channel size the lift increases slowly at first, then more rapidly, reaching a peak for a ratio of chord to channel height between $\frac{10}{7}$ and $\frac{10}{9}$ after which there is an abrupt and marked reduction in lift to values, in general lower for the minimum size of channel than for the full four foot channel.

The drag, on the other hand, decreases with reduction in channel height, slowly at first, then more rapidly until a minimum is reached at a chord: height ratio of about $\frac{2}{3}$ beyond which there is a large and rapid increase in drag to values much in excess of those for the four foot channel.

This effect, it will be observed, is quite the opposite of that observed for the drag of cylinders, the latter increasing with channel height reduction.

The above statements apply to the higher vl 's only, as at lower vl 's and near the critical angle, the critical points in these curves occur at larger channel heights. This corresponds to the observed vl effect on the critical point in the cylinder drag curves.

If the two Plates, 27 and 28, are superimposed so that the 1.5 ft. channel height for the 3×18 aerofoil (chord: height = $1/6$) coincides with the 1.0 ft. channel height for the 2×12 aerofoil (chord: height = $1/6$) the agreement between the two sets of curves will be made very clear. The curves for the 2×12 model will then appear as the continuation of those for the 3×18 and the critical points will also appear in good agreement.

These curves show clearly that the reduction in interference to be secured by increasing the channel height beyond 4 ft. for the 3×18 aerofoil can be but little or, on the other hand, that a channel of smaller height (say 2 ft.) may be used without seriously increasing the interference effects. Or again, larger models, say 6×36 , can be successfully used in the 4×4 channels. This practice has already been followed at Gottingen (refer McKinnon Wood, Int. Air Congress, London, 1923, p. 341).

Plates 29 to 30. These plates contain plottings of the observed coefficients at various angles, on a vl base, for different channel heights.

The curves thus show the combined effect of channel wall and scale or velocity.

Plates 31 to 32. The observed coefficients for the 4×4 , $4 \times 1\frac{3}{4}$, $4 \times 1\frac{1}{2}$, 4×1 and $4 \times \frac{1}{2}$ channels were corrected to free air conditions, using the Prandtl corrections, and are plotted on these plates. The curves drawn are average curves for each group of points for any one speed. It will be noted that the theoretical corrections secure satisfactory agreement of the different observations for channel heights as low as 1.0 ft. for the 2×12 and 1.5 ft. for the 3×18 models, that is a ratio of $c/h = 1/6$. Points for channel heights lower than these values were neglected in drawing the curves. The theoretical corrections are seen to be non-applicable for the 4×0.5 channel. Beyond the above limits, theory yields a definite "over" correction since the sequence of plotted points for lift progresses to the right and on drag upward and to the right.

These curves show also the speed or so called vl effect since spindle and wall interferences having been corrected, all other conditions except speed are identical throughout. Attention is directed to the effect on the lift curve at negative incidences, resulting in the characteristic sagging, and to the reduction in critical angle with reduction in speed.

Plate 33. The curves on this plate have been plotted using figures taken from Plates 31 and 32, and hence show the scale effect on lift and drag coefficient. The curves are of the usual form. The agreement between the results for the 2×12 and 3×18 aerofoils is good. The fact that the coefficients for the 2×12 models are everywhere higher than those for the 3×18 , while possibly due to experimental errors, may indicate the presence of another factor, as for instance, roughness of model surface. While both models were finished with the same polish, that on the 3×18 would be relatively finer than that on the 2×12 .

Plates 34—35. These plates show graphically the agreement between theoretical and experimental channel wall interference corrections. The theoretical corrections to bring the results of tests on the 4×4 , $4 \times 1\frac{3}{4}$, $4 \times 1\frac{1}{2}$, and 4×1 channels to free air conditions were first calculated and by subtraction the corrections necessary to bring the results of the smaller channels to those of the 4×4 channel were found. The theoretical quantities used in the computations are given in Table 7.

From Plates 15 to 22 the angle corrections necessary to bring the results to those for the 4×4 channel were measured, and similarly from Plates 23 to 26 the corresponding drag corrections were scaled.

The theoretical and experimental angle corrections are plotted on Plate 34, the full curve representing the theoretical corrections and the plotted points the experimental corrections. It will be seen that the

agreement between the theoretical and experimental corrections is good for the 3×18 aerofoil for channels as small as $4 \times 1\frac{1}{2}$ and for the 2×12 aerofoils practically for channels as small as 4×1 .

On the other hand, the drag corrections, similarly plotted on Plate 35 are not in as good agreement. The lack of agreement here is probably due to the very small absolute magnitude of the quantities involved together with the method of determining them, namely by subtraction. The agreement, however, is fairly good for lift coefficients between zero and 0.3. Glauert states (See R. & M. 723, p. 20) that for lift coefficients below 0.3- "the difference (between results in 4 and 7 ft. channels) might be too small to determine reliably, and at higher lift coefficients the drag tends to become unsteady, but experimental confirmation of the theory might be obtained over the central region"

General conclusions

1. The influence of the wind channel walls on the wind forces exerted on a symmetrical body, such as a cylinder or strut, (and presumably on a stream line form such as a dirigible envelope) is to increase the drag. The drag will be read higher than the free air drag. The increase is slight provided the model is small relatively to the channel, but becomes appreciable for large models in relatively small channels.
2. The use of excessively large models in small channels is accompanied by instability of air flow in the channel, rendering measurements difficult and results unreliable.
3. The flow of air through a restricted space may be one of two kinds, depending apparently on shape and size of the space, curvature of walls and air speed. As the space is reduced the air originally flowing through at high speed is suddenly retarded.
4. The channel wall effects on the aerofoil characteristics are found to be as predicted by theory, for the closed channel the incidence and drag for a given lift are both measured too low.
5. The experimental corrections for wall interference, on angle and drag, are found to be in reasonable agreement with those predicted by theory for channels of fair size relatively to the model.
6. The channel walls reduce the critical or stalling angle of aerofoils.

7. The results demonstrate the possibility of using relatively much larger aerofoil models in the existing wind channels than is done at present without introducing excessive wall corrections. This will in turn permit greater accuracy in the measurement of the greater forces exerted on the larger models. The investigation shows the extent to which this practice may be carried.
8. The behaviour of an aerofoil in an excessively restricted stream is shown, and the magnitude of the boundary corrections determined for channel sizes beyond the limits of applicability of theory.

LIST OF SYMBOLS

L — measured lift. K_L lift coefficient = $\frac{L}{\rho SV^2}$

D — measured total drag. K_D drag coefficient = $\frac{D}{\rho SV^2}$

D_0 — profile drag. K_{D_0} profile drag coefficient.

D_1 — induced drag due to finite span. K_{D_1} induced drag coefficient.

D^1 — induced drag due to channel walls. K_{D^1} coefficient.

c — wing chord.

s — semi-span of wing.

S — wing area.

A — aspect ratio of wing = $\frac{2s}{c} = \frac{4s^2}{S}$

R — radius of air stream or channel.

C — cross sectional area of air stream or channel.

h — channel dimension perpendicular to wing span.

b — channel dimension parallel to wing span.

V — air speed.

w — induced velocity—unlimited stream.

w_1 — induced velocity due to stream boundaries.

α — measured angle of incidence—radians.

α_0 — effective angle of incidence = $\alpha - \frac{w}{V}$

Aeronautical Research Paper No. 17.
UNIVERSITY OF TORONTO
CHANNEL WALL INTERFERENCE
CYLINDER DRAG COEFFICIENTS

TABLE No. 1.

Aug. 24, 1926 *gaf*

Cylinder: 1"x18"
Channel Width: 4.0 ft.

Cylinder: 2"x18"
Channel Width: 4.0 ft.

Channel Height-ft.	Air Speed				Channel Height-ft.	Air Speed			
	20	30	40	50		20	30	40	50
4.000	0.464	0.473	0.477	0.483	4.000	0.424	0.422	0.430	0.432
1.583	0.461	0.471	0.484	0.487	1.500	0.441	0.450	0.453	0.455
1.056	0.463	0.471	0.484	0.494	1.166	0.445	0.447	0.453	0.456
0.637	0.476	0.488	0.491	0.504	0.916	0.458	0.469	0.458	0.470
0.488	0.480	0.482	0.494	0.497	0.750	0.469	0.479	0.477	0.484
0.352	0.500	0.524	0.518	0.534	0.666	0.480	0.484	0.488	0.500
0.284	0.521	0.527	0.548	0.549	0.607	0.491	0.496	0.510	0.518
0.249	0.539	0.564	0.570	0.578	0.502	0.507	0.520	0.544	0.540
0.227	0.544	0.556	0.566	0.576	0.412	0.550	0.539	0.563	0.553
0.208	0.573	0.586	0.575	0.581	0.384	0.547	0.542	0.562	0.551
0.205	0.592	0.589	0.580	0.597	0.293	0.594	0.590	0.626	0.623
0.204	0.566	0.574	0.577	0.592	0.268	0.642	0.653	0.660	0.667
0.195	0.546	0.552	0.591	0.598	0.246	0.674	0.681	0.692	0.701
0.192	0.450	0.500	0.527	0.580	0.235	0.706	0.682	0.712	0.709
0.179	0.447	0.450	0.453	0.460	0.231	0.717	0.693	0.717	0.718
0.174	0.430	0.413	0.419	0.443	0.219	0.744	0.738	0.739	0.739
0.161	0.444	0.426	0.429	0.442	0.211	0.736	0.731	0.749	0.749
0.157	0.452	0.438	0.430	0.441	0.204	0.782	0.797	0.823	—
0.150	0.454	0.440	0.442	0.446	0.196	0.648	0.640	0.657	0.650
					0.192	0.655	0.644	0.669	0.679
					0.180	0.696	0.710	0.717	0.721
					0.166	0.788	0.796	0.799	0.822

Cylinder: 2"x18"

Channel width 1.812 ft.

Channel width 1.708 ft.

Channel width 1.812 ft.					Channel width 1.708 ft.				
4.000	0.433	0.420	0.42	0.430	4.000	0.451	0.452	0.453	0.457
1.000	0.485	0.463	0.460	0.455	1.000	0.506	0.500	0.506	0.504
0.333	0.632	0.640	0.620	0.613	0.332	0.639	0.661	0.687	0.665
0.210	0.806	0.816	0.813	0.827	0.219	0.825	0.791	0.828	0.839
0.197	0.725	0.729	0.734	0.748	0.193	0.736	0.752	0.785	0.809
0.166	0.876	0.882	0.869	0.874	0.166	0.910	0.906	0.888	0.873
Channel Width: 1.666 ft.					Channel Width: 1.625 ft.				
4.000	0.464	0.446	0.458	0.465	4.000	0.468	0.462	0.469	0.469
1.000	0.500	0.496	0.508	0.502	1.000	0.520	0.519	0.523	0.522
0.333	0.647	0.671	0.685	0.640	0.348	0.666	0.699	0.697	0.676
0.288	0.711	0.708	0.729	0.743	0.324	0.527	0.526	0.556	0.546
0.261	0.737	0.746	0.765	0.761	0.310	0.549	0.542	0.557	0.552
0.233	0.789	0.794	0.811	0.817	0.281	0.746	0.709	0.768	0.742
0.213	0.860	0.850	0.868	0.867	0.263	0.523	0.507	0.505	0.544
0.196	0.787	0.774	0.796	0.814	0.166	0.931	0.960	0.970	0.942
0.166	0.922	0.911	0.914	0.891					

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UNIVERSITY OF TORONTO.

CHANNEL WALL INTERFERENCE.

CYLINDER DRAG COEFFICIENTS.**TABLE No 2.**

Aug. 25, 1926

Cylinder 2×18

Channel Width 1.583 ft.

Channel Width 1.542 ft.

Channel Height-ft.	Air Speed.				Channel Height-ft.	Air Speed.			
	20	30	40	50		20	30	40	50
4.000	0.488	0.483	0.475	0.485	4.000	0.525	0.529	0.529	0.523
1.000	0.525	0.540	0.537	0.530	1.166	0.573	0.573	0.582	0.583
0.348	0.672	0.676	0.700	0.677	0.500	0.678	0.678	0.695	0.679
0.324	0.523	0.523	0.549	0.541	0.419	0.732	0.739	0.741	0.720
0.311	0.542	0.541	0.557	0.551	0.379	0.759	0.759	0.768	0.749
0.281	0.521	0.510	0.500	0.509	0.344	0.565	0.562	0.580	0.574
0.166	0.994	1.034	1.042	1.010	0.310	0.584	0.591	0.610	0.606
					0.281	0.507	0.506	0.510	0.513
					0.251	0.820	0.803	0.810	0.826
					0.243	0.758	0.748	0.715	0.712
					0.230	0.869	1.113	1.111	1.079
					0.203	1.118	0.959	0.900	1.149
					0.170	1.516	1.507	1.533	1.520

Channel Width 1.500 ft.

4.000	0.602	0.577	0.570	0.574
1.166	0.644	0.633	0.632	0.623
0.500	0.756	0.756	0.767	0.758
0.419	0.795	0.792	0.805	0.792
0.379	0.802	0.791	0.805	0.721
0.344	0.518	0.524	0.524	0.530
0.308	0.521	0.527	0.528	0.532
0.281	0.557	0.562	0.580	0.577
0.252	0.843	0.821	0.828	0.848
0.243	1.041	0.762	0.738	0.700
0.203	1.187	1.175	1.184	1.149
0.170	1.435	1.366	1.377	1.330

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UNIVERSITY OF TORONTO.
CHANNEL WALL INTERFERENCE.
AEROFOIL COEFFICIENTS.
Aerofoil-2x12
Air Speed-20 fps.

TABLE No. 3.

Aug. 25, 1926
gmp.**Lift Coefficient.**

Incid deg	Channel Height-ft.									
	4.00	1.75	1.50	1.00	0.500	0.333	0.250	0.166	0.125	0.104
-8	-0.1618	-0.1485	-0.1510	-0.1517	-0.1586		-0.1618	-0.1681		-0.1833
-4	-0.0489	-0.0467	-0.0430	-0.0474	-0.0500		-0.0872	-0.0999		-0.1574
0	+0.1513	+0.1466	+0.1554	+0.1504	+0.1536		+0.1681	+0.1485		+0.1175
+4	0.3385	0.3344	0.3236	0.3325	0.3698		0.4114	0.4228		0.3483
8	0.4535	0.4541	0.4602	0.4696	0.4766		0.4152	0.4216		0.4197
10	0.5186				0.4051		0.4165	0.4178		
12	0.4110	0.3862	0.3906	0.3950	0.3849		0.4216			
14		0.3773	0.3837	0.3653						

Drag Coefficient.

-8	.0581	.0521	.0474	.0525	.0556		.0600	.0676		.0689
-4	.0347	.0287	.0297	.0322	.0322		.0348	.0373		.0329
0	.0293	.0265	.0272	.0291	.0316		.0190	.0303		.0291
+4	.0360	.0326	.0360	.0367	.0367		.0329	.0386		.0493
8	.0502	.0502	.0506	.0556	.0537		.0657	.0777		.0828
10	.0651				.0758		.0853	.0916		
12	.0951	.0900	.0916	.0917	.0910		.1011			
14		.1024	.1093	.1043						

Lift Coefficient.**Air Speed-30 fps.**

-8	-0.1612	-0.1543	-0.1556	-0.1610	-0.1598		-0.1691	-0.1667		-0.1845
-4	-0.0318	-0.0296	-0.0365	-0.0331	-0.0421		-0.0736	-0.0770		-0.1438
0	+0.1956	+0.1916	+0.1891	+0.1899	+0.2143		+0.2394	+0.2495		+0.2048
+4	0.3477	0.3502	0.3512	0.3480	0.3643		0.4253	0.4806		0.4430
8	0.4931	0.4778	0.4882	0.4803	0.4972		0.5885	0.6446		0.4441
10	0.5401				0.5770		0.4677	0.4469		
12	0.5949	0.5826	0.5806	0.4705	0.4486		0.4433			
14	0.4459	0.4237	0.4290	0.4098						

Drag Coefficient.

-4	.0525	.0490	.0525	.0520	.0539		.0559	.0615		.0660
-4	.0296	.0281	.0286	.0286	.0275		.0317	.0368		.0348
0	.0251	.0213	.0239	.0227	.0244		.0199	.0227		.0298
+4	.0293	.0289	.0298	.0292	.0267		.0222	.0270		.0416
8	.0483	.0427	.0441	.0444	.0388		.0360	.0433		.0786
10	.0551	.0655			.0494		.0739	.0854		
12	.0661	.0655	.0632	.0843	.0803		.0975			
14	.1042	.1014	.1020	.0994						

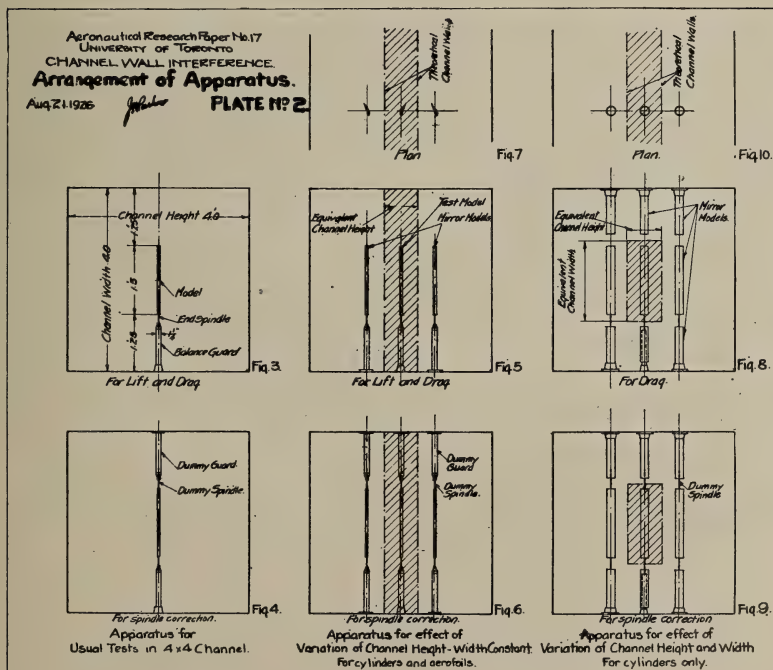
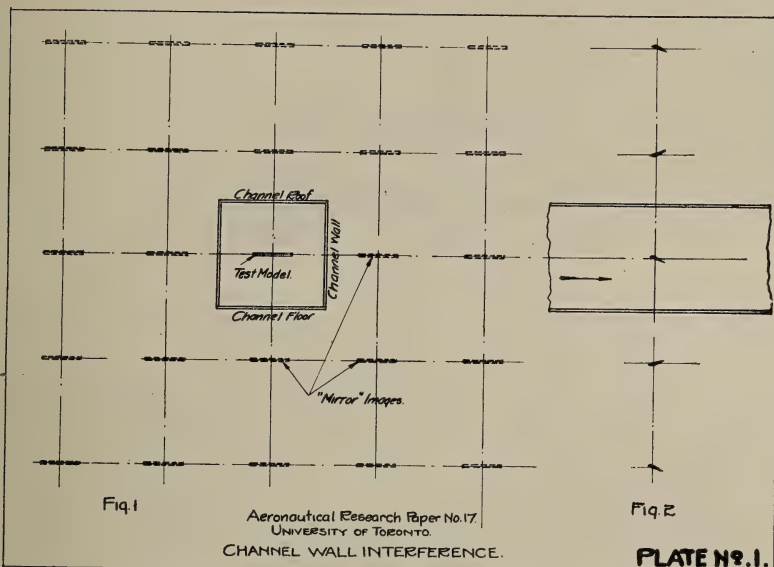
Aeronautical Research Paper No. 17.
UNIVERSITY OF TORONTO.
CHANNEL WALL INTERFERENCE.

TABLE No. 7.

Aug. 25, 1926.

**Numerical Values of Quantities Appearing
in
Theoretical Relations for Channel Wall Corrections
for
Conditions of Tests.**

Channel Characteristics					Aerofoil Area $S=25c$		$\delta \frac{S}{C}$		$57.3 \delta \frac{S}{C}$		
Height h		Width b ft	$\lambda = \frac{h}{b}$	δ	Area $C=hb$ sq.ft.	Aerofoil		Aerofoil			
ins	ft.					2x12	3x18	2x12	3x18	2x12	3x18
48	4.000	4.00	1.0000	0.274	16.000	0.166	0.375	0.00285	0.00642	0.163	0.368
21	1.750	"	0.4375	0.305	7.000	"	"	0.00726	0.01634	0.416	0.936
18	1.500	"	0.3750	0.351	6.000	"	"	0.00975	0.02194	0.559	1.257
12	1.000	"	0.2500	0.524	4.000	"	"	0.0218	0.0491	1.249	2.813
6	0.500	"	0.1250	1.048	2.000	"	"	0.0873	0.1964	5.002	11.25
4	0.333	"	0.0833	1.560	1.333	"	"	0.1950	0.439	11.17	25.15
3	0.250	"	0.0625	2.090	1.000	"	"	0.348	0.785	19.94	44.98
2	0.166	"	0.0416	3.140	0.666	"	"	0.785	1.77	44.98	101.42
1.5	0.125	"	0.0312	4.190	0.500	"	"	1.396	3.14	79.99	179.9
1.25	0.104	"	0.0260	5.040	0.416	"	"	2.019	4.54	115.7	260.1



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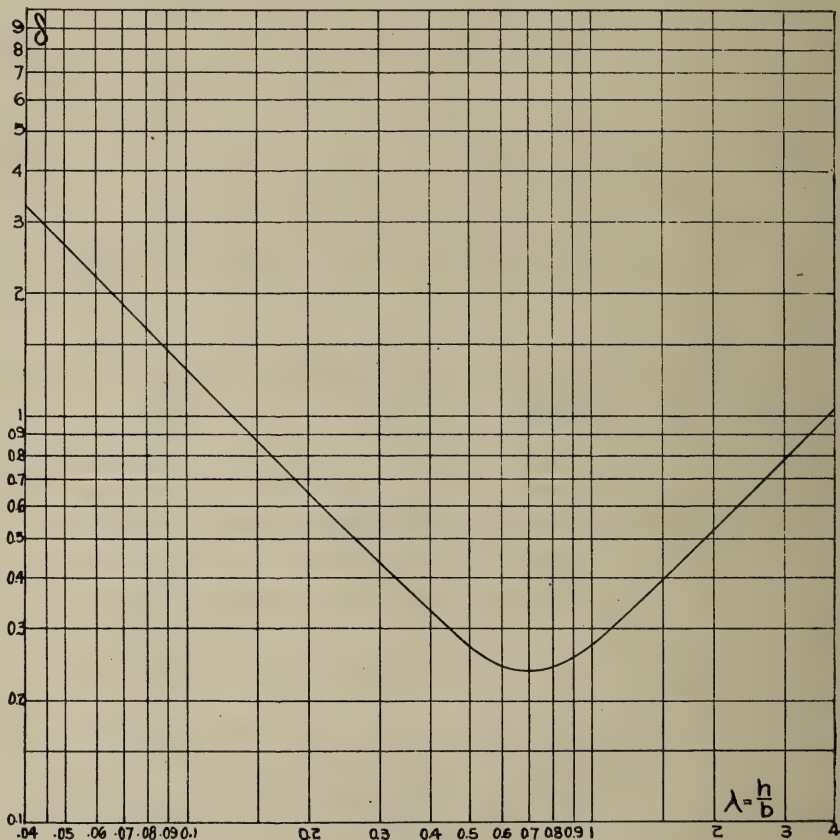
CHANNEL WALL INTERFERENCE.

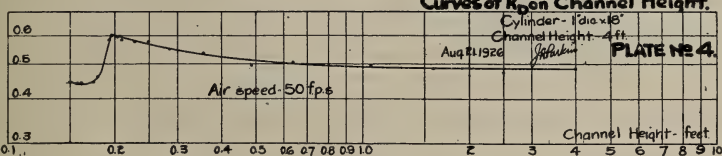
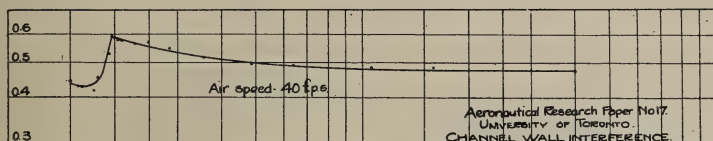
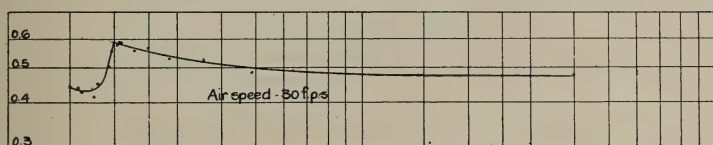
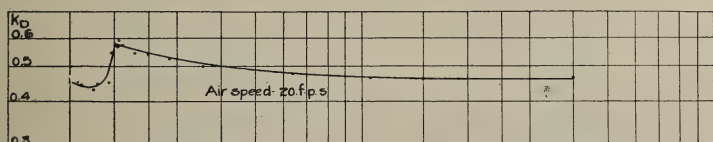
Values of Coefficient - δ .

Aug. 21. 1926.

J. H. Parkin

PLATE No. 3.





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CHANNEL WALL INTERFERENCE.

Curves of K_p on Channel Height.

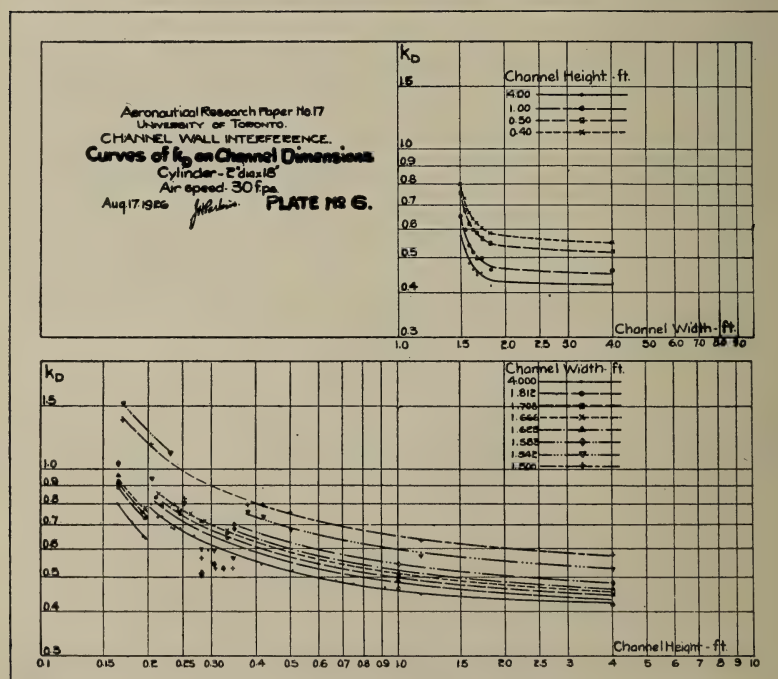
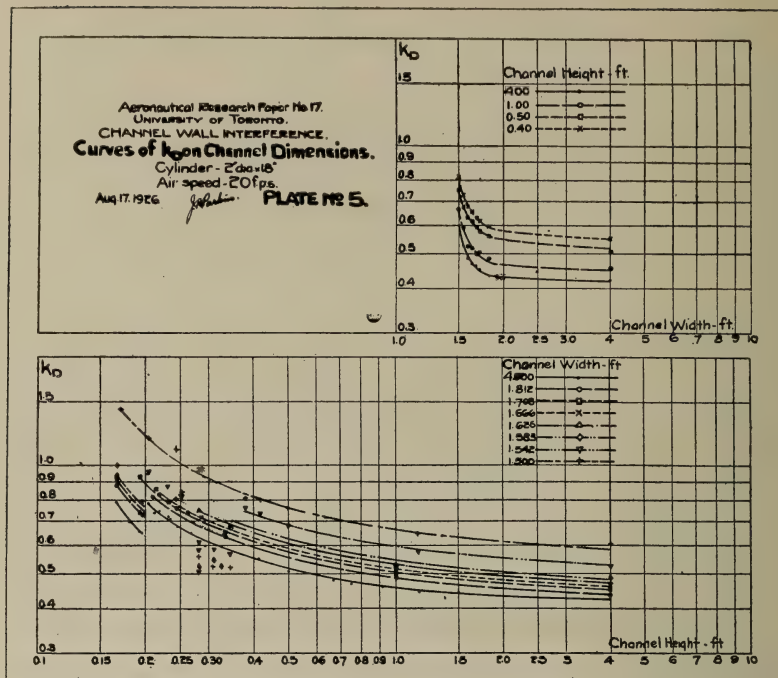
Cylinder - 1" dia. x 6"
Channel Height - 4 ft.

Aug. 21, 1926

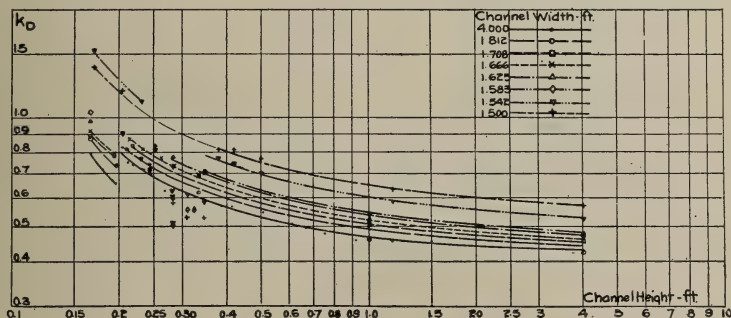
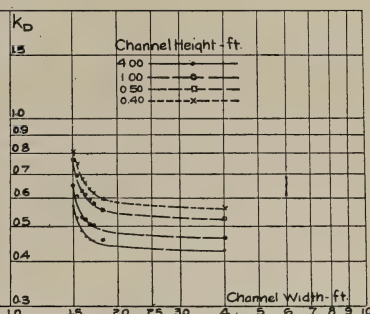
Johnston

PLATE No. 4.

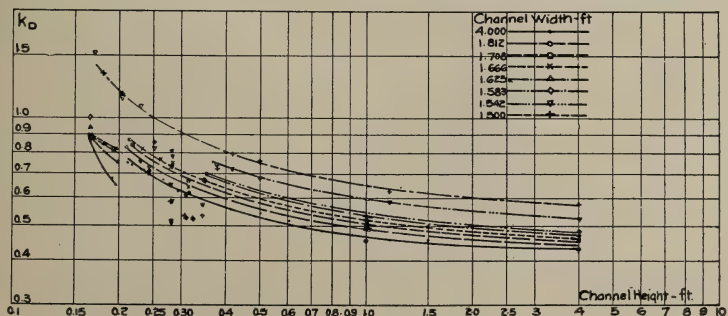
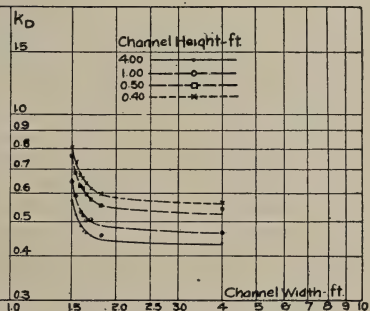
Channel Height - feet



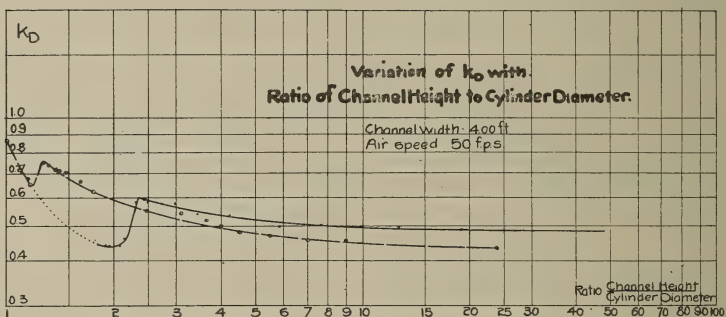
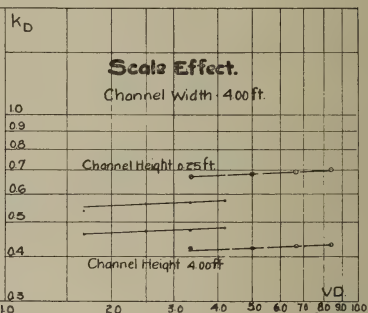
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UNIVERSITY OF TORONTO.
CHANNEL WALL INTERFERENCE.
Curves of k_D on Channel Dimensions.
Cylinder - $2d \times 18'$
Air speed - 40 f.p.s.
Aug. 16, 1926. *J. G. Johnson* **PLATE No. 7.**

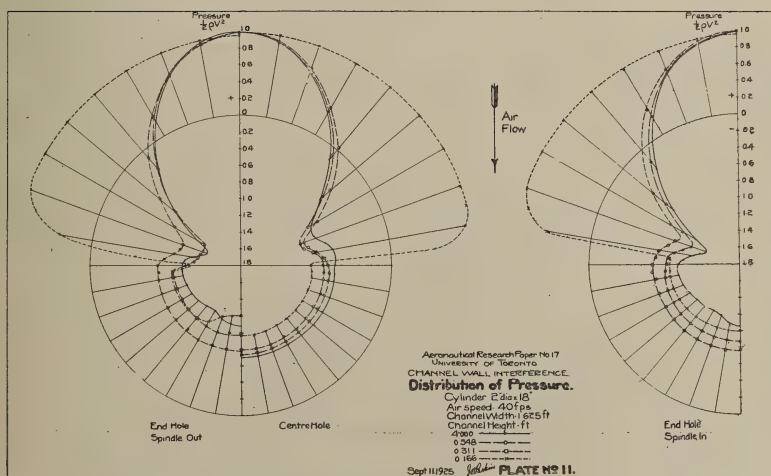
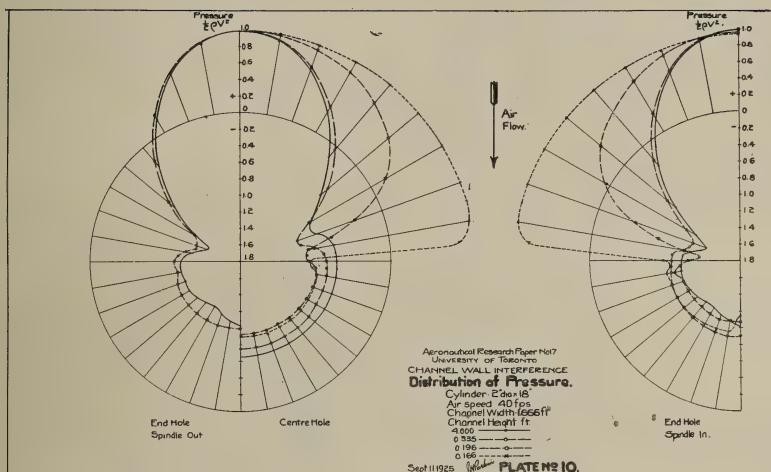


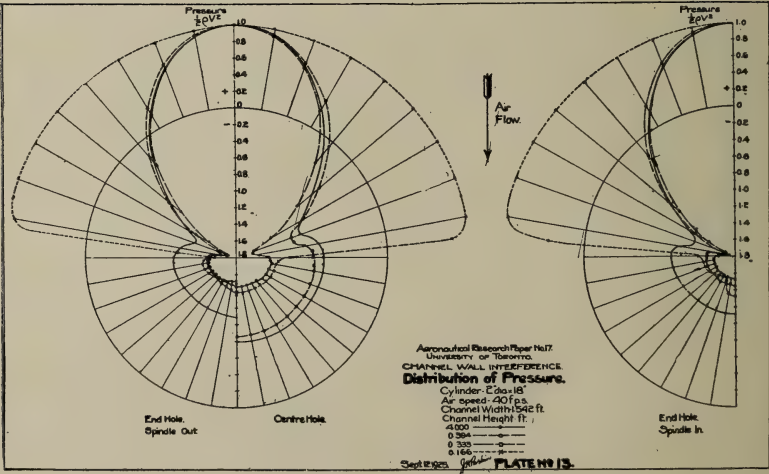
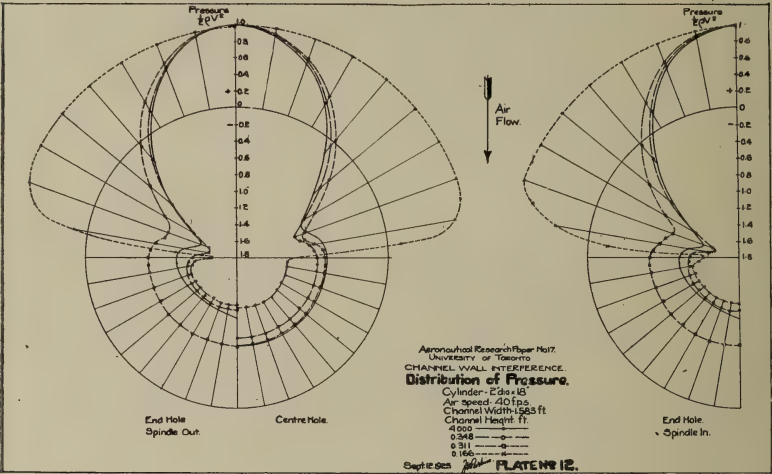
Aeronautical Research Paper No. 17.
UNIVERSITY OF TORONTO.
CHANNEL WALL INTERFERENCE.
Curves of k_D on Channel Dimensions.
Cylinder - $2d \times 18'$
Air speed - 50 f.p.s.
Aug. 16, 1926. *J. G. Johnson* **PLATE No. 8.**

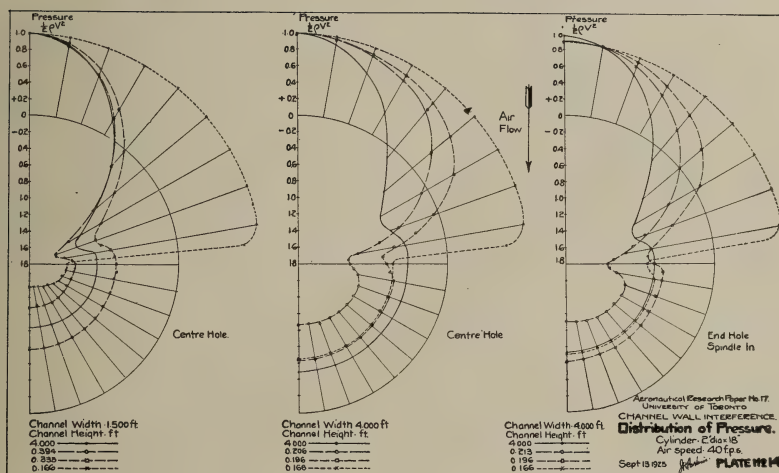


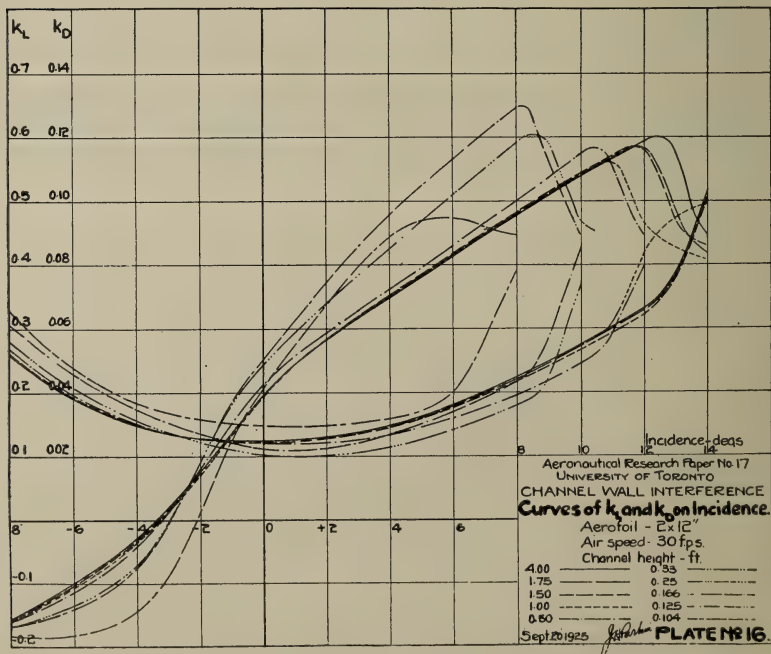
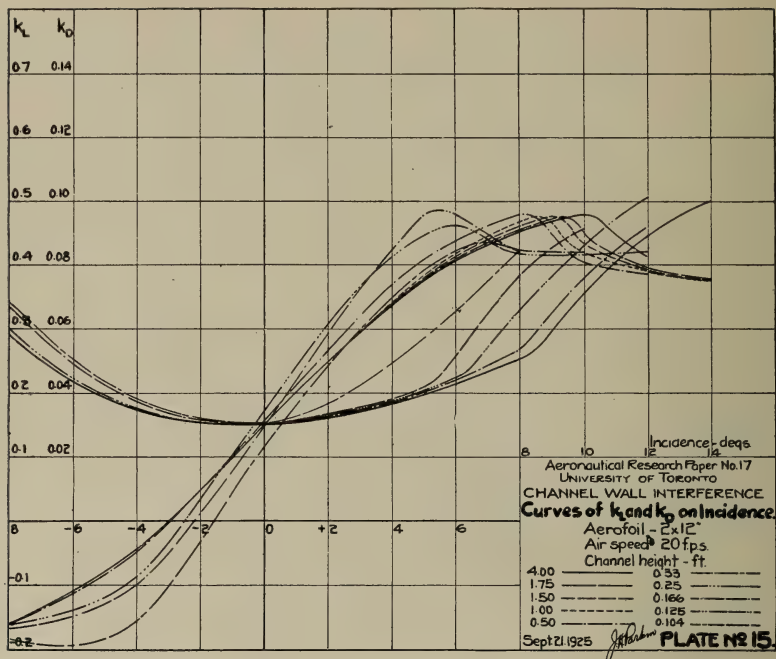
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UNIVERSITY OF TORONTO
CHANNEL WALL INTERFERENCE.
Variation of k_D for Cylinders.
Cylinder 1' dia x 18"
Cylinder 2' dia x 18"
Aug. 21 1926 *Jeffery* **PLATE No. 9.**

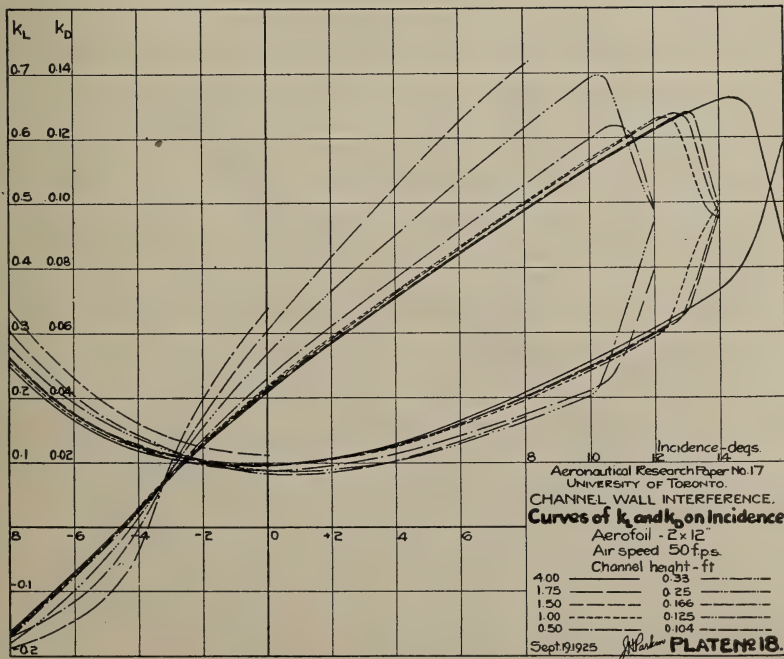
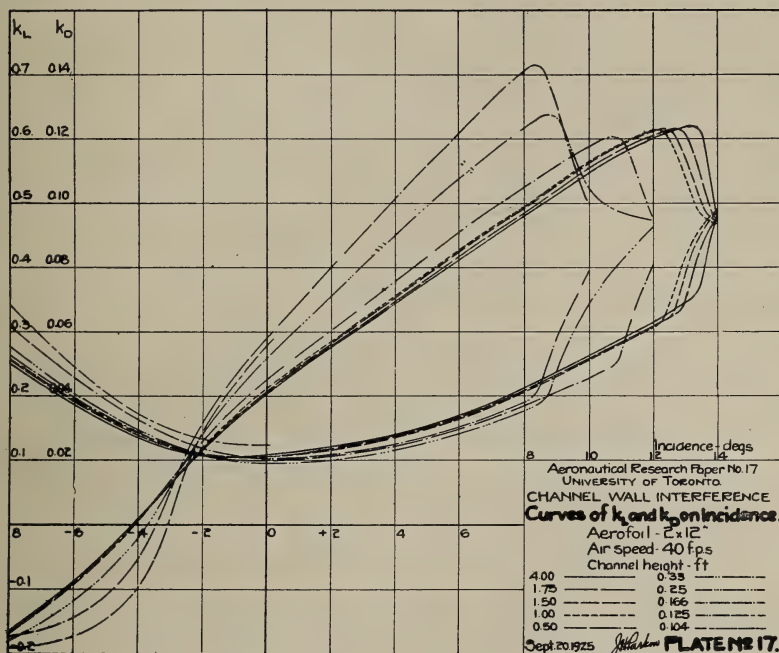


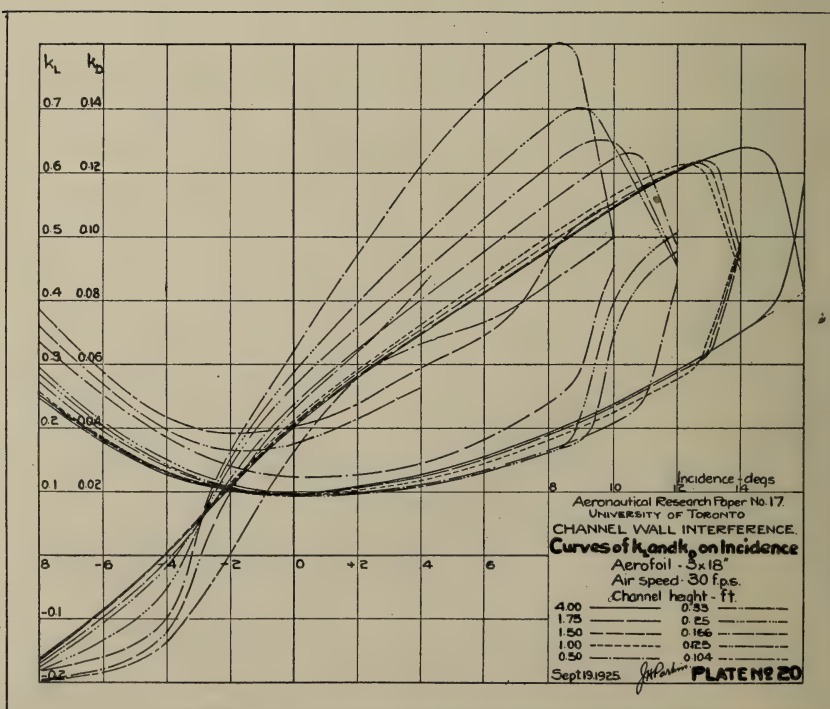
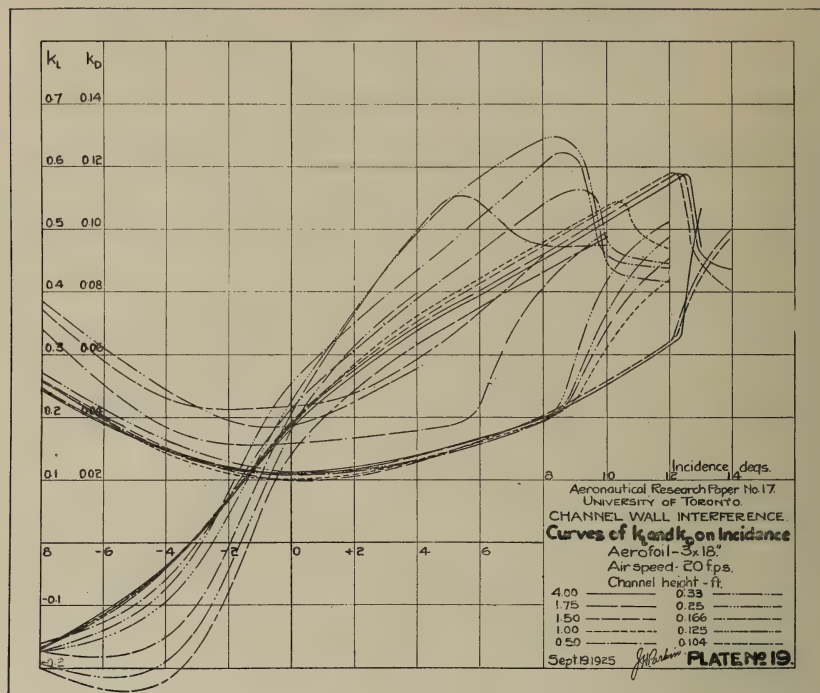


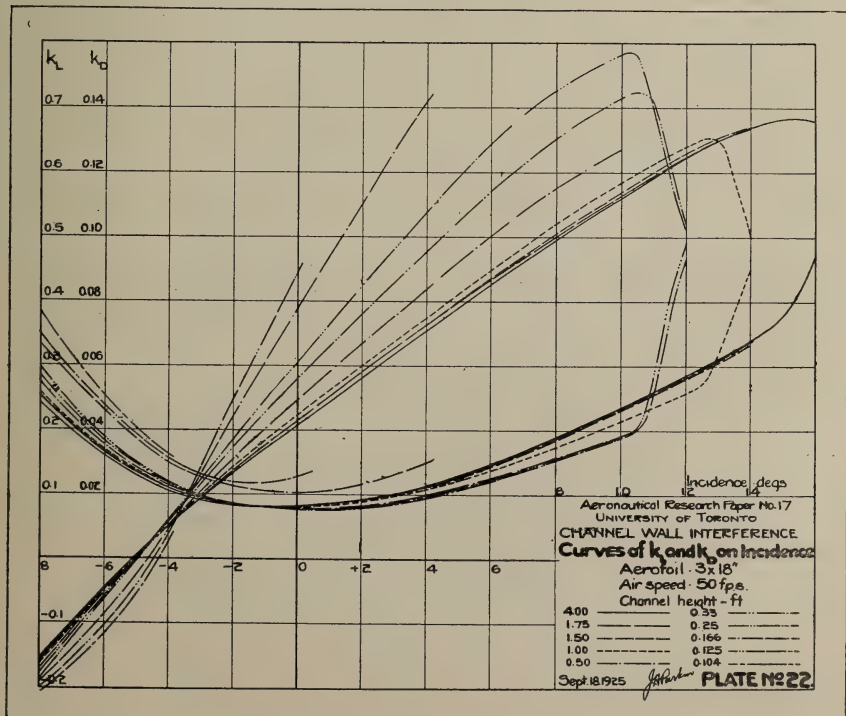
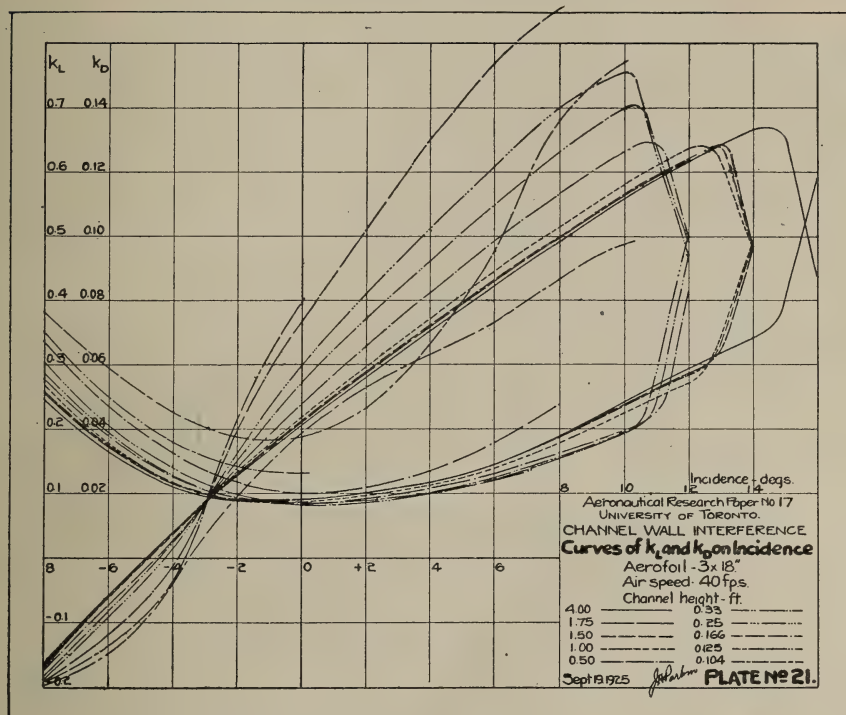


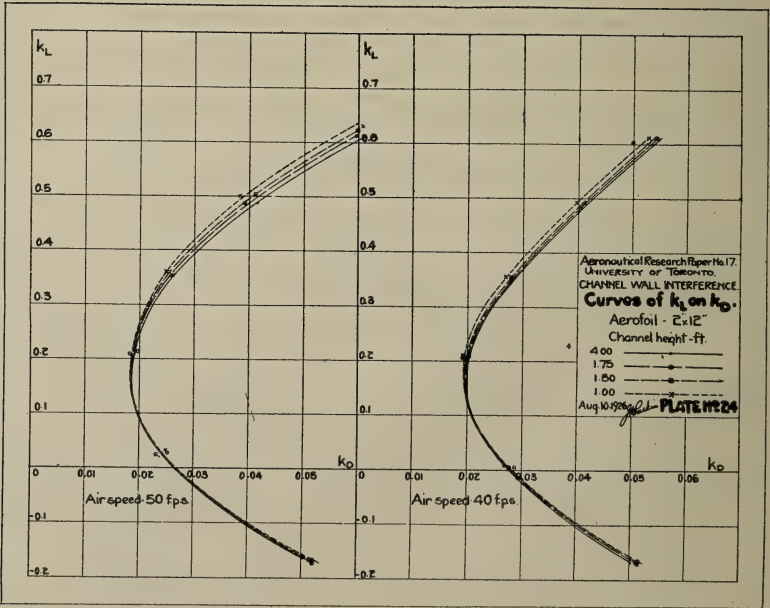
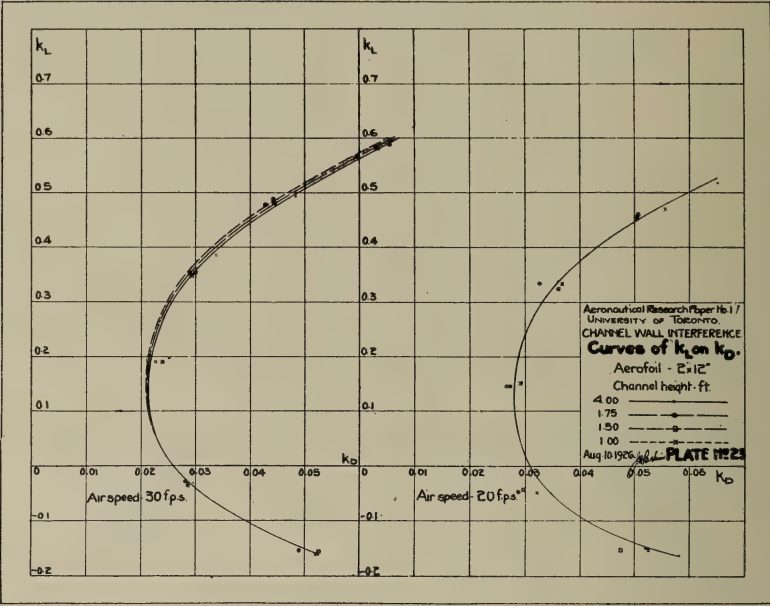


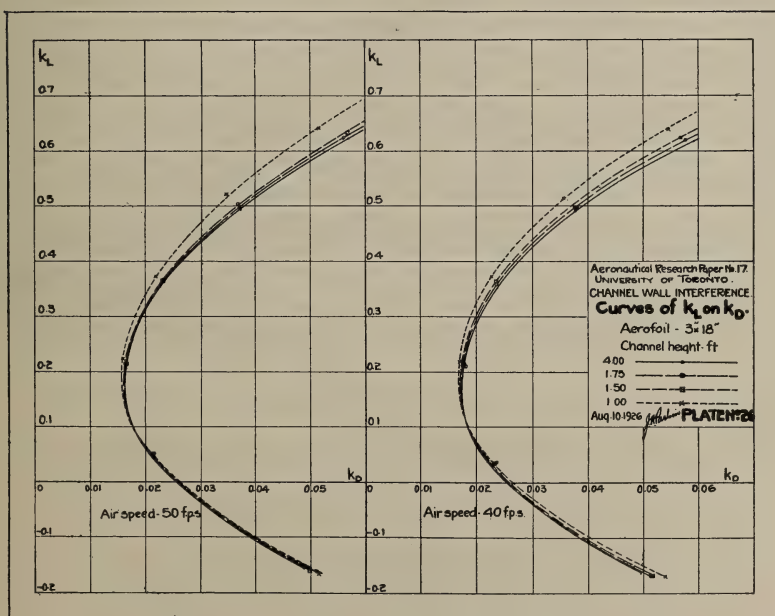
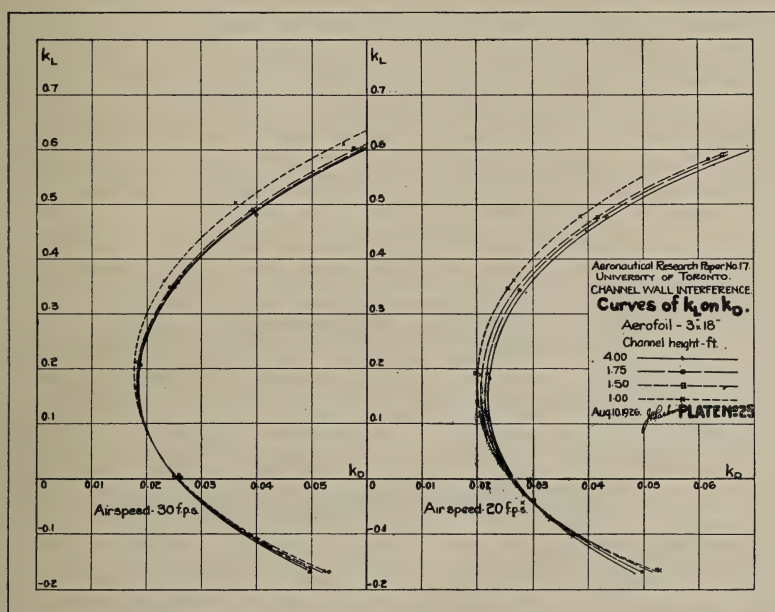


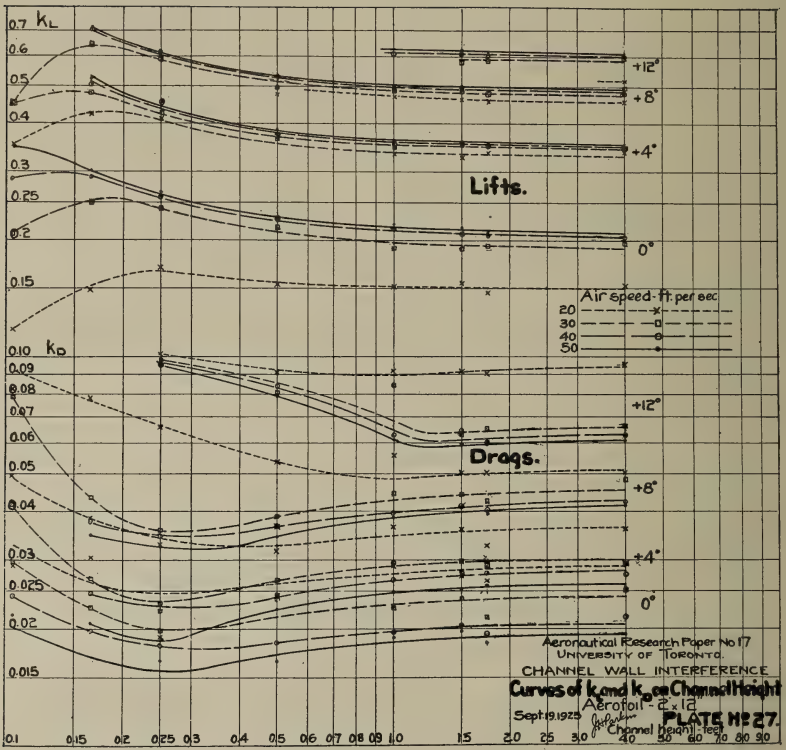


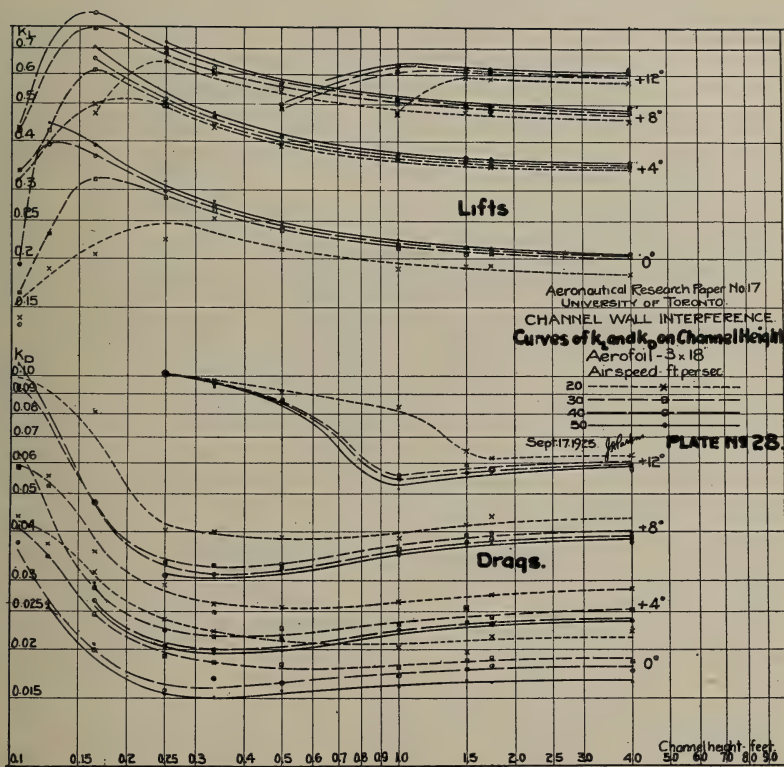


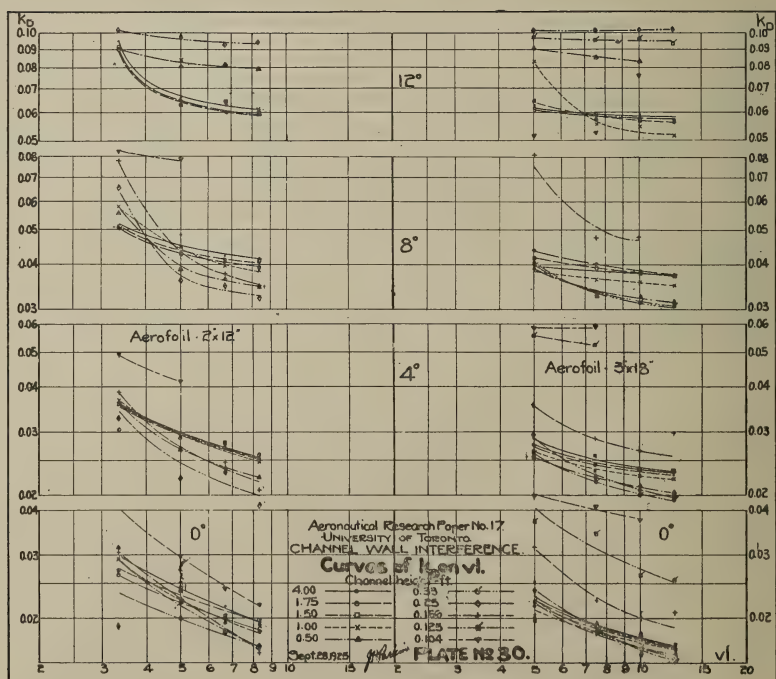
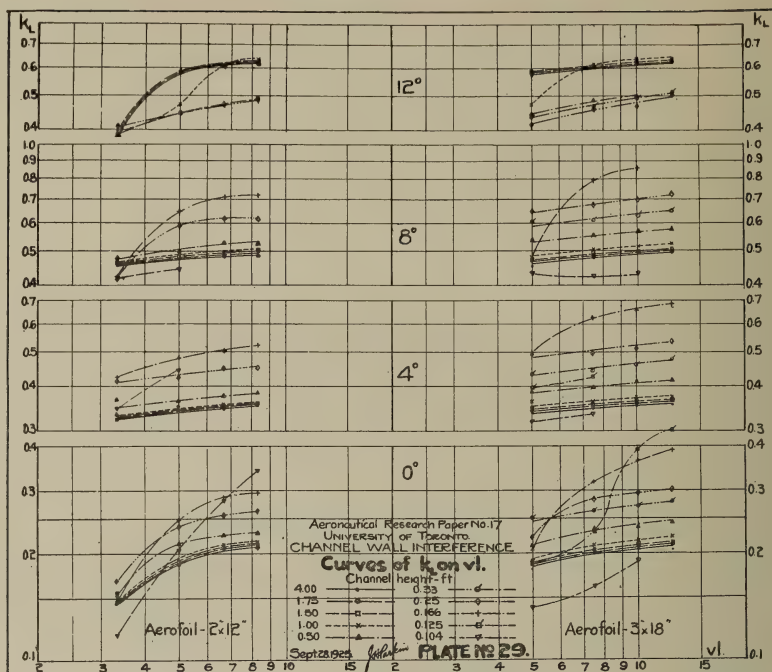


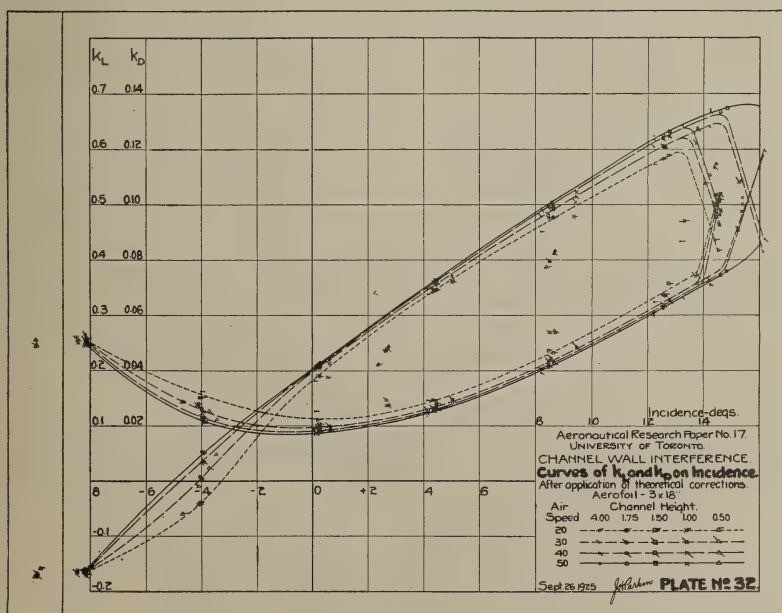
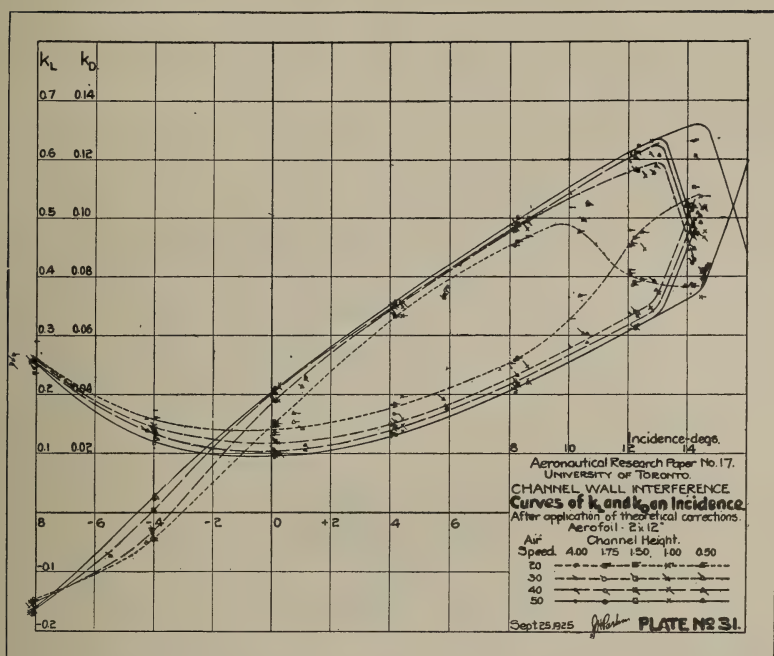


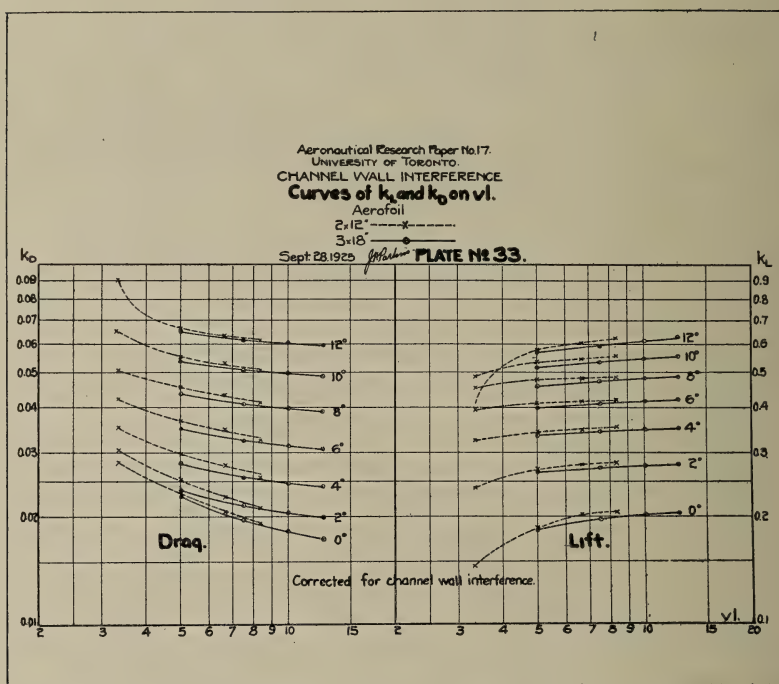


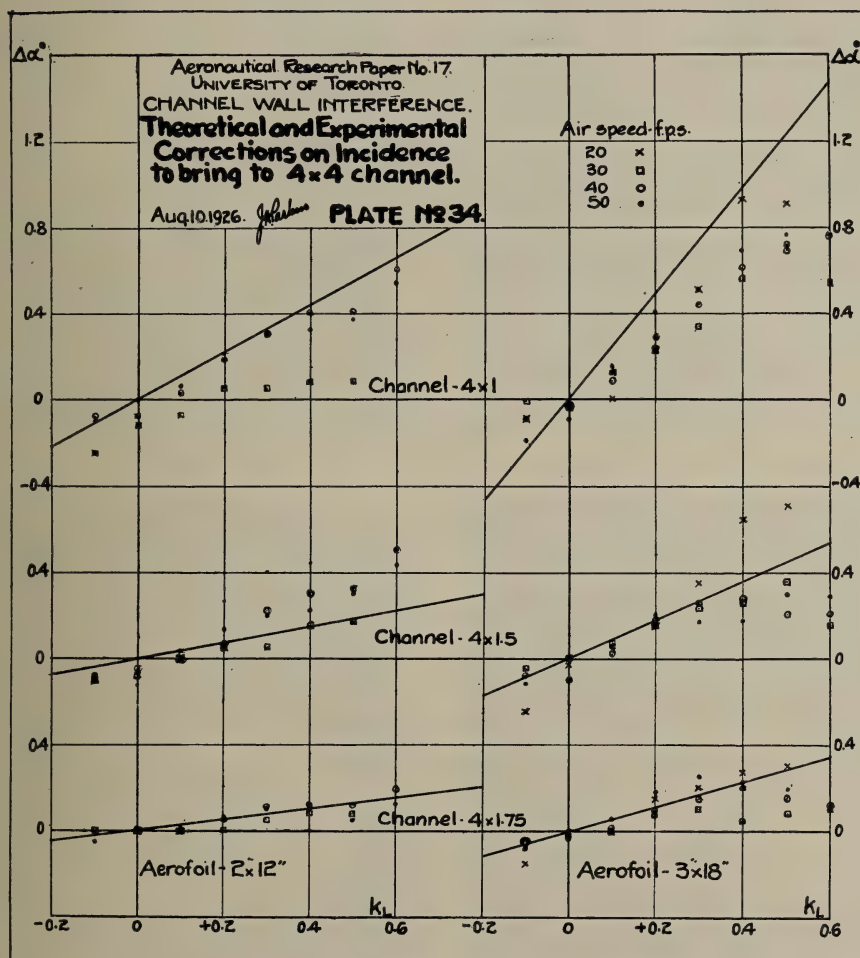


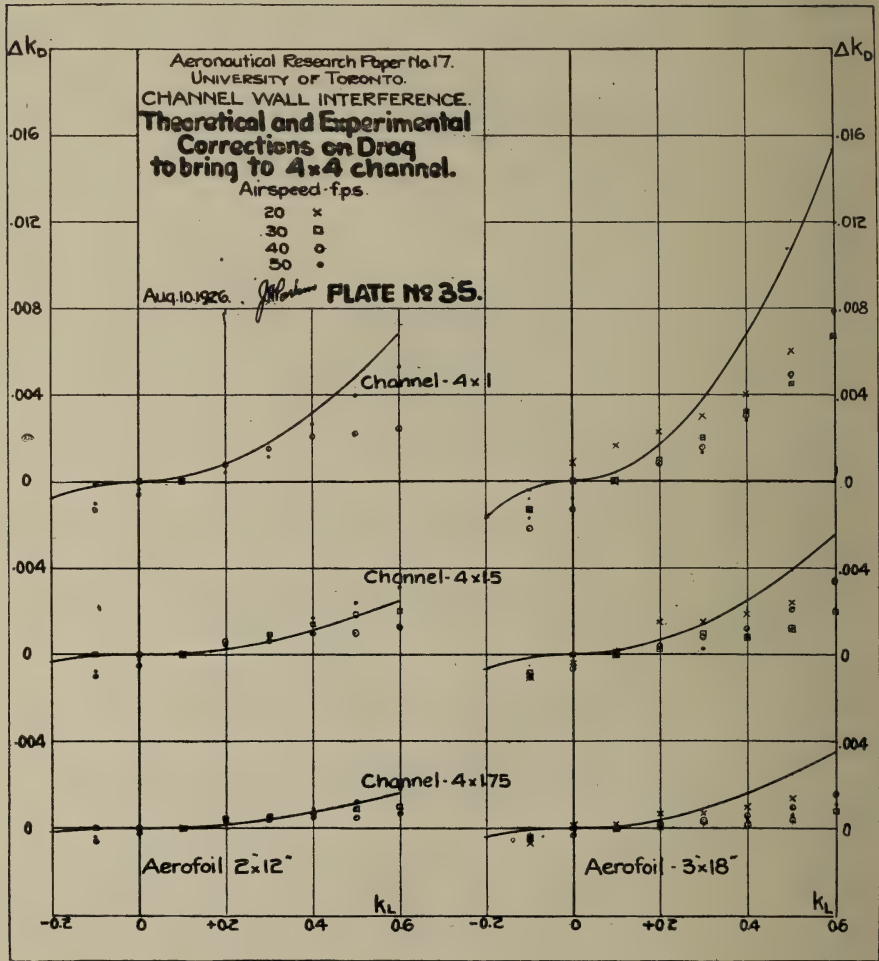


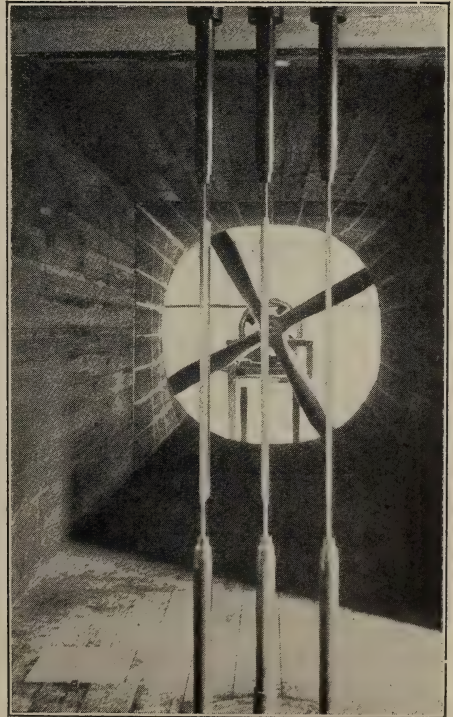
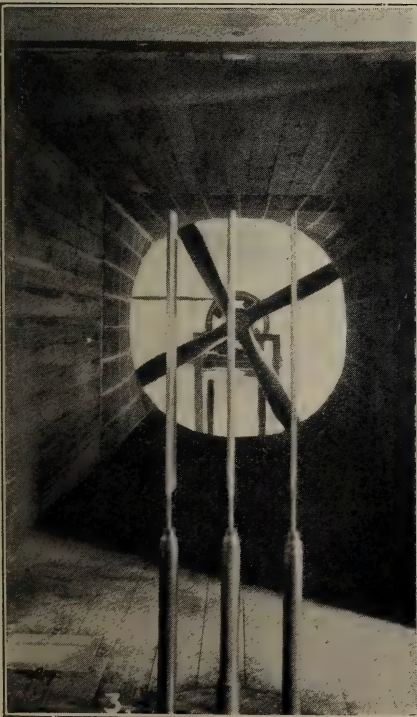
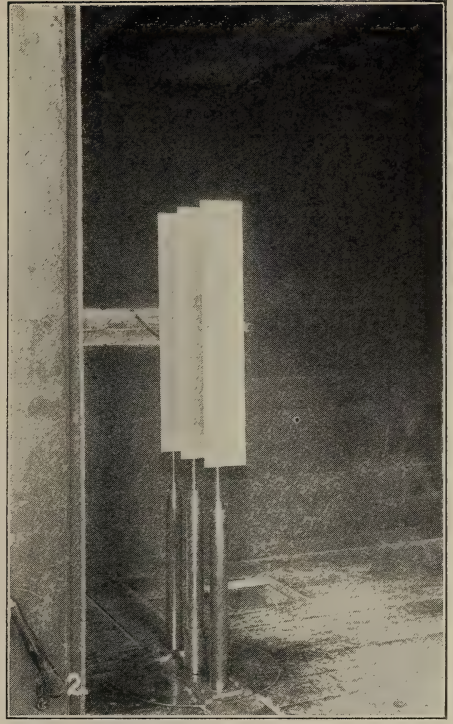
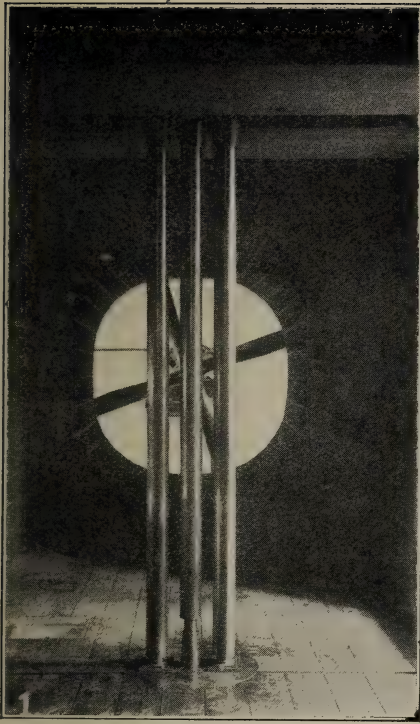












PRESSURE DISTRIBUTION OVER U.S.A. 27 AEROFOIL WITH SQUARE WING TIPS*

By J. H. PARKIN, M.E., F.R.Ae.S., J. E. B. SHORTT, B.A.Sc.,
and C. G. HEARD

SUMMARY

This investigation was undertaken to determine the distribution of loading over a U.S.A. 27 Aerofoil, with square wing tips, as a monoplane and in different biplane combinations.

The tests were conducted in the 4 foot wind channel of the University of Toronto.

Pressure measurements were made at six sections on the semi-span of a brass aerofoil of 3 in. chord and 18 in. span, arranged for pressure plotting. The range of incidence covered was from -4° to $+18^\circ$ (corresponding roughly to the zero to maximum lift range for this section). An air speed of 40 f.p.s. was used for all the tests. The pressures are expressed in terms of ρV^2 .

A duralumin dummy plane, identical in plan form and section, with the pressure plotting plane, was employed in securing the biplane measurements.

The wing combinations tested were:

Monoplane;

Biplane with gap equal to chord;

Zero stagger;

Upper wing 15° ahead of lower wing;

Upper wing 30° ahead of lower wing.

The effect of stagger on the distribution of loading is considerable. As the stagger is increased, the load on the upper wing increases, while the load on the lower wing decreases, the ratio of the load on the upper wing to the load on the lower wing varying from 1.02 with no stagger, to 1.52 with 30° stagger at 0° incidence.

*Portion of a thesis submitted by Mr. C. G. Heard for the degree of B.A.Sc.

INTRODUCTION—REASON FOR INVESTIGATION

Due to its excellent aerodynamic characteristics, the U.S.A. 27 wing section has been used in a number of recent aeroplanes, and is being incorporated in the designs of machines now in preparation.

Although thus used to a considerable extent, the nature of the distribution of pressure over the surfaces of this aerofoil has not heretofore been published.

The present investigation was, therefore, undertaken with the object of supplying aircraft designers and constructors with much needed information relative to the distribution of loading over the surfaces of the U.S.A. 27 aerofoil, as a monoplane and in the usual biplane combinations.

The aerofoils and multiple manometer were purchased with funds supplied by the Department of National Defence of Canada, through the Associate Air Research Committee of the National Research Council and the work was carried out in the Aerodynamic Laboratory of the Department of Mechanical Engineering, University of Toronto.

It is proposed to complete the investigation later by the determination of the aerodynamic characteristics of this aerofoil in the same biplane combinations.

DESCRIPTION OF APPARATUS

(a) Channel:

The tests were conducted in the new 4 ft. Royal Aircraft Establishment type wind channel at the University of Toronto.¹

(b) Pressure Plotting Model:

The model over which the pressure distributions were measured, was of brass, of 3 in. chord and 18 in. span. The section and chord were uniform along the span and the ends were finished off square with the leading edge and the tangent plane of the under-surface (plate 1 and Photos. 1 and 2). It was made in the shops of Mr. W. H. Nichols at Waltham, Mass., using the Nichols wing cutting machine, which is essentially an automatic horizontal cam profiler. The machine produces a smooth true model that is accurate to within two-thousandths of an inch on any ordinate.²

¹The New Aerodynamic Laboratory of the University of Toronto, Aero Research Paper No. 16 (in preparation.) The New Aerodynamic Laboratory of the University of Toronto, by J. H. Parkin, M.E., F.R.AeS. *Transactions and Year Book of University of Toronto Engineering Society*, April 1924.

²N.A.C.A. Technical Note 172. December 1923. *The Nichols Wing Cutting Equipment*, by James B. Ford, Massachusetts Institute of Technology.

The tubes into which the pressure openings were drilled, were German silver, 0.06 in. outside diameter and 0.04 in bore. Eight grooves were milled in the lower surface of the aerofoil blank, parallel to the leading edge, and the tubes soldered in place in the bottom of the grooves before the aerofoil was cut in shape. This gave a perfectly smooth under surface, without any of the irregularities which would be certain to occur if the tubes were soldered in after the aerofoil was cut to its finished dimensions.

The tubes were finished flush with the end of the model on the end on which the pressures were to be measured, and left open so that the tubes could be cleaned with a wire if they became stopped in any way. At the other end of the model the tubes were allowed to project 1 in.

The pressure openings were drilled into the tubes from both surfaces of the aerofoil with a 0.02 in. drill. There were six sections (*A* to *F*) of the openings on the semi-span, one section being at mid-span, and the one nearest the wing tip being $\frac{1}{4}$ in. (0.083 chord) from the end of the aerofoil. With the exception of the first three rows from the leading edge on the upper surface, the pressure openings were drilled normal to the tangent plane of the lower surface. The first three rows from the leading edge on the upper surface were drilled at the angles shown in Plate 1, as near as practicable normal to the finished surface of the aerofoil at the opening.³ The openings were spaced closer together where the pressure gradient is steepest (near the leading edge) and near the wing tip where the distribution of load along the span changes more rapidly than it does along the central portion of the aerofoil.

A socket, as shown, was provided at each end of the aerofoil to fit the steel spindle, which is used for holding models in the chuck of the N.P.L. balance.

(c) *Dummy Plane:*

The dummy aerofoil which was used with the brass pressure plotting plane for biplane combinations, was duralumin (made by W. H. Nichols). It was identical in plan form and section with the brass plane, but without tubes and pressure openings. The sockets in the ends for the spindle were provided as in the brass plane.

(d) *Manometer:*

A multiple slanting manometer, of the Krell type, was used, containing 10 tubes, which allowed the 8 pressures on one surface at a section to be measured simultaneously (see Photo. 3). The upper ends of the two outside tubes were connected directly to the top of the reser-

³N.A.C.A. Report No. 74, 1920. Construction of Models for Tests in Wind Tunnels.

voir. These two tubes indicated at all times the level of the liquid in the reservoir and, hence, the zero readings of the central tubes which were connected to the model.

The tubes (glass barometer tubing) were held in V-shaped grooves, milled in a brass plate, by boxwood scales (graduated in millimeters) fastened to the plate with small brass screws (see Photos 4 and 5).

The sensitivity of the manometer was changed by raising or lowering the upper end of the plate on a screw, the lower end of the plate being pivoted in a U-shaped support rigidly mounted on the wooden base. The correct slope (1 in 10 or 1 in 20) was indicated by levels mounted on wedges at the side of the plate. The lower ends of the glass tubes were connected to the reservoir, which could be raised and lowered to change the zero of the liquid in the glass tubes. A tube was also sweated into the top of the reservoir to enable it to be connected with the side plate in the channel. The manometer thus indicated the differences in pressure between the side of the channel and points on the surface of the aerofoil.

The liquid used in the manometer was alcohol, coloured with red dye, the meniscus of which showed up well against the white paper laid between the plate and the tubes.

The manometer was calibrated in inches of water against the Modified Chattock gauge,⁴ used for speed regulation. The result of the calibration was as follows:

Slope of gauge 1 in 10

1 mm. net reading = 0.00330 inches of water

= 0.01715 pds. per sq. ft.

= 0.004517 multiples of ρV^2 pds. per sq. ft. at
40 f.p.s. ($\rho = 0.002373$)

Capacity of manometer = 500 mm. reading = 1.65 inches of water.

SET UP

The pressure openings on the aerofoil, with the exception of those in one surface at the section being investigated, were closed with narrow strips of thin paper fastened over each row with seccotine. The ends of the inserted tubes were closed in the same manner where they were flush with the end of the aerofoil. The rubber tubes, used to join the manometer to the aerofoil were then connected to the aerofoil. Thick walled rubber tubing, with a bore equal to the outside diameter of the German silver tubes, was first placed over the ends of the tubes projecting from the aerofoil, and the larger tubing placed over this smaller

⁴For description see Aero. Research Paper No. 16 (in preparation).

tubing, sealing all the joints with seccotine. The small tubes were made fairly short to keep the damping effect of the small air passages as small as possible (see Photos. 1 and 2).

The pressure plotting aerofoil was mounted in a vertical position on a tapered spindle in the chuck of the N.P.L. balance, as it would be for lift and drag tests. The balance was kept locked during these tests, however, and used only to adjust the aerofoil for incidence.

The model was lined up with the chord (the tangent plane to the lower surface) parallel to the wind direction, using the optical alignment device of the channel,⁵ and checked for plumb. Careful checking during the tests disclosed no alteration of position of the aerofoil.

For biplane combinations the dummy plane was mounted vertically on a spindle, held in a chuck screwed into one of three holes drilled and tapped in a turn-table set in the floor of the channel, concentric with the vertical axis of the balance. The holes in the turn-table were so spaced that when both planes were lined up with the wind direction, the dummy plane formed the upper plane of one of the required biplane combinations, with the gap equal to 3 inches, and the stagger either 0° , 15° or 30° , as required. The dummy plane was changed to the lower plane of the biplane combinations by turning the turn-table through 180° and again lining up the dummy plane in its proper position. In this way it was not necessary to change the setting of the plane on which the pressures were being measured to obtain the desired combinations of the wings. Care was taken to see that the dummy plane was at the same level and parallel to the pressure plotting plane each time it was set up. Thin brass strips, narrowed off to about $1/16$ in. wide, near the pressure plotting plane, were used to keep the two planes in proper relative position at the top. Strips of three lengths were used, one for each stagger, the ends being fastened to the top of the aerofoils by flat headed screws placed in the spindle sockets provided. It was considered that with this arrangement, the interference with the air flow at the wing tip would be a minimum, and the required rigidity given to the models.

METHOD OF TEST

The method followed was to determine the pressure distribution on the upper surface at the median section (all the other openings being sealed, as previously explained), for the monoplane at angles of incidence from -4° to $+18^\circ$. The dummy plane was then set up as the upper plane of a biplane with 0° stagger, and the pressure distribution at all the angles again determined (the turntable being turned through the angle of incidence in the same direction as the spindle of the balance,

⁵For description see Aero Research Paper No. 16 (in preparation).

to keep the two planes in the correct relative position to the wind direction). The chuck holding the dummy plane was then moved to the next hole in the turntable, the dummy plane lined up and the distribution of pressure at this row of holes obtained for the lower wing of a biplane with 15° stagger, and then to the next hole for 30° stagger. The holes in the turn-table not in use were closed with set-screws. When the distribution on the upper surface at the median section for the lower wings was completed, the turn-table was turned through 180° and the distribution over the upper surface of the upper wings at the median section determined.

When the pressure distribution along this row of holes had been determined for all the angles of incidence for each of the structural arrangements, the holes were closed with a narrow strip of thin paper. The paper was then removed from the next row of holes and the same method followed at the new section. This process was repeated until the whole wing was tested.

The wind speed was kept constant for all the tests at 40 f.p.s.

The manometer slope used was 1 in 10. The height of the reservoir was adjusted until the reading was about 125 mm. or $\frac{1}{4}$ of the length of the scales from the lower ends. It was found that with a few exceptions (for the high suction at the leading edge of the upper surfaces of the monoplane and upper wings) the reservoir did not have to be raised or lowered, as either positive or negative pressures could be measured with this setting. Although the level of the liquid in the reservoir changed considerably in some cases, it was considered easier to determine new zeros for the tubes than to adjust the level of the liquid to its old height. After the planes had been set in their new positions, a certain length of time was allowed for the liquid to come to its final position before the readings were taken each time.

Typical manometer readings for the two surfaces of the aerofoil, at a high angle of incidence, are shown in Photos. 4 and 5.

The difference in pressure registered on the manometer was the difference between the pressure at the observation point on the aerofoil, and the pressure at the side-plate (to which the reservoir of the manometer was connected for convenience). The difference in pressure required was that between the point on the surface of the aerofoil, and that same point in the undisturbed air stream. The difference between the pressure at the side plate and the pressure in the region which was occupied by the model, with an air speed of 40 f.p.s., was determined after the test, using a Krell slanting manometer and a Pitot tube, and was found to be equivalent to a reading of about $\frac{1}{3}$ mm. on the multiple slanting manometer, with a slope of 1 in 10. As this was closer than

the readings could be checked, due to slight variations in wind speed, no correction was applied to the observed readings.

PRESENTATION OF RESULTS

The observed pressures in terms of ρV^2 pds. per sq. ft. (V in ft. per sec. $\rho = .002373$) are tabulated in Table 7 and plotted in Plates 2 to 9. Each plate is a complete record of the distribution for one angle of incidence. The pressures measured in this manner are assumed to act normally to the surface of the aerofoil at the observation point.⁶

A departure from the usual method of plotting pressure distributions was used in presenting the results, positive pressures being plotted downward, and negative pressures or suction upward. In this way the distribution over each surface was in its correct relative position on the biplane diagrams, which helped to show more clearly the effect of the various biplane constructions on the pressure distributions along the chords of the upper and lower wings.

The force normal to the chord at each section was determined from the normal force diagrams in Plates 2 to 9. Although the pressures at the observation points are measured normal to the aerofoil surface, and not normal to the chord, the force normal to the chord may be found by means of a simple construction.⁷ The observation points are projected normal to the chord and ordinates measured off from the chord to represent the pressures at the observation points. The area of the resulting figure (found by planimeter) then represents the total force normal to the chord at the section, and from this the average normal force per square foot of projected area along the chord at the section, in terms of ρV^2 pds. per sq. ft., that is the *normal force coefficient* may be readily determined. The values of the normal force coefficients of the sections determined in this way are given in Table III, and are plotted in Plates 10 and 13. Due to the small areas of the diagrams as plotted, it was rather difficult to check the areas very closely, and the average of several readings is given in each case.

The distributions of the load along the semi-span for each angle of incidence are plotted in Plate 10 for each of the structural arrangements investigated.

DISCUSSION OF RESULTS

As is generally found with square tipped aerofoils, an area of high suction at the trailing edge of the wing tip is very apparent at angles of

⁶*Applied Aerodynamics*, by L. Bairstow, pp. 109, 164, 168.

⁷R & M 60—1911-12. Investigation of the Pressure Distribution in a median section, over the upper and lower surfaces of three specially chosen aerofoils (p. 64).

incidence above about 8° for the monoplane and the upper wings of the biplane combinations. On the lower wings the high suction in this area is partially suppressed, the effect of positive stagger of the wings being to reduce this suction still further.

The load along the span decreases from the mid-span to the wing tip for the lower wings at all angles of incidence tested. On the upper wings and on the monoplane the load decreases from mid-span to the wing tip for angles of incidence up to about 8° . For higher angles of incidence the load decreases from mid-span to a section about 0.25 of the chord from the wing tip, and then increases suddenly between this section and the wing tip, due to the area of high suction at the trailing edge of the wing tip (see Plates 10 and 13).

The normal force coefficients K_z for the complete wings, are given in Table IV, where the ratio of the loading of the upper wing to the loading on the lower wing for each angle of incidence of the three biplane combinations, are given. With no stagger of the wings the proportion of the load carried by the upper wing increases, as the angle of incidence is increased until at 18° the load on the upper wing is 1.18 times the load on the lower wing. At low angles of incidence the loads on the two wings are nearly equal.

With 15° stagger the upper wing takes from 1.16 to 1.23 times the load taken by the lower wings.

With 30° stagger the load on the upper wing varies from 1.52 to 1.23 times the load on the lower wing, the ratio decreasing as the angle of incidence is increased.

To show the effect of biplane construction and stagger on the pressure distribution over the median section at high angles of incidence, the normal force diagrams for this section at 16° incidence are plotted to a large scale in Plate 11.

Lower Wing. The pressure distributions on the lower surface of the lower wings, and on the lower surface of the monoplane, are nearly coincident on the combined diagram from the leading edge to close to the trailing edge. The suctions on the upper surfaces of the lower wings are decreased by the presence of the upper wings, particularly in the region of high suction at the leading edge. The effect of stagger on the normal force coefficient of the median section is slight, although the maximum suction at the leading edge is decreased by increasing the positive stagger of the wings.

Upper Wing. The effect of the lower wing on the under surface of the upper wing is to reduce the pressures very materially, the pressures on the rear third of the upper wing being negative, whereas the pressure on the under surface of the monoplane is positive along practically the full chord. Positive stagger is advantageous in this respect, as the

distribution of pressure approaches more nearly that on the under surface of the monoplane as the upper wing is moved ahead of the lower wing. The suction on the upper surfaces of the upper wings are greater than on the upper surface of the monoplane. The effect of positive stagger is slight, except at the leading edge, where the suction increases considerably with positive stagger of the wings.

The increase in the value of the maximum suction near the leading edge of the upper surface of the upper wing, as the positive stagger is increased, is noticeable at all the angles of incidence tested (except at low angles where the pressure is positive for a short distance from the leading edge) and particularly at the high angles of incidence.

BIBLIOGRAPHY ON PRESSURE DISTRIBUTION OVER AEROFOILS

1. The Resistance of the Air and Aviation. G. Eiffel. Translated by Jerome C. Hunsaker.
2. B.A.C.A. 1911-12. R & M 60. Pressure Distribution over a Median Section of a Flat Plate and Two Aerofoils.
3. B.A.C.A. 1912-13. R & M 73. Pressure Distribution over the Entire Surface of an Aerofoil.
4. B.A.C.A. 1916-17. Vol. II. R & M 353. Pressure Distribution on the Wings of a Biplane of R.A.F. 15 Section with Raked Wing Tips.
5. B.A.C.A. 1916-17. Vol. II. R & M 287. Pressure Distribution over the Wing of an Aeroplane in Flight.
6. B.A.C.A. 1917-18. Vol. I. R & M 347. Pressure Distribution on Model F.E. 9 Wings.
7. B.A.C.A. 1917-18. Vol. I. R & M 355. The Distribution of Pressure on the Upper and Lower Wings of a Biplane. (R.A.F. 15).
8. B.A.C.A. 1919-20. Vol. I. R & M 661. Pressure Distribution over a Tail Plane.
9. B.A.C.A. 1920-21. Vol. I. R & M 709. Pressure Distribution over Aerofoil with Balanced Aileron. Square Horn Type.
10. B.A.C.A. 1923-24. Vol. I. R & M 891. Pressure Distribution over the Wings of a Model B.E. 2 C. Biplane with Raked Wing Tips. (R.A.F. 15).
11. B.A.C.A. 1923-24. Vol. II. R & M 894. Full Scale and Model Measurements of Pressure Distribution round Two Ribs of a B.E. 2 E. Aeroplane with R.A.F. 19 Section.
12. B.A.C.A. 1923-24. Vol. II. R & M 895. Full Scale and Model Measurement of Pressure Distribution round one rib of a B.E. 2 C. Aeroplane with R.A.F. 15 Section.
13. N.A.C.A. 1922. Report No. 150. Pressure Distribution over Thick Aerofoils—Model Tests.
14. N.A.C.A. 1923. Report. No. 161. The Distribution of Lift over Wing Tips and Ailerons.

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TABLE I

ORDINATES OF U.S.A. 27 SECTION

All dimensions expressed in per cent. of chord

Distance from Leading Edge	Lower Ordinate	Upper Ordinate
0.00	1.77	1.77
1.25	0.50	3.80
2.50	0.33	5.07
5.00	0.17	6.93
7.50	0.10	8.22
10.00	0.00	9.17
15.00	0.10	10.50
20.00	0.35	11.33
30.00	0.95	11.90
40.00	1.17	11.57
50.00	0.90	10.77
60.00	0.25	9.52
70.00	0.10	8.00
77.00	0.00	
80.00	0.05	6.03
90.00	0.15	3.65
100.00	0.67	0.67
Radius of Leading Edge		1.05
Radius of Trailing Edge		0.21

TABLE II

POSITIONS OF SECTIONS AND OBSERVATION POINTS

All distances given as multiples of the chord

Section	Distance from Wing Tip	Observation Point	Distance from Leading Edge
A	3.000 Mid-span	1 Upper	0.02
		1 Lower	0.04
B	2.030	2 Upper	0.06
		2 Lower	0.08
C	1.000	3 Upper	0.13
		3 Lower	0.15
D	.500	4	0.25
		5	0.40
E	.250	6	0.55
		7	0.70
F	.083	8	0.90

TABLE III
NORMAL FORCE COEFFICIENTS K_z AT EACH SECTION

Model 3"×18" Wind Speed—40 f.p.s.

MONOPLANE

Angle of Incidence	Section					
	A	B	C	D	E	F
0	.257	.217	.203	.173	.117	.085
2	.323	.329	.270	.209	.173	.108
4	.404	.387	.337	.280	.227	.149
8	.537	.529	.463	.411	.329	.309
12	.664	.646	.582	.516	.449	.450
16	.767	.778	.710	.600	.534	.582
18	.723	.760	.717	.664	.603	.690

BIPLANE—GAP = CHORD—Zero Stagger

UPPER WING

0	.203	.204	.176	.147	.107	.087
2	.287	.277	.241	.204	.160	.133
4	.337	.336	.297	.257	.213	.183
8	.483	.486	.419	.357	.330	.302
12	.591	.584	.507	.467	.403	.454
16	.708	.741	.654	.541	.493	.600
18	.786	.760	.663	.570	.564	.666

LOWER WING

0	.200	.195	.163	.149	.113	.085
2	.267	.257	.229	.184	.160	.129
4	.323	.316	.294	.267	.212	.183
8	.422	.417	.393	.330	.314	.260
12	.540	.512	.480	.450	.397	.354
16	.618	.626	.581	.528	.470	.461
18	.623	.615	.584	.555	.511	.510

BIPLANE—GAP = CHORD—15° Stagger

UPPER WING

Angle of Incidence	A	B	Section C	D	E	F
0	.231	.233	.187	.160	.133	.097
2	.322	.306	.257	.226	.184	.137
4	.370	.359	.320	.280	.260	.193
8	.523	.504	.444	.384	.334	.320
12	.674	.636	.574	.520	.429	.463
16	.780	.765	.666	.595	.534	.604
18	.790	.805	.720	.630	.577	.720

LOWER WING

0	.187	.173	.155	.140	.115	.066
2	.231	.220	.220	.213	.151	.116
4	.303	.306	.291	.249	.196	.147
8	.396	.400	.378	.338	.282	.237
12	.516	.501	.490	.433	.376	.331
16	.605	.573	.560	.497	.440	.400
18	.654	.623	.598	.533	.475	.493

BIPLANE—GAP = CHORD—30° Stagger

UPPER WING

0	.253	.253	.227	.163	.143	.087
2	.338	.338	.311	.253	.190	.167
4	.420	.421	.367	.316	.257	.225
8	.583	.566	.502	.417	.356	.344
12	.707	.707	.617	.518	.457	.507
16	.852	.818	.729	.654	.564	.640
18	.777	.774	.756	.680	.614	.730

LOWER WING

0	.160	.160	.141	.117	.091	.053
2	.227	.223	.211	.183	.137	.107
4	.303	.273	.263	.241	.187	.129
8	.383	.383	.370	.338	.260	.229
12	.485	.476	.463	.410	.375	.304
16	.595	.590	.546	.506	.440	.427
18	.640	.644	.610	.543	.484	.462

TABLE IV

NORMAL FORCE COEFFICIENTS K_z —COMPLETE WINGS

Model 3"×18" Wind Speed—40 f.p.s.

MONOPLANE

Incidence	K_z
0	0.199
2	0.280
4	0.343
8	0.475
12	0.592
16	0.712
18	0.713

BIPLANE—GAP = CHORD—No Stagger

Angle of Incidence	Upper Wing	Lower Wing	Biplane	$\frac{K_z \text{ Upper Wing}}{K_z \text{ Lower Wing}}$
0	.177	.174	.176	1.02
2	.245	.233	.239	1.05
4	.306	.291	.299	1.05
8	.431	.382	.407	1.12
12	.530	.480	.505	1.10
16	.664	.580	.622	1.14
18	.700	.593	.647	1.18

BIPLANE—GAP = CHORD + 15° Stagger

0	.197	.157	.177	1.25
2	.266	.208	.237	1.28
4	.323	.278	.301	1.16
8	.452	.368	.410	1.22
12	.581	.470	.526	1.23
16	.688	.545	.617	1.26
18	.735	.588	.662	1.25

BIPLANE—GAP = CHORD + 30° Stagger

0	.215	.141	.178	1.52
2	.297	.206	.252	1.44
4	.373	.254	.314	1.47
8	.502	.356	.429	1.41
12	.627	.442	.535	1.42
16	.744	.545	.645	1.36
18	.739	.599	.669	1.23

TABLE V

AERODYNAMIC CHARACTERISTICS

(DIRECT FORCE MEASUREMENTS)

Model 3"×18"—Duralumin—Wind Speed 40 f.p.s.

Corrected for spindle interference. Not corrected for channel interference

Incidence	Cy.	Cx.	L/D
-4	+ .0184	.0211	0.87
0	+ .1949	.0153	12.75
2	+ .2701	.0174	15.53
4	+ .3407	.0223	15.29
6	+ .4082	.0287	14.24
8	+ .4796	.0369	13.00
10	+ .5446	.0453	12.04
12	+ .6044	.0553	10.94
16	+ .6941	.0758	9.16
18	+ .6784	.0876	7.75

TABLE VI

Effect of Stagger—Median Section—16° Incidence

NORMAL FORCE COEFFICIENTS K_z

MONOPLANE = 0.767

BIPLANE—GAP = CHORD

Upper Wing	Lower Wing
0° Stagger = 0.708	0° Stagger = 0.618
15° Stagger = 0.780	15° Stagger = 0.605
30° Stagger = 0.852	30° Stagger = 0.595

Maximum observed Suctions on the Upper Surface, near the Leading Edge
Suctions in multiples of ρV^2 pds. per sq. ft.

MONOPLANE = 1.98

BIPLANE—GAP = CHORD

Upper Wing	Lower Wing
0° Stagger = 1.79	0° Stagger = 1.62
15° Stagger = 2.26	15° Stagger = 1.43
30° Stagger = 2.42	30° Stagger = 1.25

TABLE VII
PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL
Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

MONOPLANE

Section A—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.395	+.056	-.163	-.246	-.212	-.158	-.126	-.108
0	+.199	-.217	-.391	-.395	-.321	-.239	-.217	-.059
2	+.056	-.368	-.508	-.467	-.375	-.289	-.239	-.047
4	-.117	-.529	-.623	-.529	-.436	-.336	-.192	-.047
8	-.542	-.894	-.858	-.655	-.607	-.282	-.219	-.050
12	-1.050	-1.254	-1.080	-.876	-.488	-.321	-.217	-.050
16	-1.813	-1.565	-1.344	-.838	-.538	-.303	-.181	-.077
18	-1.983	-1.633	-1.245	-.718	-.402	-.226	-.187	-.174

Section A—Lower Surface

-4	-.544	-.544	-.323	-.097	-.023	-.036	-.036	-.014
0	-.167	-.095	-.054	.000	+.061	+.016	+.002	+.020
2	+.002	+.002	+.011	+.043	+.081	+.038	+.014	+.023
4	+.124	+.088	+.070	.088	.110	.056	.027	.027
8	+.312	.239	.181	.163	.165	.092	.056	.032
12	.431	.354	.273	.235	.217	.131	.081	.032
16	.483	.425	.339	.285	.250	.154	.092	.005
18	.485	.431	.343	.287	.250	.146	.072	-.047

Section B—Upper Surface

-4	+.397	+.050	-.145	-.267	-.214	-.167	-.131	-.110
0	+.201	-.217	-.359	-.409	-.316	-.241	-.212	-.056
2	+.070	-.345	-.407	-.461	-.375	-.291	-.230	-.043
4	-.106	-.544	-.603	-.551	-.434	-.345	-.192	-.047
8	-.538	-.899	-.831	-.680	-.601	-.285	-.208	-.045
12	1.027	-1.245	-1.041	-.890	-.465	-.318	-.205	-.043
16	-1.786	-1.563	-1.323	-.982	-.522	-.307	-.187	-.074
18	-2.015	-1.635	-1.267	-.833	-.472	-.271	-.199	-.151

Section B—Lower Surface

-4	-.544	-.544	-.264	-.081	-.009	-.036	-.034	-.009
0	-.154	-.136	-.052	.000	+.061	+.014	+.002	+.020
2	-.002	.000	+.011	+.045	.086	.036	.016	.029
4	+.128	+.092	.074	.086	.117	.056	.033	.029
8	.309	.237	.178	.163	.167	.090	.056	.034
12	.427	.350	.269	.230	.212	.124	.084	.038
16	.490	.427	.336	.280	.248	.151	.097	+.016
18	.499	.440	.354	.294	.257	.147	.086	-.016

PRESSURE DISTRIBUTION ON U.S.A. -27 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

MONOPLANE

Section C—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.379	+.041	-.185	-.253	-.208	-.172	-.126	-.106
0	+.212	-.192	-.379	-.381	-.296	-.235	-.196	-.045
2	+.144	-.303	-.472	-.438	-.341	-.278	-.217	-.036
4	-.063	-.465	-.574	-.492	-.381	-.321	-.185	-.036
8	-.416	-.777	-.772	-.603	-.529	-.267	-.185	-.032
12	-.856	-1.104	-.958	-.780	-.434	-.300	-.192	-.034
16	-1.371	-1.407	-1.183	-.874	-.470	-.305	-.174	-.050
18	-1.689	-1.518	-1.256	-.804	-.479	-.294	-.172	-.086

Section C—Lower Surface

-4	-.540	-.544	-.226	-.074	-.014	-.045	-.032	-.004
0	-.203	-.108	-.070	-.009	+.047	-.009	-.004	+.020
2	-.034	-.023	-.011	+.032	.065	+.018	+.009	.025
4	+.074	+.052	+.036	.063	.090	.032	.018	.025
8	.267	.199	.138	.136	.133	.063	.043	.032
12	.390	.312	.221	.194	.178	.097	.063	.032
16	.463	.390	.298	.250	.217	.124	.086	.025
18	.483	.420	.318	.267	.228	.131	.088	.009

Section D—Upper Surface

-4	+.381	+.043	-.176	-.246	-.199	-.158	-.117	-.097
0	+.246	-.154	-.332	-.345	-.267	-.210	-.172	-.041
2	+.140	-.271	-.416	-.395	-.300	-.241	-.196	-.034
4	+.025	-.386	-.499	-.443	-.339	-.289	-.167	-.038
8	-.280	-.657	-.673	-.540	-.454	-.248	-.178	-.050
12	-.632	-.930	-.838	-.661	-.438	-.285	-.196	-.054
16	-1.036	-1.192	-.987	-.802	-.420	-.312	-.205	-.059
18	-1.222	-1.278	-1.054	-.847	-.434	-.314	-.205	-.068

Section D—Lower Surface

-4	-.497	-.497	-.241	-.068	-.006	-.020	-.025	.000
0	-.246	-.154	-.077	-.014	+.038	+.004	-.009	+.016
2	-.081	-.052	-.032	+.014	.045	.018	-.002	.016
4	+.029	+.020	+.016	.043	.065	.034	+.007	.018
8	.212	.154	.106	.104	.101	.059	.023	.018
12	.343	.267	.187	.163	.142	.088	.045	.023
16	.479	.352	.257	.214	.176	.115	.063	.023
18	.456	.377	.285	.235	.192	.124	.074	.016

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL
Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

MONOPLANE

Section E—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.370	+.038	+.287	-.230	-.183	-.147	-.115	-.088
0	+.267	-.122	-.285	-.300	-.232	-.187	-.154	-.038
2	+.176	-.223	-.363	-.348	-.264	-.219	-.165	-.036
4	+.077	-.318	-.425	-.381	-.296	-.241	-.160	-.052
8	-.163	-.526	-.558	-.454	-.372	-.239	-.181	-.092
12	-.452	-.765	-.696	-.551	-.413	-.273	-.228	-.122
16	-.763	-.991	-.829	-.668	-.388	-.318	-.267	-.142
18	-.917	-1.070	-.890	-.723	-.402	-.336	-.282	-.151

Section E—Lower Surface

	1	2	3	4	5	6	7	8
-4	-.490	-.492	-.208	-.059	-.014	-.023	-.029	-.002
0	-.287	-.201	-.081	-.018	+.020	-.007	-.025	.000
2	-.133	-.083	-.054	-.004	.034	+.007	-.018	.004
4	-.032	-.027	-.020	+.016	.043	.011	-.016	-.011
8	+.142	+.090	+.056	.065	.072	.032	-.002	-.007
12	.273	.192	.126	.110	.119	.070	+.011	-.009
16	.372	.280	.192	.158	.126	.077	.025	-.007
18	.413	.318	.214	.174	.140	.088	.036	-.002

Section F—Upper Surface

	1	2	3	4	5	6	7	8
-4	+.359	+.027	-.136	-.181	-.145	-.122	-.104	-.050
0	+.259	-.097	-.226	-.232	-.183	-.145	-.128	-.079
2	+.196	-.163	-.271	-.262	-.219	-.172	-.158	-.131
4	+.119	-.244	-.325	-.303	-.239	-.194	-.187	-.201
8	-.059	-.395	-.420	-.377	-.294	-.253	-.312	-.388
12	-.269	-.565	-.533	-.463	-.343	-.352	-.529	-.563
16	-.490	-.723	-.641	-.526	-.411	-.506	-.786	-.679
18	-.596	-.788	-.693	-.551	-.449	-.598	-.896	-.709

Section F—Lower Surface

	1	2	3	4	5	6	7	8
-4	-.503	-.470	-.095	-.050	-.002	-.020	-.029	-.007
0	-.259	-.174	-.077	-.029	+.007	-.018	-.034	-.023
2	-.147	-.090	-.065	-.023	.009	-.016	-.036	-.032
4	-.068	-.050	-.045	-.009	.014	-.014	-.038	-.041
8	+.061	+.027	-.002	+.011	.014	-.018	-.045	-.054
12	.174	.101	+.045	.032	.023	-.016	-.047	-.068
16	.259	.169	.081	.047	.025	-.018	-.052	-.077
18	.294	.196	.101	.056	.027	-.014	-.047	-.072

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—Zero Stagger

Upper Wing—Section A—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.366	+.023	-.199	-.278	-.239	-.192	-.151	-.131
0	+.194	-.237	-.418	-.425	-.352	-.280	-.244	-.092
2	+.054	-.401	-.553	-.512	-.420	-.334	-.289	-.092
4	-.090	-.551	-.659	-.574	-.476	-.395	-.248	-.097
8	-.463	-.892	-.896	-.698	-.623	-.336	-.269	-.099
12	-.914	-1.263	-1.143	-.878	-.589	-.399	-.296	-.108
16	-1.418	-1.612	-1.364	-1.143	-.616	-.436	-.294	-.115
18	-1.773	-1.791	-1.511	-1.045	-.641	-.429	-.280	-.126

Upper Wing—Section A—Lower Surface

-4	-.596	-.601	-.217	-.101	-.046	-.073	-.074	-.038
0	-.255	-.174	-.126	-.074	-.009	-.056	-.063	-.025
2	-.099	-.089	-.081	-.038	+.005	-.036	-.056	-.022
4	+.008	-.022	-.041	-.011	-.017	-.037	-.055	-.026
8	.183	+.055	.040	+.032	.043	-.026	-.054	-.036
12	.312	.210	.116	.081	.072	-.010	-.045	-.045
16	.409	.300	.186	.126	.099	+.004	-.041	-.062
18	.446	.344	.222	.156	.117	+.017	-.036	-.073

Upper Wing—Section B—Upper Surface

-4	+.388	+.018	-.181	-.300	-.241	-.194	-.154	-.131
0	+.208	-.239	-.390	-.447	-.352	-.278	-.241	-.090
2	+.074	-.381	-.499	-.522	-.407	-.327	-.282	-.088
4	-.079	-.556	-.625	-.596	-.470	-.388	-.241	-.097
8	-.438	-.867	-.845	-.720	-.625	-.357	-.267	-.101
12	-.874	-1.245	-1.088	-.910	-.592	-.386	-.282	-.106
16	-1.391	-1.608	-1.328	-1.181	-.612	-.429	-.289	-.113
18	-1.712	-1.748	-1.481	-1.136	-.630	-.420	-.276	-.126

Upper Wing—Section B—Lower Surface

-4	-.589	-.589	-.183	-.090	-.032	-.072	-.070	-.034
0	-.246	-.169	-.126	-.070	-.007	-.059	-.063	-.023
2	-.099	-.086	-.079	-.038	+.007	-.045	-.054	-.018
4	+.009	-.018	-.034	-.011	.025	-.038	-.052	-.023
8	.181	+.101	+.041	+.032	.045	-.029	-.052	-.034
12	.309	.205	.115	.077	.072	-.016	-.045	-.043
16	.449	.294	.176	.122	.097	-.004	-.041	-.059
18	.443	.341	.214	.149	.117	+.007	-.038	-.072

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—Zero Stagger

Upper Wing—Section C—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.366	+.014	-.212	-.280	-.232	-.192	-.145	-.124
0	+.219	-.190	-.397	-.407	-.323	-.262	-.217	-.077
2	+.099	-.330	-.501	-.470	-.368	-.305	-.248	-.070
4	-.050	-.488	-.614	-.538	-.422	-.359	-.226	-.059
8	-.357	-.786	-.813	-.650	-.553	-.318	-.239	-.086
12	-.723	-1.084	-.994	-.781	-.567	-.352	-.248	-.090
16	-1.156	-1.405	-1.196	-1.021	-.531	-.393	-.267	-.104
18	-1.373	-1.540	-1.278	-1.079	-.560	-.397	-.262	-.110

Upper Wing—Section C—Lower Surface

-4	-.562	-.565	-.185	-.097	-.038	-.083	-.063	-.025
0	-.289	-.167	-.136	-.072	-.011	-.072	-.059	-.016
2	-.122	-.099	-.092	-.038	+.002	-.050	-.047	-.014
4	-.025	-.043	-.059	-.018	.011	-.047	-.047	-.016
8	+.138	+.065	+.007	+.018	.029	-.043	-.050	-.029
12	.273	.174	.074	.056	.052	-.036	-.050	-.041
16	.372	.262	.142	.099	.077	-.023	-.043	-.050
18	.413	.303	.174	.122	.088	-.016	-.043	-.061

Upper Wing—Section D—Upper Surface

-4	+.375	+.025	-.196	-.264	-.217	-.178	-.131	-.065
0	+.237	-.172	-.357	-.372	-.289	-.232	-.190	-.063
2	+.133	-.289	-.445	-.427	-.325	-.267	-.217	-.063
4	+.023	-.402	-.524	-.472	-.363	-.312	-.203	-.070
8	-.235	-.650	-.689	-.567	-.472	-.296	-.208	-.083
12	-.558	-.923	-.860	-.682	-.553	-.318	-.237	-.101
16	-.919	-1.190	-1.025	-.854	-.463	-.361	-.259	-.113
18	-1.129	-1.341	-1.113	-.964	-.497	-.384	-.273	-.122

Upper Wing—Section D—Lower Surface

-4	-.551	-.551	-.192	-.083	-.032	-.054	-.054	-.020
0	-.300	-.199	-.128	-.068	-.007	-.043	-.052	-.016
2	-.149	-.113	-.095	-.045	-.009	-.038	-.052	-.018
4	-.050	-.054	-.061	-.023	+.004	-.032	-.050	-.018
8	+.108	+.047	+.007	+.014	.020	-.025	-.052	-.029
12	.241	.151	.070	.050	.041	-.016	-.047	-.038
16	.348	.241	.136	.095	.065	-.002	-.043	-.047
18	.390	.282	.167	.115	.079	+.004	-.038	-.054

PRESSURE DISTRIBUTION ON U.S.A.—27 AREOFOIL
Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—Zero Stagger

Upper Wing—Section E—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.368	+.025	-.176	-.237	-.190	-.154	-.117	-.095
0	+.248	-.142	-.300	-.316	-.246	-.201	-.167	-.054
2	+.167	-.244	-.377	-.366	-.280	-.237	-.187	-.056
4	+.074	-.339	-.452	-.411	-.321	-.267	-.185	-.079
8	-.145	-.535	-.583	-.485	-.397	-.273	-.210	-.119
12	-.404	-.756	-.725	-.580	-.465	-.303	-.257	-.151
16	-.718	-1.009	-.869	-.700	-.445	-.361	-.309	-.187
18	-.876	-1.108	-.935	-.756	-.456	-.393	-.327	-.205

Upper Wing—Section E—Lower Surface

-4	-.517	-.522	-.167	-.072	-.029	-.047	-.056	-.020
0	-.316	-.221	-.124	-.059	-.007	-.043	-.059	-.023
2	-.167	-.122	-.092	-.045	-.009	-.036	-.059	-.032
4	-.070	-.068	-.065	-.027	-.002	-.038	-.061	-.038
8	+.065	+.018	-.014	.000	+.009	-.034	-.063	-.050
12	.201	.113	+.045	+.034	.025	-.023	-.061	-.056
16	.303	.196	.101	.068	.045	-.014	-.056	-.065
18	.352	.237	.131	.088	.056	-.004	-.050	-.068

Upper Wing—Section F—Upper Surface

-4	+.341	+.004	-.163	-.205	-.160	-.133	-.115	-.061
0	+.241	-.126	-.257	-.259	-.214	-.167	-.145	-.088
2	+.169	-.205	-.307	-.296	-.246	-.194	-.174	-.138
4	+.095	-.282	-.363	-.336	-.269	-.221	-.205	-.199
8	-.072	-.434	-.461	-.418	-.327	-.276	-.316	-.368
12	-.257	-.596	-.571	-.515	-.395	-.381	-.508	-.540
16	-.526	-.768	-.700	-.612	-.465	-.547	-.741	-.675
18	-.696	-.856	-.761	-.650	-.512	-.637	-.863	-.734

Upper Wing—Section F—Lower Surface

-4	-.538	-.483	-.099	-.068	-.023	-.041	-.050	-.023
0	-.282	-.185	-.104	-.054	-.016	-.047	-.061	-.041
2	-.176	-.099	-.097	-.050	-.018	-.047	-.063	-.052
4	-.108	-.088	-.079	-.038	-.018	-.050	-.068	-.063
8	+.009	-.025	-.045	-.018	-.020	-.052	-.074	-.077
12	.113	+.045	-.011	-.002	-.016	-.054	-.081	-.092
16	.205	.108	+.023	+.018	-.011	-.050	-.079	-.104
18	.248	.142	.045	.027	-.009	-.047	-.079	-.108

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—Zero Stagger

Lower Wing—Section A—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.390	+.041	-.192	-.280	-.248	-.190	-.149	-.133
0	+.237	-.160	-.332	-.350	-.285	-.210	-.181	-.063
2	+.128	-.259	-.404	-.381	-.303	-.226	-.199	-.027
4	+.007	-.377	-.476	-.411	-.327	-.250	-.172	-.023
8	-.307	-.632	-.632	-.515	-.413	-.196	-.145	-.009
12	-.716	-.921	-.786	-.596	.332	-.210	-.140	.000
16	-1.269	-1.138	-.980	-.592	-.357	-.203	-.117	+.002
18	-1.617	-1.319	-.985	-.565	-.341	-.174	-.088	-.016

Lower Wing—Section A—Lower Surface

-4	-.544	-.549	-.273	-.086	-.014	-.031	-.029	-.002
0	-.230	-.126	-.059	-.002	+.016	+.023	-.007	+.029
2	-.036	-.017	+.002	+.038	.082	.044	+.025	.040
4	+.088	+.070	.061	.081	.116	.065	.042	.049
8	.278	.217	.168	.160	.171	.106	.073	.060
12	.403	.333	.259	.228	.221	.146	.106	.073
16	.472	.418	.336	.287	.264	.183	.125	.080
18	.492	.447	.366	.307	.282	.194	.138	.067

Lower Wing—Section B—Upper Surface

-4	+.393	+.029	-.174	-.298	-.241	-.194	-.151	-.128
0	+.244	-.156	-.300	-.366	-.278	-.214	-.178	-.056
2	+.133	-.273	-.381	-.402	-.303	-.232	-.199	-.025
4	+.007	-.386	-.454	-.434	-.330	-.264	-.165	-.018
8	-.296	-.643	-.621	-.501	-.416	-.201	-.140	-.007
12	-.689	-.908	-.756	-.619	-.321	-.214	-.133	+.002
16	-1.296	-1.156	-.948	-.643	-.341	-.208	-.113	+.011
18	-1.578	-1.303	-.941	-.592	-.325	-.178	-.088	+.007

Lower Wing—Section B—Lower Surface

-4	-.542	-.547	-.253	-.077	-.009	-.032	-.032	.000
0	-.192	-.115	-.056	.000	+.061	+.014	+.007	+.025
4	-.034	-.014	+.007	+.041	.081	.041	.027	.043
2	+.092	+.072	.065	.086	.117	.061	.043	.047
8	.278	.219	.172	.163	.172	.104	.074	.061
12	.404	.332	.259	.232	.221	.147	.108	.074
16	.472	.413	.336	.289	.267	.178	.133	.079
18	.494	.443	.363	.312	.280	.187	.140	.068

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—Zero Stagger

Lower Wing—Section C—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.375	+.025	-.212	-.285	-.237	-.194	-.149	-.122
0	+.241	-.145	-.334	-.348	-.273	-.217	-.176	-.050
2	+.142	-.253	-.397	-.379	-.294	-.232	-.190	-.027
4	+.023	-.366	-.479	-.418	-.323	-.255	-.174	-.023
8	-.255	-.598	-.614	-.485	-.393	-.219	-.147	-.016
12	-.623	-.858	-1.196	-.578	-.345	-.223	-.145	-.004
16	-1.034	-1.093	-.899	-.700	-.341	-.226	-.128	+.002
18	-1.359	-1.205	-.976	-.596	-.345	-.212	-.113	-.004

Lower Wing—Section C—Lower Surface

-4	-.542	-.547	-.235	-.081	-.014	-.045	-.029	.000
0	-.262	-.131	-.077	-.009	+.047	-.009	-.004	+.025
2	-.063	-.036	-.018	+.029	.065	+.023	+.014	.034
4	+.061	+.045	+.034	.068	.095	.041	.032	.041
8	.237	.181	.133	.136	.140	.108	.059	.052
12	.370	.296	.219	.194	.140	.104	.081	.056
16	.454	.384	.294	.253	.228	.142	.110	.063
18	.476	.418	.330	.282	.246	.160	.122	.065

Lower Wing—Section D—Upper Surface

-4	+.381	+.029	-.144	-.269	-.221	-.176	-.133	-.110
0	+.267	-.117	-.298	-.323	-.246	-.192	-.156	-.050
2	+.174	-.217	-.359	-.350	-.267	-.212	-.176	-.027
4	+.074	-.314	-.429	-.386	-.289	-.239	-.165	-.023
8	-.172	-.520	-.549	-.436	-.350	-.217	-.140	-.023
12	-.476	-.745	-.675	-.520	-.372	-.210	-.145	-.023
16	-.833	-.978	-.795	-.680	-.312	-.226	-.149	-.023
18	-1.009	-1.041	-.849	-.664	-.318	-.226	-.142	-.023

Lower Wing—Section D—Lower Surface

-4	-.512	-.512	-.239	-.068	-.011	-.023	-.025	.000
0	-.278	-.187	-.077	-.016	+.034	.000	-.014	+.011
2	-.106	-.063	-.036	+.009	.047	+.023	.000	.025
4	+.011	+.011	+.011	+.043	.070	.036	+.011	.027
8	.192	.140	.101	.104	.108	.063	.032	.032
12	.330	.255	.185	.163	.151	.097	.059	.043
16	.427	.348	.257	.219	.190	.128	.083	.047
18	.456	.384	.287	.241	.212	.147	.097	.050

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—Zero Stagger

Lower Wing—Section E—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.375	+.034	-.172	-.239	-.196	-.156	-.117	-.092
0	+.273	-.108	-.273	-.298	-.223	-.178	-.147	-.043
2	+.201	-.192	-.327	-.323	-.246	-.199	-.160	-.032
4	+.110	-.276	-.372	-.350	-.262	-.214	-.151	-.036
8	-.095	-.445	-.488	-.399	-.314	-.214	-.156	-.056
12	-.348	-.646	-.601	-.467	-.372	-.214	-.174	-.070
16	-.628	-.818	-.691	-.553	-.318	-.250	-.201	-.081
18	-.781	-.914	-.729	-.596	-.327	-.262	-.205	-.086

Lower Wing—Section E—Lower Surface

-4	-.497	-.499	-.205	-.061	-.014	-.023	-.032	-.002
0	-.309	-.244	-.074	-.020	+.020	-.007	-.025	+.004
2	-.142	-.086	-.052	-.002	.036	+.011	-.011	.007
4	-.043	-.032	-.018	+.020	.050	.018	-.009	.004
8	+.131	+.086	+.059	.068	.079	.041	+.007	.009
12	.267	.192	.128	.117	.110	.065	.027	.016
16	.381	.291	.203	.169	.145	.095	.052	.025
18	.409	.323	.232	.190	.160	.108	.063	.026

Lower Wing—Section F—Upper Surface

-4	+.352	+.018	-.147	-.147	-.158	-.131	-.113	-.054
0	+.264	-.083	-.212	-.221	-.187	-.145	-.124	-.074
2	+.210	-.147	-.244	-.239	-.208	-.151	-.138	-.099
4	+.142	-.208	-.285	-.269	-.226	-.167	-.160	-.145
8	-.016	-.339	-.359	-.318	-.239	-.203	-.212	-.232
12	-.208	-.481	-.445	-.379	-.289	-.250	-.327	-.341
16	-.422	-.625	-.531	-.420	-.339	-.343	-.461	-.416
18	-.531	-.691	-.583	-.436	-.357	-.397	-.524	-.416

Lower Wing—Section F—Lower Surface

-4	-.520	-.483	-.101	-.059	-.004	-.025	-.032	-.009
0	-.273	-.190	-.074	-.029	+.007	-.016	-.029	-.014
2	-.158	-.095	-.065	-.020	.014	-.011	-.032	-.020
4	-.072	-.050	-.041	-.004	.020	-.007	-.029	-.027
8	+.063	+.032	+.006	+.018	.025	-.004	-.032	-.034
12	.174	.106	.052	.041	.034	-.002	-.029	-.038
16	.271	.183	.099	.065	.041	+.004	-.023	-.034
18	.305	.210	.117	.074	.045	.009	-.020	-.032

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE+15° Stagger

Upper Wing—Section A—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.368	+.011	-.208	-.282	-.239	-.190	-.154	-.131
0	+.158	-.278	-.449	-.443	-.366	-.289	-.255	-.099
2	+.009	-.479	-.574	-.515	-.429	-.345	-.287	-.092
4	-.167	-.621	-.702	-.589	-.492	-.402	-.253	-.101
8	-.598	-1.005	-.969	-.745	-.691	-.366	-.291	-.119
12	-1.118	-1.407	-1.227	-.964	-.583	-.429	-.316	-.128
16	-1.771	-1.782	-1.497	-1.086	-.675	-.447	-.307	-.145
18	-2.260	-1.904	-1.644	-1.005	-.671	-.420	-.280	-.169

Upper Wing—Section A—Lower Surface

-4	-.540	-.551	-.169	-.077	-.025	-.062	-.068	-.040
0	-.142	-.103	-.077	-.029	+.016	-.033	-.052	-.024
2	-.013	-.026	-.031	.000	.032	-.026	-.054	-.028
4	+.088	+.042	+.013	+.025	.045	-.025	-.056	-.035
8	-.255	-.171	+.099	.074	.072	-.013	-.052	-.050
12	.383	.289	.181	.131	.103	+.002	-.050	-.064
16	.448	.363	.244	.172	.126	.011	-.053	-.092
18	.468	.396	.276	.192	.139	.016	-.059	-.119

Upper Wing—Section B—Upper Surface

-4	+.368	+.007	-.178	-.294	-.237	-.192	-.151	-.133
0	+.160	-.280	-.418	-.465	-.366	-.287	-.253	-.092
2	+.020	-.431	-.535	-.538	-.420	-.381	-.273	-.088
4	+.002	-.616	-.666	-.619	-.488	-.395	-.246	-.099
8	-.553	-.960	-.901	-.754	-.659	-.390	-.272	-.101
12	-1.036	-1.343	-1.156	-.971	-.560	-.409	-.296	-.119
16	-1.781	-1.736	-1.441	-1.143	-.641	-.434	-.296	-.133
18	-2.169	-1.868	-1.572	-1.054	-.652	-.413	-.271	-.158

Upper Wing—Section B—Lower Surface

-4	-.521	-.517	-.131	-.061	-.009	-.056	-.061	-.034
0	-.133	-.097	-.065	-.023	+.023	-.032	-.047	-.020
2	-.009	-.018	-.023	+.004	.034	-.027	-.047	-.025
4	+.095	+.047	+.020	.029	.047	-.025	-.052	-.032
8	.253	.172	.104	.077	.077	-.016	-.052	-.047
12	.381	.282	.169	.124	.104	-.002	-.050	-.061
16	.454	.359	.246	.174	.128	+.007	-.050	-.086
18	.479	.434	.267	.187	.133	.004	-.059	-.110

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE+15° Stagger

Upper Wing—Section C—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.357	+.004	-.214	-.278	-.228	-.187	-.133	-.122
0	+.183	-.235	-.411	-.411	-.323	-.262	-.217	-.081
2	+.047	-.381	-.522	-.481	-.379	-.312	-.248	-.079
4	-.097	-.524	-.619	-.538	-.427	-.366	-.230	-.088
8	-.445	-.833	-.842	-.664	-.578	-.323	-.246	-.097
12	-.867	-1.174	-1.039	-.829	-.526	-.375	-.267	-.106
16	-1.310	-1.472	-1.238	-1.061	-.558	-.404	-.276	-.122
18	-1.612	-1.640	-1.380	-.958	-.585	-.407	-.271	-.131

Upper Wing—Section C—Lower Surface

-4	-.531	-.533	-.097	-.014	-.020	-.070	-.059	-.027
0	-.174	-.115	-.092	-.036	+.007	-.052	-.052	-.023
2	-.043	-.043	-.047	-.007	.023	-.041	-.047	-.020
4	+.050	+.014	-.011	+.014	.032	-.041	-.050	-.027
8	.212	.133	+.061	.054	.052	-.036	-.054	-.043
12	.384	.237	.133	.099	.077	-.029	-.052	-.054
16	.413	.316	.199	.140	.097	-.023	-.054	-.077
18	.456	.359	.228	.158	.108	-.020	-.056	-.092

Upper Wing—Section D—Upper surface

-4	+.368	+.014	-.203	-.271	-.221	-.178	-.138	-.115
0	+.210	-.201	-.372	-.384	-.300	-.241	-.205	-.070
2	+.108	-.314	-.461	-.438	-.339	-.280	-.228	-.072
4	-.020	-.445	-.553	-.490	-.381	-.327	-.201	-.079
8	-.312	-.714	-.732	-.594	-.501	-.298	-.221	-.095
12	-.668	-1.002	-.908	-.725	-.510	-.336	-.253	-.113
16	-1.068	-1.276	-1.079	-.919	-.488	-.381	-.278	-.128
18	-1.287	-1.414	-1.167	-1.000	-.515	-.397	-.287	-.136

Upper Wing—Section D—Lower Surface

-4	-.517	-.515	-.156	-.065	-.018	-.047	-.056	-.025
0	-.230	-.149	-.101	-.384	+.002	-.038	-.054	-.023
2	-.092	-.070	-.061	-.018	.009	-.032	-.052	-.023
4	+.004	-.014	-.027	.000	.018	-.029	-.054	-.027
8	.172	+.101	+.045	+.038	.032	-.023	-.059	-.045
12	.294	.203	.115	.081	.056	-.016	-.059	-.056
16	.390	.287	.176	.119	.077	-.004	-.056	-.070
18	.434	.330	.205	.142	.088	.000	-.054	-.079

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s

BIPLANE+15° Stagger

Upper Wing—Section E—Upper Surface

Incidence degs.	Pressure Opening															
	1		2		3		4		5		6		7		8	
-4	+	.370	+	.025	-	.178	-	.241	-	.190	-	.156	-	.124	-	.097
0	+	.239	-	.156	-	.321	-	.334	-	.255	-	.212	-	.176	-	.059
2	+	.147	-	.262	-	.393	-	.375	-	.300	-	.250	-	.196	-	.065
4	+	.041	-	.366	-	.470	-	.422	-	.334	-	.278	-	.192	-	.088
8	-	.199	-	.585	-	.616	-	.508	-	.425	-	.285	-	.228	-	.133
12	-	.497	-	.836	-	.772	-	.619	-	.472	-	.327	-	.278	-	.169
16	-	.809	-	1.059	-	.919	-	.743	-	.465	-	.388	-	.332	-	.208
18	-	.987	-	1.176	-	1.002	-	.820	-	.492	-	.420	-	.352	-	.226

Upper Wing—Section E—Lower Surface

-4	-	.488	-	.488	-	.145	-	.054	-	.018	-	.041	-	.052	-	.020
0	-	.253	-	.158	-	.095	-	.041	+	.004	-	.032	-	.054	-	.025
2	-	.126	-	.090	-	.070	-	.025	-	.007	-	.029	-	.056	-	.034
4	-	.036	-	.041	-	.041	-	.007	-	.014	-	.027	-	.059	-	.043
8	+	.122	+	.061	+	.020	+	.023	-	.025	-	.023	-	.061	-	.056
12	-	.244	-	.156	-	.081	-	.059	-	.043	-	.014	-	.059	-	.065
16	-	.341	-	.237	-	.138	-	.095	-	.059	-	.004	-	.054	-	.074
18	-	.388	-	.278	-	.165	-	.113	-	.070	+	.002	-	.052	-	.079

Upper Wing—Section F—Upper Surface

-4	+	.341	-	.009	-	.154	-	.196	-	.163	-	.136	-	.117	-	.063
0	+	.223	-	.126	-	.246	-	.250	-	.214	-	.169	-	.154	-	.099
2	+	.163	-	.210	-	.303	-	.296	-	.248	-	.199	-	.187	-	.158
4	+	.088	-	.285	-	.357	-	.336	-	.267	-	.226	-	.219	-	.221
8	-	.097	-	.447	-	.465	-	.420	-	.334	-	.280	-	.339	-	.404
12	-	.309	-	.666	-	.587	-	.520	-	.395	-	.390	-	.531	-	.576
16	-	.547	-	.801	-	.720	-	.603	-	.476	-	.562	-	.790	-	.736
18	-	.684	-	.896	-	.792	-	.646	-	.531	-	.659	-	.923	-	.806

Upper Wing—Section F—Lower Surface

-4	-	.494	-	.452	-	.090	-	.054	-	.016	-	.034	-	.047	-	.025
0	-	.241	-	.154	-	.095	-	.045	-	.007	-	.038	-	.061	-	.045
2	-	.142	-	.097	-	.077	-	.032	-	.007	-	.034	-	.061	-	.056
4	-	.074	-	.063	-	.061	-	.023	-	.009	-	.041	-	.068	-	.068
8	+	.045	+	.004	-	.025	-	.007	-	.011	-	.045	-	.077	-	.088
12	-	.147	-	.070	+	.009	+	.009	-	.009	-	.052	-	.086	-	.106
16	-	.237	-	.138	-	.045	-	.027	-	.007	-	.052	-	.088	-	.119
18	-	.276	-	.167	-	.065	-	.036	-	.002	-	.047	-	.083	-	.117

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE +15° Stagger

Lower Wing—Section A—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.386	+.034	-.194	-.273	-.230	-.176	-.142	-.122
0	+.244	-.147	-.314	-.334	-.271	-.203	-.172	-.077
2	+.163	-.237	-.375	-.361	-.291	-.217	-.192	-.038
4	+.045	-.339	-.452	-.397	-.318	-.241	-.196	-.025
8	-.232	-.571	-.589	-.449	-.384	-.235	-.149	-.016
12	-.587	-.818	-.727	-.529	-.404	-.210	-.151	-.007
16	-1.000	-1.059	-.878	-.680	-.348	-.221	-.140	+.004
18	-1.434	-1.167	-.994	-.580	-.379	-.226	-.131	+.004

Lower Wing—Section A—Lower Surface

-4	-.535	-.540	-.259	-.077	-.009	-.028	-.027	+.001
0	-.241	-.163	-.058	+.002	+.060	+.016	-.007	+.029
2	-.059	-.028	-.004	+.034	.080	.043	+.026	.040
4	+.064	+.053	+.052	.079	.116	.069	.045	.052
8	.249	.196	.155	.149	.167	.105	.079	.065
12	.389	.323	.254	.223	.216	.145	.106	.076
16	.467	.410	.326	.282	.261	.182	.136	.082
18	.484	.439	.358	.309	.278	.194	.146	.079

Lower Wing—Section B—Upper Surface

-4	+.386	+.025	-.174	-.294	-.237	-.183	-.147	-.119
0	+.255	-.140	-.287	-.352	-.271	-.208	-.169	-.065
2	+.167	-.235	-.348	-.381	-.291	-.226	-.190	-.029
4	+.047	-.348	-.427	-.427	-.314	-.244	-.190	-.018
8	-.244	-.592	-.567	-.481	-.381	-.239	-.145	-.014
12	-.565	-.811	-.716	-.574	-.411	-.219	-.151	-.002
16	-1.025	-1.052	-.869	-.723	-.343	-.228	-.138	-.007
18	-1.434	-1.163	-.969	-.646	-.370	-.228	-.126	-.002

Lower Wing—Section B—Lower Surface

-4	-.540	-.544	-.246	-.072	-.007	-.029	-.025	+.002
0	-.246	-.147	-.101	+.002	+.014	+.011	+.004	.029
2	-.061	-.029	-.002	.036	+.036	.038	.025	.038
4	+.063	+.050	+.052	.077	.110	.059	.041	.047
8	.248	.194	.154	.151	.160	.095	.070	.059
12	.379	.309	.244	.219	.210	.136	.099	.070
16	.461	.397	.321	.276	.255	.169	.128	.079
18	.481	.427	.350	.298	.269	.185	.142	.077

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL
Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE +15° Stagger

Lower Wing—Section C—Upper Surface

Incidence degs.	Pressure Opening															
	1	2	3	4	5	6	7	8								
-4	+	.361	+	.014	-	.217	-	.285	-	.232	-	.185	-	.145	-	.113
0	+	.248	-	.131	-	.323	-	.341	-	.264	-	.208	-	.167	-	.056
2	+	.160	-	.235	-	.431	-	.372	-	.289	-	.230	-	.190	-	.029
4	-	.059	-	.325	-	.447	-	.402	-	.312	-	.250	-	.187	-	.025
8	-	.199	-	.547	-	.587	-	.470	-	.377	-	.257	-	.149	-	.018
12	-	.508	-	.765	-	.705	-	.535	-	.438	-	.228	-	.156	-	.011
16	-	.896	-	1.018	-	.842	-	.702	-	.348	-	.246	-	.151	-	.007
18	-	1.140	-	1.113	-	.937	-	.662	-	.370	-	.246	-	.145	-	.011

Lower Wing—Section C—Lower Surface

-4	-	.542	-	.547	-	.212	-	.074	-	.011	-	.043	-	.027	+	.004
0	-	.280	-	.149	-	.079	-	.011	+	.043	-	.011	-	.004	-	.025
2	-	.088	-	.047	-	.025	+	.025	+	.068	+	.020	+	.018	-	.034
4	+	.038	+	.032	+	.027	+	.061	+	.092	+	.038	+	.029	-	.041
8	-	.223	-	.169	-	.124	-	.128	-	.140	-	.074	-	.052	-	.050
12	-	.354	-	.282	-	.205	-	.190	-	.183	-	.108	-	.083	-	.061
16	-	.445	-	.372	-	.285	-	.246	-	.221	-	.140	-	.110	-	.065
18	-	.470	-	.407	-	.316	-	.273	-	.241	-	.158	-	.122	-	.068

Lower Wing—Section D—Upper Surface

-4	+	.381	+	.034	-	.192	-	.259	-	.210	-	.167	-	.101	-	.101
0	+	.273	-	.106	-	.287	-	.309	-	.237	-	.187	-	.147	-	.056
2	+	.196	-	.199	-	.350	-	.343	-	.257	-	.205	-	.174	-	.029
4	+	.099	-	.285	-	.404	-	.372	-	.280	-	.228	-	.174	-	.025
8	-	.128	-	.476	-	.520	-	.422	-	.334	-	.244	-	.136	-	.025
12	-	.404	-	.687	-	.641	-	.422	-	.409	-	.210	-	.147	-	.025
16	-	.738	-	.908	-	.750	-	.619	-	.316	-	.235	-	.156	-	.025
18	-	.926	-	1.007	-	.822	-	.711	-	.332	-	.246	-	.158	-	.029

Lower Wing—Section D—Lower Surface

-4	-	.517	-	.517	-	.221	-	.063	-	.009	-	.025	-	.025	+	.002
0	-	.291	-	.203	-	.074	-	.016	+	.032	-	.000	-	.014	-	.014
2	-	.117	-	.068	-	.041	+	.007	+	.045	+	.020	+	.000	-	.020
4	-	.002	+	.002	+	.007	+	.038	+	.068	+	.036	+	.009	-	.027
8	+	.174	-	.126	-	.092	-	.097	-	.104	-	.063	-	.032	-	.032
12	-	.312	-	.241	-	.174	-	.156	-	.145	-	.095	-	.054	-	.038
16	-	.407	-	.332	-	.250	-	.212	-	.185	-	.124	-	.081	-	.045
18	-	.454	-	.377	-	.280	-	.235	-	.201	-	.140	-	.092	-	.047

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL
Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE +15° Stagger

Lower Wing—Section E—Upper Surface

Incidence degs.	Pressure Opening															
	1	2	3	4	5	6	7	8								
-4	+	.379	+	.034	-	.172	-	.235	-	.183	-	.147	-	.110	-	.086
0	+	.282	-	.088	-	.255	-	.276	-	.208	-	.165	-	.136	-	.043
2	+	.208	-	.167	-	.305	-	.300	-	.223	-	.181	-	.151	-	.032
4	+	.140	-	.244	-	.352	-	.325	-	.244	-	.201	-	.147	-	.034
8	-	.061	-	.413	-	.456	-	.372	-	.294	-	.212	-	.142	-	.054
12	-	.294	-	.589	-	.551	-	.425	-	.352	-	.201	-	.163	-	.068
16	-	.571	-	.781	-	.657	-	.524	-	.303	-	.235	-	.192	-	.079
18	-	.716	-	.865	-	.714	-	.587	-	.309	-	.253	-	.210	-	.088

Lower Wing—Section E—Lower Surface

-4	-	.490	-	.492	-	.196	-	.059	-	.011	-	.023	-	.029	+	.002
0	-	.316	-	.253	-	.070	-	.018	+	.025	-	.002	-	.020	-	.011
2	-	.151	-	.088	-	.057	-	.004	+	.036	+	.011	-	.011	-	.011
4	-	.047	-	.032	-	.020	+	.018	+	.050	+	.020	-	.007	-	.009
8	+	.117	+	.074	+	.052	+	.068	+	.079	+	.041	+	.009	+	.011
12	+	.253	+	.183	+	.124	+	.115	+	.108	+	.065	+	.027	+	.018
16	+	.352	+	.269	+	.190	+	.158	+	.138	+	.088	+	.045	+	.025
18	+	.399	+	.314	+	.219	+	.181	+	.154	+	.101	+	.059	+	.027

Lower Wing—Section F—Upper Surface

-4	+	.345	+	.018	-	.142	-	.181	-	.154	-	.126	-	.106	-	.050
0	+	.264	-	.083	-	.208	-	.214	-	.178	-	.140	-	.119	-	.070
2	+	.219	-	.136	-	.244	-	.237	-	.196	-	.145	-	.136	-	.095
4	+	.151	-	.199	-	.278	-	.259	-	.214	-	.163	-	.156	-	.136
8	-	.002	-	.325	-	.350	-	.314	-	.232	-	.192	-	.203	-	.208
12	-	.002	-	.458	-	.431	-	.375	-	.282	-	.232	-	.296	-	.305
16	-	.386	-	.598	-	.517	-	.427	-	.334	-	.327	-	.422	-	.388
18	-	.488	-	.657	-	.560	-	.434	-	.363	-	.386	-	.492	-	.420

Lower Wing—Section F—Lower Surface

-4	-	.506	-	.472	-	.097	-	.054	+	.002	-	.020	-	.029	-	.007
0	-	.287	-	.201	-	.072	-	.027	+	.009	-	.016	-	.029	-	.014
2	-	.167	-	.099	-	.068	-	.023	+	.011	-	.011	-	.032	-	.020
4	-	.079	-	.057	-	.045	-	.009	+	.018	-	.009	-	.029	-	.025
8	+	.052	+	.025	+	.002	+	.018	+	.029	-	.004	-	.029	-	.032
12	+	.156	+	.092	+	.041	+	.036	+	.029	-	.007	-	.034	-	.041
16	+	.259	+	.169	+	.092	+	.063	+	.038	+	.002	-	.025	-	.032
18	+	.296	+	.201	+	.108	+	.068	+	.041	+	.004	-	.020	-	.036

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL
 Pressures expressed in terms of ρV^2 Air Speed 40f.p.s.

BIPLANE +30° Stagger

Upper Wing—Section A—Upper Surface

Incidence degs.	Pressure Opening															
	1	2	3	4	5	6	7	8								
-4	+	.368	+	.014	-	.208	-	.273	-	.239	-	.190	-	.154	-	.133
0	+	.133	-	.291	-	.458	-	.438	-	.370	-	.289	-	.262	-	.095
2	-	.027	-	.463	-	.592	-	.517	-	.436	-	.348	-	.271	-	.095
4	-	.219	-	.648	-	.720	-	.587	-	.506	-	.438	-	.255	-	.104
8	-	.734	-	1.059	-	1.005	-	.756	-	.700	-	.372	-	.298	-	.124
12	-	1.276	-	1.483	-	1.271	-	1.023	-	.601	-	.440	-	.323	-	.140
16	-	2.191	-	1.859	-	1.612	-	1.005	-	.698	-	.447	-	.307	-	.172
18	-	2.420	-	1.986	-	1.527	-	.946	-	.589	-	.366	-	.289	-	.262

Upper Wing—Section A—Lower Surface

-4	-	.497	-	.501	-	.138	-	.047	+	.003	-	.033	-	.043	-	.026
0	-	.090	-	.063	-	.041	+	.002	+	.043	-	.003	-	.031	-	.016
2	+	.043	+	.020	+	.014	+	.034		.065	+	.002	-	.028	-	.022
4	-	.146	-	.095	-	.060		.061		.080	+	.007	-	.033	-	.034
8	-	.305	-	.230	-	.154		.128		.121		.027	-	.025	-	.046
12	-	.420	-	.336	-	.236		.183		.154		.044	-	.023	-	.065
16	-	.470	-	.409	-	.299		.223		.177		.050	-	.032	-	.108
18	-	.476	-	.420	-	.308		.223		.172		.032	-	.068	-	.172

Upper Wing—Section B—Upper Surface

-4	+	.377	+	.007	-	.187	-	.303	-	.244	-	.194	-	.156	-	.133
0	+	.142	-	.300	-	.427	-	.470	-	.370	-	.291	-	.259	-	.092
2	-	.020	-	.465	-	.553	-	.547	-	.429	-	.348	-	.264	-	.092
4	-	.196	-	.641	-	.675	-	.623	-	.501	-	.390	-	.246	-	.101
8	-	.666	-	1.050	-	.960	-	.795	-	.689	-	.368	-	.291	-	.117
12	-	1.236	-	1.452	-	1.226	-	1.057	-	.578	-	.425	-	.309	-	.126
16	-	2.128	-	1.828	-	1.545	-	1.073	-	.666	-	.443	-	.294	-	.163
18	-	2.359	-	1.913	-	1.472	-	1.018	-	.637	-	.411	-	.298	-	.235

Upper Wing—Section B—Lower Surface

-4	-	.499	-	.494	-	.124	-	.047	+	.004	-	.038	-	.047	-	.027
0	-	.097	-	.061	-	.038	+	.004		.038	-	.011	-	.034	-	.018
2	+	.038	+	.023	+	.018	+	.038		.065	+	.002	-	.029	-	.020
4	-	.145	-	.097	-	.065		.068		.081		.007	-	.029	-	.029
8	-	.305	-	.223	-	.151		.126		.113		.020	-	.029	-	.047
12	-	.418	-	.330	-	.235		.181		.147		.036	-	.025	-	.065
16	-	.483	-	.407	-	.294		.223		.169		.043	-	.032	-	.104
18	-	.483	-	.416	-	.305		.232		.167		.027	-	.063	-	.149

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE +30° Stagger

Upper Wing—Section C—Upper Surface

Incidence degs.	1	2	3	Pressure Opening 4	5	6	7	8
-4	+.348	.000	-.217	-.280	-.232	-.187	-.147	-.126
0	+.167	-.235	-.420	-.416	-.327	-.267	-.230	-.077
2	+.020	-.407	-.535	-.486	-.386	-.321	-.244	-.077
4	-.140	-.556	-.650	-.556	-.449	-.375	-.228	-.088
8	-.531	-.901	-.874	-.684	-.614	-.332	-.255	-.101
12	-.980	-1.242	-1.088	-.881	-.512	-.384	-.276	-.113
16	-1.549	-1.581	-1.314	-.967	-.580	-.409	-.280	-.136
18	-1.907	-1.725	-1.425	-.942	-.607	-.409	-.276	-.160

Upper Wing—Section C—Lower Surface

-4	-.508	-.508	-.136	-.056	-.002	-.050	-.043	-.023
0	-.126	-.077	-.054	-.002	+.034	-.025	-.032	-.014
2	+.007	-.002	-.002	+.025	.052	-.014	-.029	-.016
4	.101	+.059	+.029	.047	.065	-.011	-.029	-.023
8	.271	.190	.117	.104	.095	+.002	-.027	-.036
12	.388	.296	.190	.754	.119	.009	-.029	-.059
16	.456	.372	.255	.194	.142	.020	-.029	-.081
18	.483	.407	.282	.212	.151	.018	-.041	-.110

Upper Wing—Section D—Upper Surface

-4	+.368	+.016	-.201	-.267	-.217	-.174	-.131	-.108
0	+.203	-.208	-.377	-.386	-.294	-.235	-.185	-.063
2	+.092	-.327	-.467	-.440	-.341	-.280	-.226	-.070
4	-.043	-.454	-.567	-.497	-.386	-.330	-.201	-.079
8	-.354	-.734	-.747	-.603	-.510	-.298	-.226	-.101
12	-.743	-1.050	-.939	-.754	-.492	-.348	-.257	-.117
16	-1.170	-1.343	-1.118	-.969	-.510	-.397	-.289	-.140
18	-1.432	-1.477	-1.220	-.941	-.547	-.416	-.303	-.154

Upper Wing—Section D—Lower Surface

-4	-.497	-.490	-.138	-.045	.000	-.025	-.032	-.014
0	-.178	-.108	-.074	-.018	+.025	-.014	-.036	-.018
2	-.056	-.041	-.034	+.007	.032	-.007	-.036	-.020
4	+.047	+.027	+.009	.029	.045	-.004	-.038	-.027
8	.217	.149	.088	.077	.063	+.007	-.038	-.041
12	.339	.246	.158	.122	.092	.018	-.034	-.056
16	.434	.336	.223	.167	.115	.029	-.034	-.072
18	.467	.372	.253	.187	.123	.036	-.034	-.081

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL.

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE +30° Stagger

Upper Wing—Section E—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.372	+.025	-.176	-.241	-.187	-.154	-.119	-.097
0	+.232	-.160	-.321	-.332	-.257	-.214	-.183	-.061
2	+.133	-.267	-.397	-.377	-.287	-.244	-.192	-.065
4	+.027	-.379	-.476	-.425	-.339	-.280	-.196	-.092
8	-.226	-.614	-.623	-.515	-.429	-.287	-.239	-.147
12	-.526	-.854	-.795	-.637	-.472	-.341	-.300	-.187
16	-.890	-1.113	-.932	-.761	-.470	-.402	-.348	-.223
18	-1.066	-1.229	-1.030	-.836	-.503	-.436	-.377	-.241

Upper Wing—Section E—Lower Surface

-4	-.472	-.474	-.136	-.041	-.004	-.023	-.038	-.014
0	-.223	-.133	-.081	-.025	+.020	-.016	-.043	-.023
2	-.099	-.068	-.050	-.007	.023	-.011	-.045	-.036
4	-.007	-.016	-.020	+.011	.027	-.011	-.050	-.045
8	-.154	+.092	+.045	.045	.043	-.004	-.050	-.056
12	.276	.185	.108	.083	.063	+.004	-.050	-.068
16	.379	.273	.172	.124	.083	.016	-.043	-.077
18	.416	.307	.196	.142	.092	.020	-.043	-.081

Upper Wing—Section F—Upper Surface

-4	+.345	+.007	-.154	-.196	-.163	-.136	-.117	-.063
0	+.235	-.131	-.248	-.257	-.212	-.167	-.153	-.108
2	+.156	-.219	-.307	-.300	-.248	-.199	-.187	-.167
4	+.077	-.294	-.361	-.341	-.271	-.235	-.223	-.241
8	-.115	-.461	-.479	-.431	-.343	-.289	-.361	-.436
12	-.345	-.655	-.607	-.538	+.402	-.411	-.587	-.643
16	-.596	-.829	-.732	-.614	-.501	-.580	-.836	-.815
18	-.720	-.919	-.813	-.659	-.556	-.687	-.967	-.872

Upper Wing—Section F—Lower Surface

-4	-.481	-.443	-.083	-.045	-.014	-.032	-.043	-.023
0	-.221	-.133	-.083	-.034	-.004	-.034	-.052	-.045
2	-.131	-.088	-.070	-.025	-.007	-.036	-.061	-.063
4	-.056	-.050	-.050	-.016	-.002	-.036	-.068	-.072
8	+.070	+.025	-.011	+.002	-.002	-.043	-.081	-.097
12	.174	.092	+.027	.016	+.002	-.047	-.086	-.115
16	.264	.160	.063	.032	-.002	-.054	-.092	-.128
18	.296	.185	.081	.038	+.002	-.054	-.092	-.128

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL

Pressures expressed in terms of ρV^2 Air speed 40 f.p.s.

BIPLANE +30° Stagger

Lower Wing—Section A—Upper Surface

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.390	+.052	-.169	-.246	-.214	-.163	-.126	-.108
0	+.278	-.095	-.278	-.305	-.253	-.190	-.158	-.099
2	+.194	-.187	-.343	-.341	-.280	-.208	-.181	-.056
4	+.097	-.296	-.420	-.377	-.307	-.230	-.203	-.029
8	-.156	-.503	-.560	-.440	-.372	-.273	-.156	-.025
12	-.463	-.738	-.693	-.508	-.465	-.221	-.163	-.016
16	-.908	-1.021	-.858	-.659	-.368	-.250	-.165	-.009
18	+1.249	-1.163	-.991	-.716	-.420	-.267	-.163	-.016

Lower Wing—Section A—Lower Surface

-4	-.542	-.549	-.282	-.086	-.015	-.029	-.031	-.004
0	-.294	-.236	-.052	-.002	+.055	+.010	+.004	+.025
2	-.090	-.047	-.017	+.025	.072	.038	.020	+.037
4	+.033	+.033	+.036	+.063	.104	.059	.037	.045
8	.218	.174	.138	.142	.157	.100	.067	.058
12	.354	.290	.228	.205	.203	.133	.097	.070
16	.443	.386	.307	.269	.253	.172	.130	.081
18	.465	.418	.341	.296	.271	.189	.139	.076

Lower Wing—Section B—Upper Surface

-4	+.386	+.034	-.156	-.271	-.214	-.169	-.128	-.108
0	+.269	-.124	-.273	-.334	-.257	-.196	-.160	-.088
2	+.199	-.203	-.325	-.325	-.282	-.217	-.181	-.045
4	+.101	-.294	-.386	-.402	-.303	-.232	-.201	-.023
8	-.151	-.512	-.540	-.479	-.370	-.273	-.154	-.018
12	-.461	-.741	-.673	-.549	-.467	-.226	-.158	-.009
16	-.869	-1.009	-.838	-.696	-.366	-.250	-.160	-.004
18	-1.242	-1.156	-.971	-.770	-.407	-.264	-.158	-.009

Lower Wing—Section B—Lower Surface

-4	-.533	-.529	-.223	-.068	-.004	-.029	-.029	.000
0	-.285	-.185	-.056	.000	+.009	+.007	+.002	+.025
2	-.079	-.038	-.011	+.027	.029	.034	.020	.036
4	+.036	+.034	+.041	.068	.104	.056	.036	.043
8	.219	.174	.131	.133	.151	.088	.061	.054
12	.357	.291	.230	.208	.203	.128	.097	.070
16	.438	.377	.303	.267	.246	.163	.124	.077
18	.472	.418	.341	.291	.264	.178	.133	.074

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE+30° Stagger

Lower Wing—Section C—Upper Surface

Incidence degs.	Pressure Opening															
	1	2	3	4	5	6	7	8								
-4	+	.377	+	.036	-	.192	-	.259	-	.208	-	.172	-	.124	-	.101
0	+	.278	-	.088	-	.285	-	.309	-	.244	-	.194	-	.154	-	.079
2	+	.196	-	.183	-	.348	-	.348	-	.269	-	.212	-	.172	-	.041
4	+	.099	-	.285	-	.411	-	.381	-	.296	-	.235	-	.194	-	.027
8	-	.124	-	.479	-	.544	-	.452	-	.354	-	.282	-	.158	-	.020
12	-	.418	-	.707	-	.673	-	.520	-	.443	-	.230	-	.163	-	.018
16	-	.777	-	.948	-	.818	-	.646	-	.363	-	.257	-	.165	-	.016
18	-	.987	-	1.063	-	.896	-	.734	-	.381	-	.269	-	.165	-	.020

Lower Wing—Section C—Lower Surface

-4	-	.538	-	.542	-	.226	-	.074	-	.011	-	.043	-	.027	+	.004
0	-	.303	-	.190	-	.077	-	.014	+	.041	-	.014	-	.004	-	.023
2	-	.113	-	.059	-	.034	+	.018	-	.061	+	.014	+	.009	-	.029
4	+	.014	+	.016	+	.016	-	.054	-	.086	-	.036	-	.027	-	.038
8	-	.194	-	.149	-	.108	-	.119	-	.128	-	.068	-	.047	-	.045
12	-	.330	-	.263	-	.194	-	.178	-	.174	-	.101	-	.074	-	.054
16	-	.422	-	.350	-	.271	-	.237	-	.217	-	.138	-	.106	-	.065
18	-	.465	-	.397	-	.309	-	.267	-	.237	-	.154	-	.117	-	.065

Lower Wing—Section D—Upper Surface

-4	+	.377	+	.032	-	.183	-	.246	-	.203	-	.160	-	.117	-	.095
0	+	.276	-	.095	-	.276	-	.300	-	.230	-	.178	-	.145	-	.059
2	+	.205	-	.181	-	.332	-	.332	-	.250	-	.199	-	.165	-	.029
4	+	.119	-	.273	-	.397	-	.368	-	.273	-	.221	-	.174	-	.025
8	-	.104	-	.456	-	.517	-	.431	-	.339	-	.271	-	.145	-	.029
12	-	.370	-	.668	-	.634	-	.492	-	.425	-	.219	-	.156	-	.034
16	-	.673	-	.865	-	.747	-	.603	-	.343	-	.246	-	.167	-	.034
18	-	.858	-	.989	-	.820	-	.693	-	.345	-	.262	-	.172	-	.036

Lower Wing—Section D—Lower Surface

4 -	-	.510	-	.508	-	.232	-	.068	-	.009	-	.023	-	.025	-	.000
0	-	.309	-	.230	-	.072	-	.014	+	.034	-	.000	-	.011	+	.016
2	-	.142	-	.081	-	.050	-	.000	-	.047	+	.016	-	.002	-	.018
4	-	.025	-	.011	-	.002	+	.034	-	.065	-	.034	+	.011	-	.029
8	+	.156	+	.115	+	.083	-	.092	-	.101	-	.059	-	.029	-	.032
12	-	.296	-	.228	-	.165	-	.149	-	.138	-	.088	-	.050	-	.034
16	-	.397	-	.323	-	.239	-	.205	-	.185	-	.124	-	.079	-	.045
18	-	.485	-	.363	-	.273	-	.230	-	.199	-	.136	-	.090	-	.047

PRESSURE DISTRIBUTION ON U.S.A.—27 AEROFOIL
 Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE +30° Stagger

Lower Wing—Section E—Upper Surface

Incidence degs.	Pressure Opening															
	1		2		3		4		5		6		7		8	
-4	+	.366	+	.038	-	.154	-	.217	-	.169	-	.133	-	.104	-	.081
0	+	.285	-	.072	-	.235	-	.257	-	.196	-	.156	-	.126	-	.052
2	+	.214	-	.145	-	.278	-	.278	-	.210	-	.172	-	.145	-	.034
4	+	.154	-	.219	-	.330	-	.307	-	.232	-	.192	-	.149	-	.036
8	-	.034	-	.384	-	.431	-	.359	-	.280	-	.219	-	.145	-	.056
12	-	.253	-	.547	-	.524	-	.411	-	.348	-	.205	-	.174	-	.079
16	-	.497	-	.716	-	.623	-	.492	-	.318	-	.237	-	.199	-	.088
18	-	.650	-	.822	-	.687	-	.560	-	.314	-	.260	-	.217	-	.095

Lower Wing—Section E—Lower Surface

-4	-	.501	-	.503	-	.196	-	.059	-	.009	-	.023	-	.029	-	.000
0	-	.316	-	.267	-	.068	-	.018	+	.025	-	.004	-	.020	+	.011
2	-	.167	-	.097	-	.061	-	.009	-	.038	+	.009	-	.011	-	.011
4	-	.056	-	.038	-	.025	+	.016	-	.050	-	.018	-	.007	-	.009
8	+	.097	+	.063	+	.041	-	.059	-	.074	-	.036	+	.004	-	.009
12	+	.237	+	.169	+	.113	-	.106	-	.101	-	.059	-	.023	-	.016
16	+	.350	+	.262	+	.178	-	.151	-	.133	-	.086	-	.043	-	.020
18	+	.388	+	.300	+	.212	-	.174	-	.149	-	.099	-	.054	-	.025

Lower Wing—Section F—Upper Surface

-4	+	.352	+	.020	-	.138	-	.176	-	.145	-	.117	-	.099	-	.047
0	-	.271	-	.072	-	.192	-	.201	-	.167	-	.133	-	.113	-	.088
2	+	.217	-	.131	-	.230	-	.223	-	.185	-	.142	-	.131	-	.092
4	+	.151	-	.196	-	.264	-	.248	-	.205	-	.156	-	.151	-	.128
8	+	.018	-	.309	-	.339	-	.303	-	.228	-	.196	-	.201	-	.190
12	-	.160	-	.443	-	.416	-	.366	-	.262	-	.230	-	.267	-	.267
16	-	.357	-	.578	-	.510	-	.436	-	.316	-	.303	-	.384	-	.359
18	-	.512	-	.652	-	.560	-	.454	-	.352	-	.363	-	.465	-	.407

Lower Wing—Section F—Lower Surface

-4	-	.508	-	.476	-	.097	-	.054	-	.000	-	.020	-	.029	-	.004
0	-	.296	-	.214	-	.072	-	.029	+	.007	-	.018	-	.029	-	.014
2	-	.172	-	.101	-	.068	-	.023	-	.014	-	.011	-	.029	-	.018
4	-	.092	-	.061	-	.050	-	.011	-	.016	-	.009	-	.032	-	.027
8	+	.036	+	.014	-	.004	+	.011	-	.023	-	.009	-	.034	-	.038
12	+	.149	+	.088	+	.041	-	.032	-	.025	-	.009	-	.036	-	.041
16	+	.239	+	.158	+	.081	-	.052	-	.032	-	.004	-	.032	-	.038
18	+	.282	+	.187	+	.099	-	.063	-	.034	-	.002	-	.027	-	.036

PLATE 1 PRESSURE DISTRIBUTION OVER U.S.A.-27 AEROFOIL

WITH SQUARE WING TIPS

MODEL - 3×18 - BRASS

SCALES - FIFTY TIMES FULL SIZE

FULL SIZE

- METHOD OF INSERTING TUBES
1. TUBES BROKE IN UNDER SURFACE
 2. TUBE IN PLACE IN BRASS
 3. TUBE SOLDERED IN PLACE BEFORE REMOVAL
- 1/2 CUT TO SHAPE

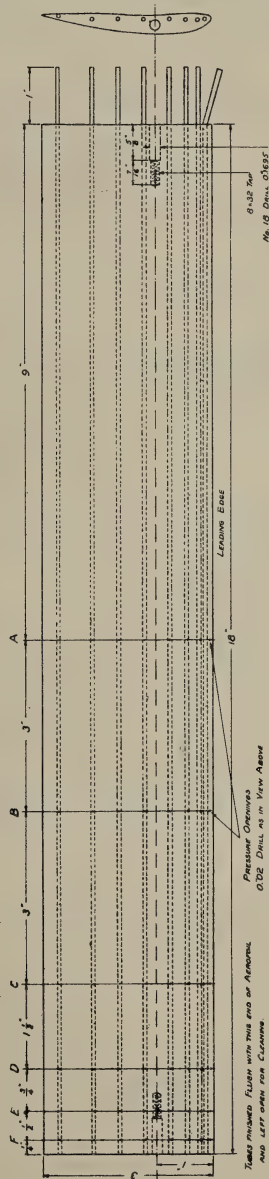
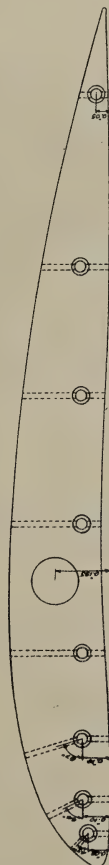


PLATE 2 PRESSURE DISTRIBUTION OVER U.S.A-27 AEROFOIL
WITH SQUARE WING TIPS
NORMAL FORCE DIAGRAMS

MODEL 3"x18" BRASS UNIT OF PRESSURE = $9V^2$ PDS. PER SQ. FT. WIND SPEED - 40 FT PER SEC.
ANGLE OF INCIDENCE = -4°

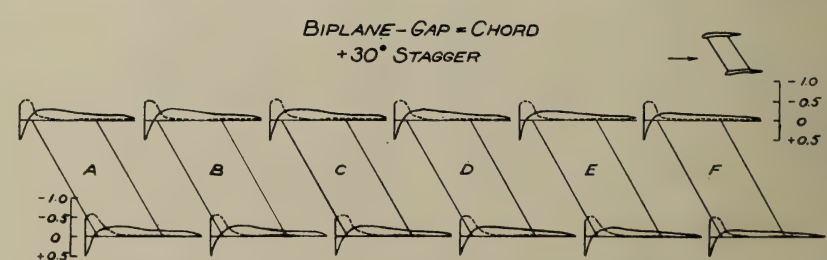
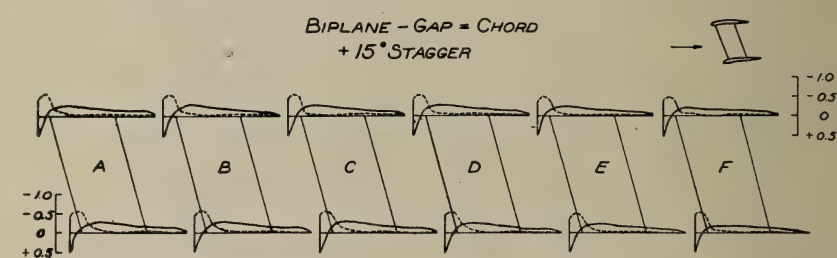
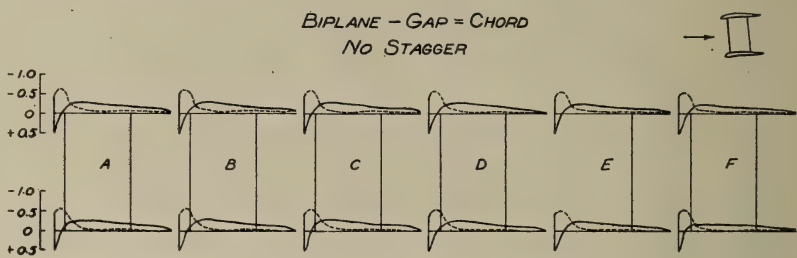
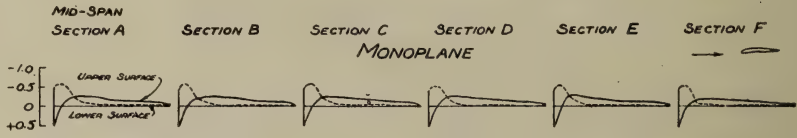
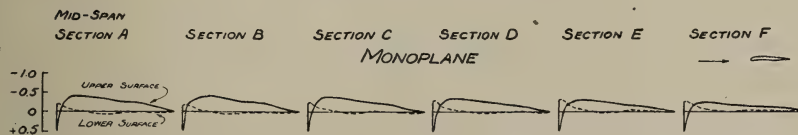


PLATE 3

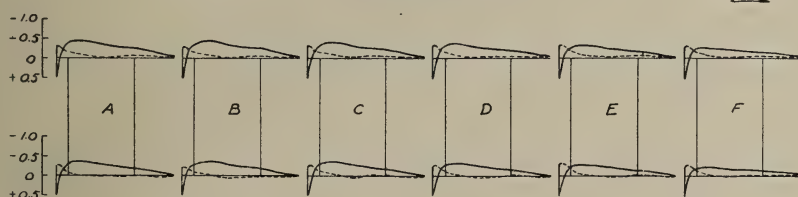
PRESSURE DISTRIBUTION OVER U.S.A-27 AEROFOIL
WITH SQUARE WING TIPS
NORMAL FORCE DIAGRAMS

MODEL 3" x 18" BRASS UNIT OF PRESSURE = QV^2 PDS. PER SQ. FT. WIND SPEED - 40 FT. PER SEC

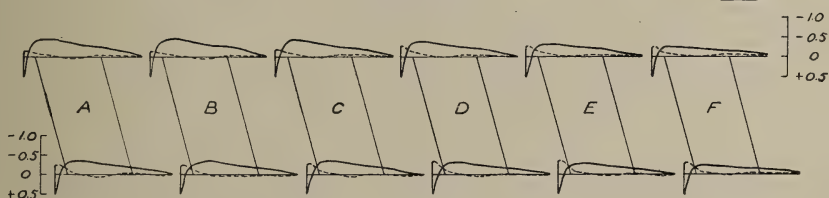
ANGLE OF INCIDENCE = 0°



BIPLANE - GAP = CHORD
NO STAGGER



BIPLANE - GAP = CHORD
+ 15° STAGGER



BIPLANE - GAP = CHORD
+ 30° STAGGER

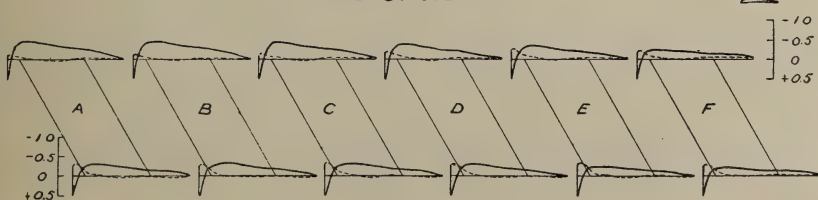


PLATE 4 PRESSURE DISTRIBUTION OVER U.S.A-27 AEROFOIL
WITH SQUARE WING TIPS
NORMAL FORCE DIAGRAMS
MODEL 3"x18" BRASS UNIT OF PRESSURE = $9V^2$ PDS. PER SQ. FT. WIND SPEED - 40 FT. PER SEC.
ANGLE OF INCIDENCE = $+2^\circ$

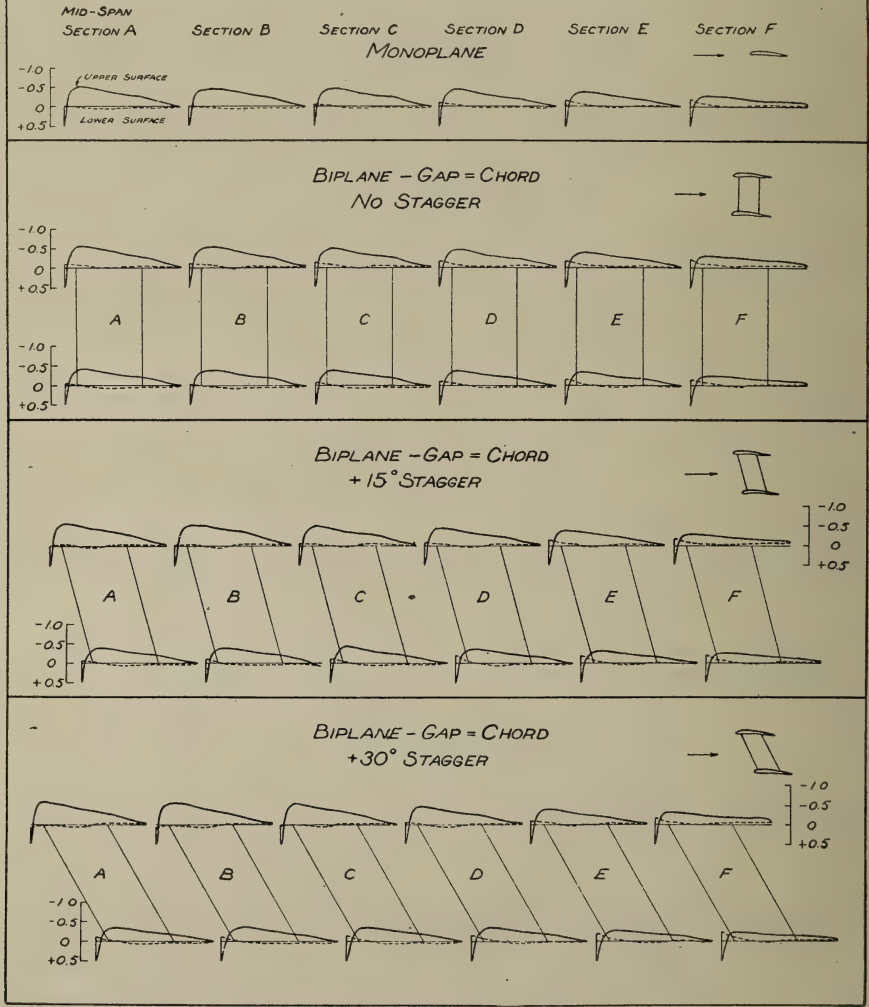
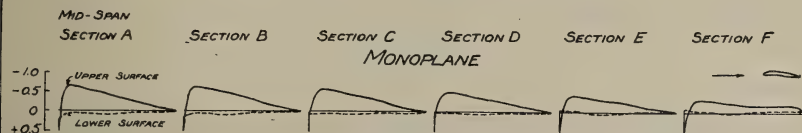


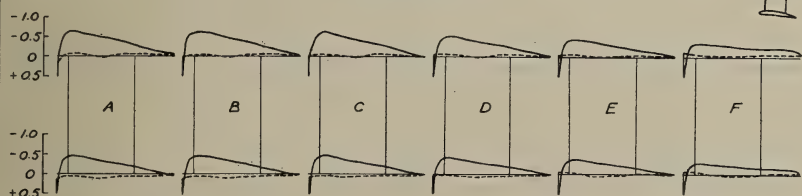
PLATE 5

PRESSURE DISTRIBUTION OVER U.S.A-27 AEROFOIL
WITH SQUARE WING TIPS

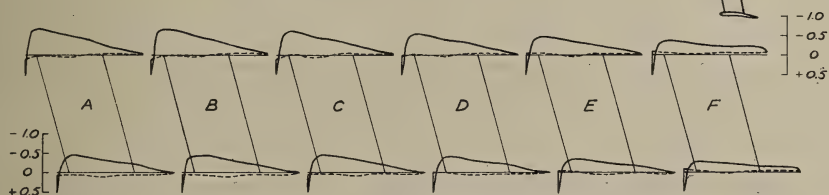
NORMAL FORCE DIAGRAMS

MODEL 3" x 18" BRASS UNIT OF PRESSURE = qV^2 PDS. PER SQ. FT. WIND SPEED - 40 FT. PER SEC.ANGLE OF INCIDENCE = $+4^\circ$ 

BIPLANE - GAP = CHORD
NO STAGGER



BIPLANE - GAP = CHORD
+ 15° STAGGER



BIPLANE - GAP = CHORD
+ 30° STAGGER

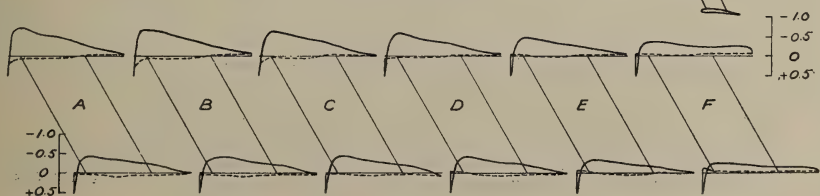


PLATE 6

PRESSURE DISTRIBUTION OVER U.S.A-27 AEROFOIL

WITH SQUARE WING TIPS

NORMAL FORCE DIAGRAMS

MODEL 3"x18" BRASS UNIT OF PRESSURE = RV^2 PDS. PER SQ. FT. WIND SPEED - 40 FT. PER SEC.

ANGLE OF INCIDENCE = +8

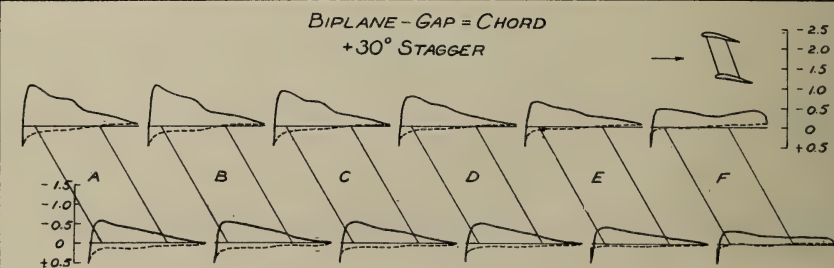
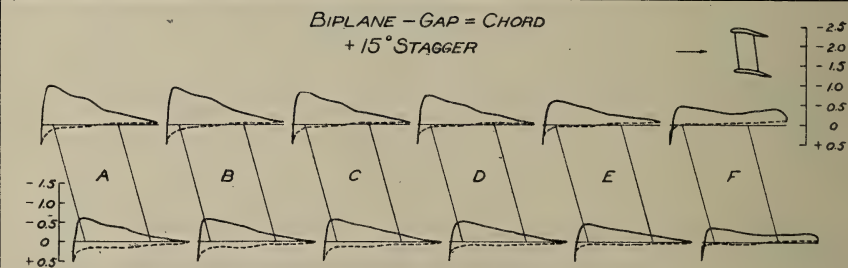
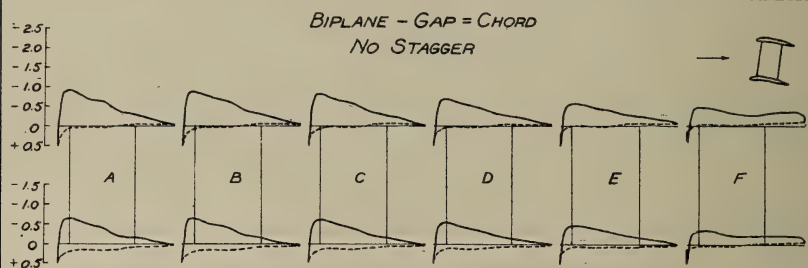
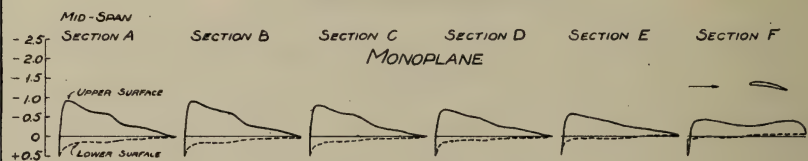


PLATE 7

PRESSURE DISTRIBUTION OVER U.S.A.-27 AEROFOIL

WITH SQUARE WING TIPS

NORMAL FORCE DIAGRAMS

MODEL 3" x 18" BRASS

UNIT OF PRESSURE = ρV^2 POS PER SQ. FT.

WIND SPEED - 40 FT. PER SEC.

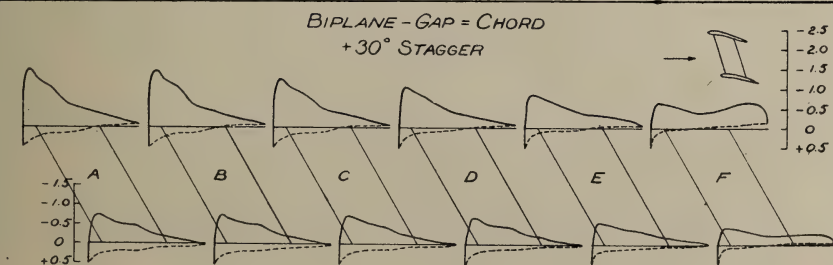
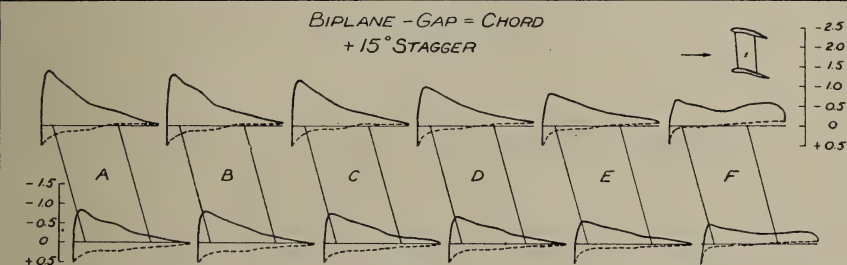
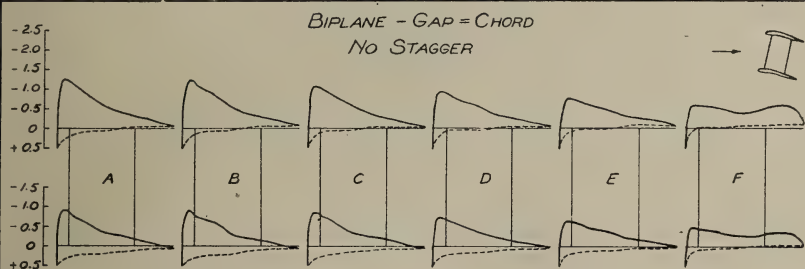
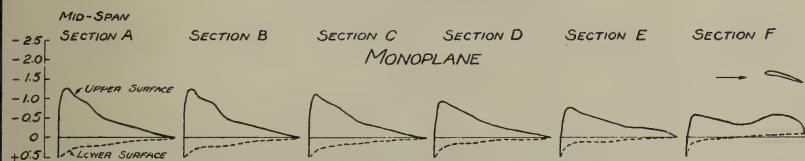
ANGLE OF INCIDENCE = $+12^\circ$ 

PLATE B

PRESSURE DISTRIBUTION OVER USA-27 AEROFOIL

WITH SQUARE WING TIPS

NORMAL FORCE DIAGRAMS

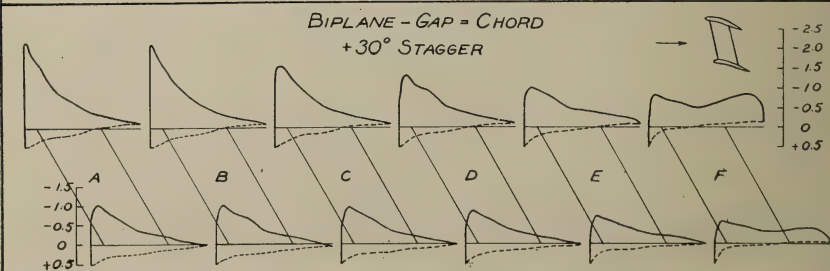
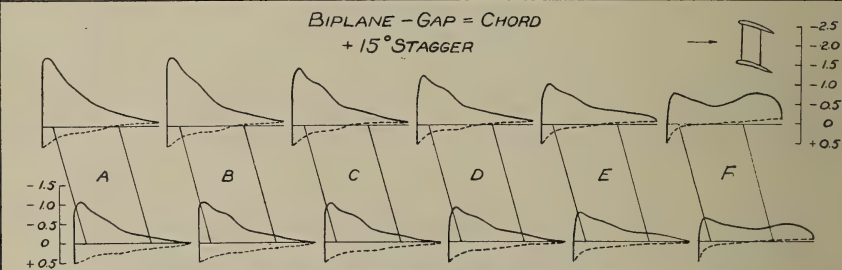
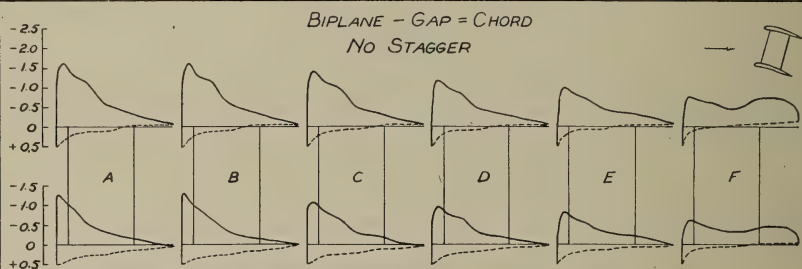
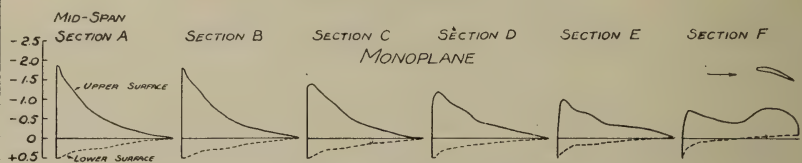
MODEL 3"x16" BRASS UNIT OF PRESSURE = $2V^2$ PDS. PER SQ. FT WIND SPEED - 40 FT PER SEC.ANGLE OF INCIDENCE = $+16^\circ$ 

PLATE 9

PRESSURE DISTRIBUTION OVER U.S.A-27 AEROFOIL
WITH SQUARE WING TIPS

NORMAL FORCE DIAGRAMS

MODEL 3" x 18" BRASS UNIT OF PRESSURE = ρV^2 PDS. PER SQ. FT. WIND SPEED - 40 FT. PER SEC.

ANGLE OF INCIDENCE = $+18^\circ$

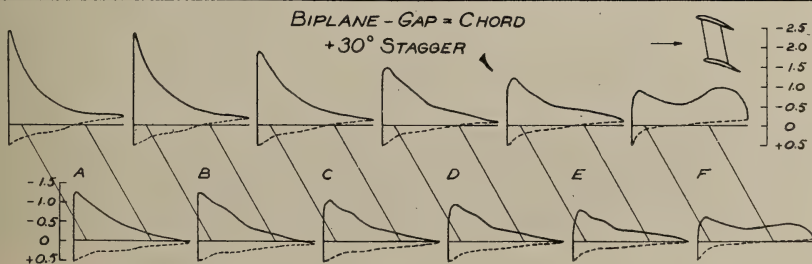
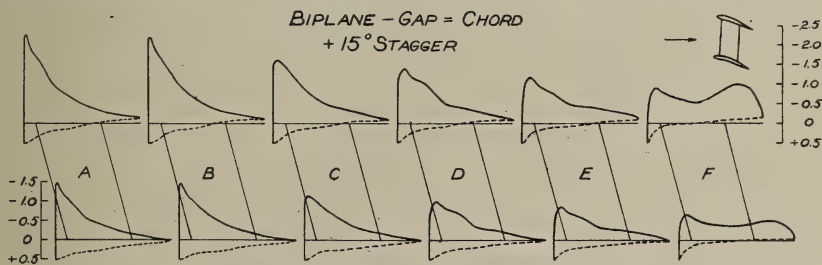
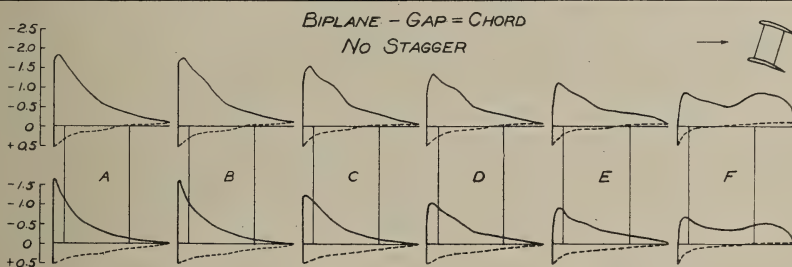
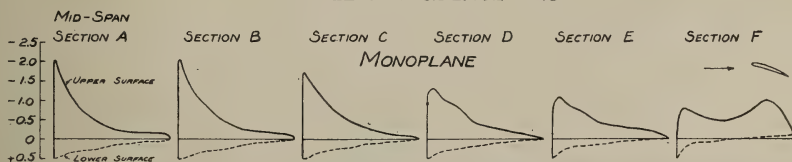
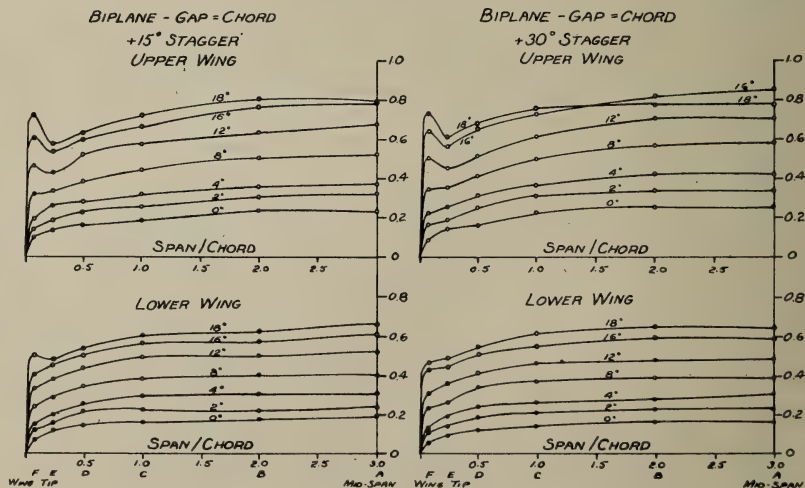
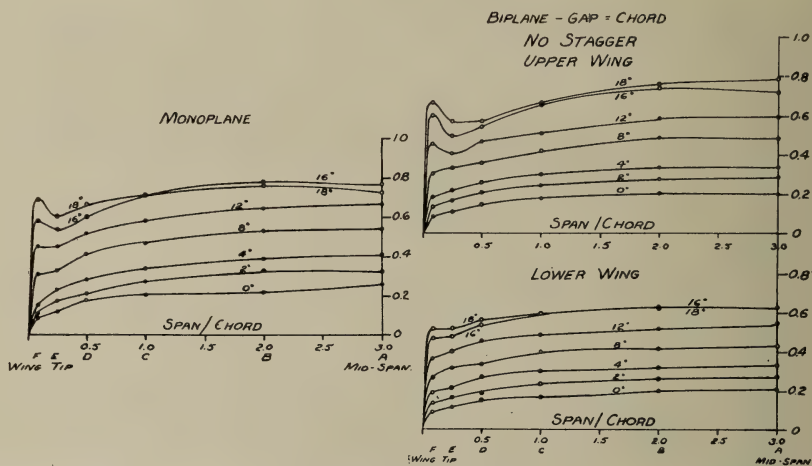


PLATE 10 PRESSURE DISTRIBUTION OVER U.S.A-27 AEROFOIL
WITH SQUARE WING TIPS
NORMAL FORCE ALONG SEMI-SPAN.

WIND SPEED - 40 FPS. UNIT OF FORCE = ρV^2 PDS. PER LINEAL FOOT FOR 1 FOOT CHORD

MODEL 3'18"



PRESSURE DISTRIBUTION OVER U.S.A.-27 AEROFOIL

WITH SQUARE WING TIPS

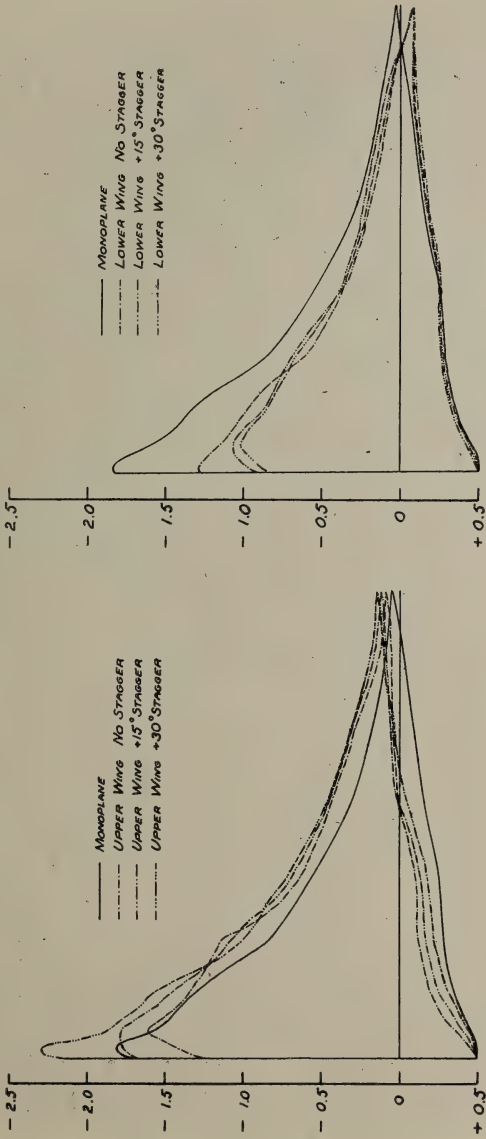
NORMAL FORCE DIAGRAMS

UNIT OF PRESSURE = ρV^2 POS. PER SQ. FT.

WIND SPEED - 40 FT. PER SEC.

ANGLE OF INCIDENCE = 16°

MEDIAN SECTION (A)



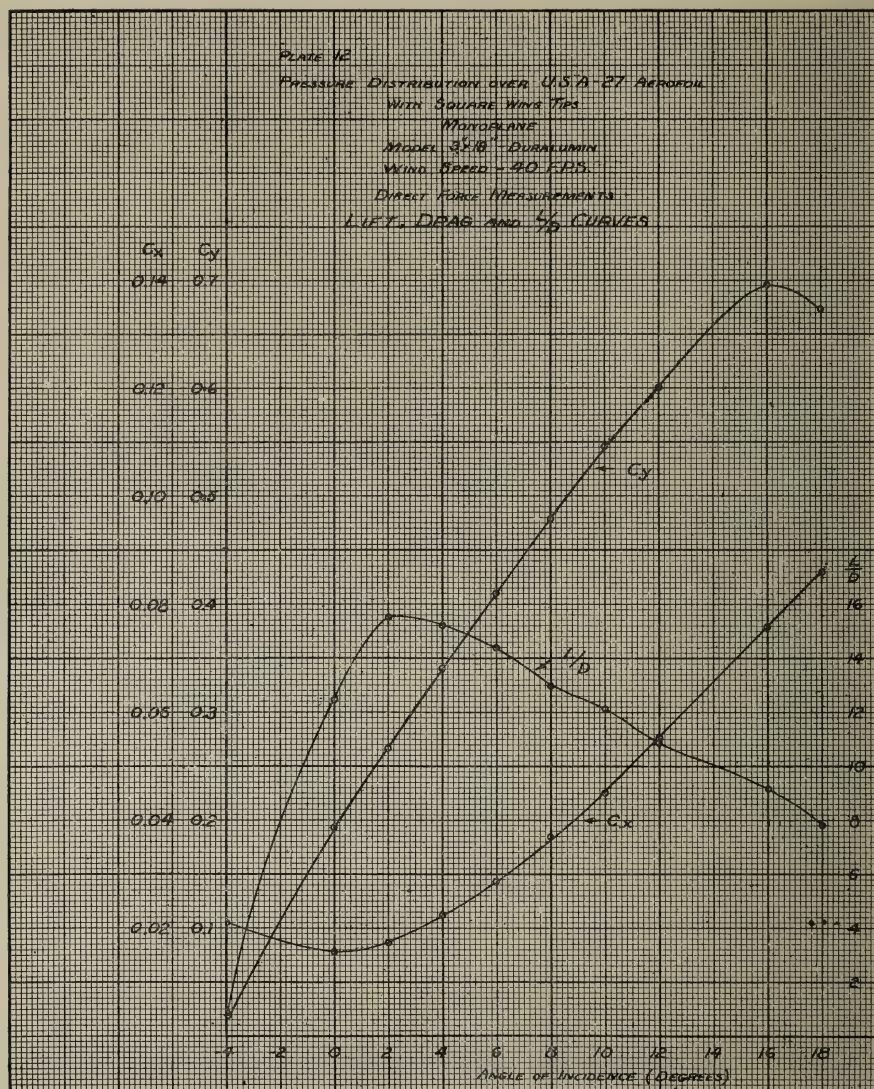


PLATE No. 12

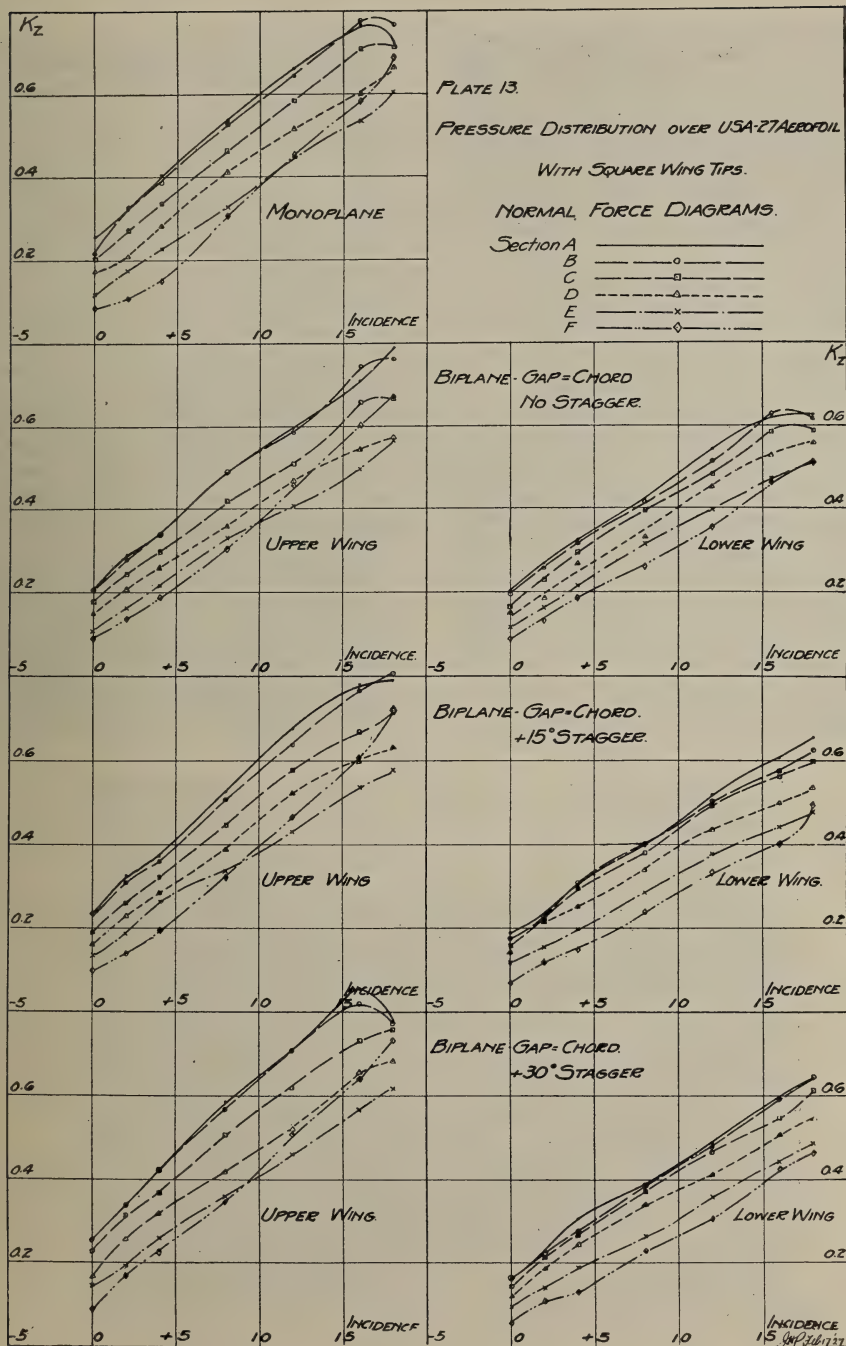




PHOTO NO. 1



PHOTO NO. 2

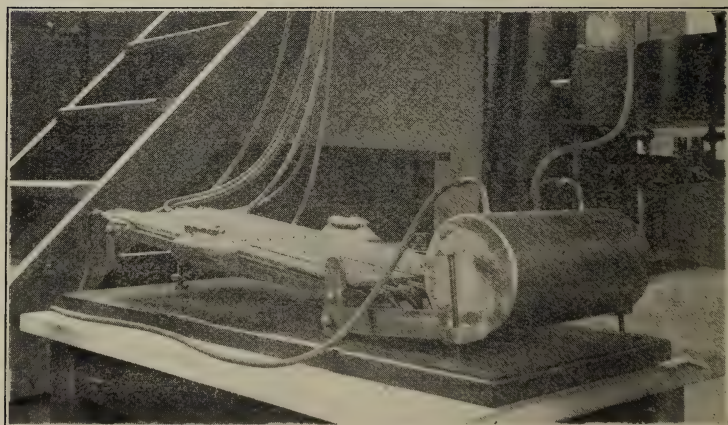


PHOTO NO. 3



Photo No. 5

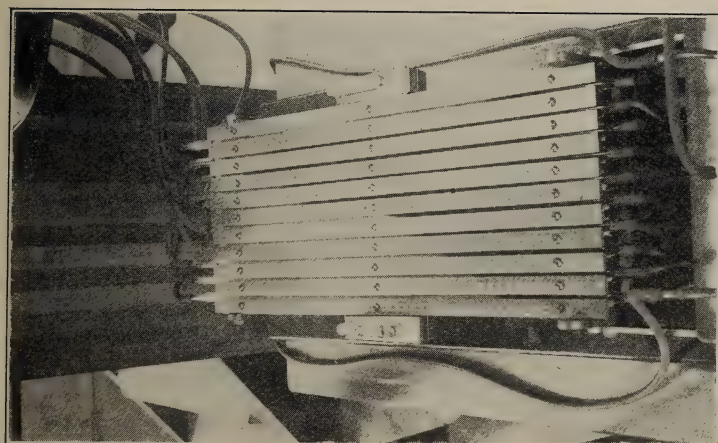


Photo No. 4

PRESSURE DISTRIBUTION OVER GOTTINGEN—387
AEROFOIL WITH SQUARE WING TIPS

By J. H. PARKIN, M.E., F.R.Ae.S., J. E. B. SHORTT, B.A.Sc., and
J. G. CADE, B.A.Sc.

SUMMARY

This investigation is similar to that described in A.R.P. No. 18, and was undertaken to determine the distribution of loading over a Gottingen -387 Aerofoil, of standard form, as a monoplane and in different usual biplane combinations.

The measurements were made in the wind channel of the University of Toronto.

Pressure measurements were made at eight points, on the upper and on the lower surfaces, at each of six sections along the semi span of a $3'' \times 18''$ brass aerofoil arranged for pressure measurements.

For the biplane measurements, a duralumin dummy aerofoil, identical in plan form and section with the pressure plotting aerofoil, was employed.

The wing combinations investigated were: monoplane, biplane with gap equal to chord and staggers of zero, 15° and 30° .

Incidence ranged from -4° to $+18^\circ$ and the air speed was 40 f.p.s. The pressures are expressed in terms of ρV^2 .

The general distribution of pressure over the Gottingen -387 Aerofoil is similar to that of the U.S.A.-27. The same increase in suction at the leading edge of the upper plane, as the stagger increases, was observed, and a similar region of high suction was found to exist on the upper surface of the upper wing at the trailing edge at the wing tips.

The change in the distribution of pressure occurring at the critical angle was found to result in the wing tips carrying the greater part of the total load at incidences above the critical.

Generally speaking, for staggered biplanes, with the Gottingen-387 section, the upper plane was found to carry a greater part of the load, than with the U.S.A.-27 section. For the Gottingen section biplane with 30° stagger the upper plane carries an average of 50% more load than the lower plane over the range of incidence tested.

INTRODUCTION—REASON FOR INVESTIGATION

The Gottingen-387 aerofoil is one of the best of the so-called thick wing sections, possessing high maximum lift, good L/D and moderate C.P. movement, together with a thickness to chord ratio of 0.154. The section has been used in several modern aeroplanes, more commonly monoplanes, but its use is now contemplated in biplanes. In machines of both types, now being designed for service in Canada, consideration is being given to the use of the Gottingen-387 section.

The present investigation was undertaken with the object of supplying the Royal Canadian Air Force and Canadian aircraft constructors with information as to the distribution of pressure over the Gottingen-387 aerofoil, not only as a monoplane, but particularly when employed in the usual biplane combinations.

Incidentally, the investigation indicated the effect of stagger in biplane arrangements. This latter information will be supplemented by an investigation of the aerodynamic characteristics for the same biplane combinations, the results of which will be given in Aeronautical Research Paper No. 19.

The aerofoils used in the investigation were purchased with funds supplied by the Department of National Defence of Canada, through the National Research Council.

DESCRIPTION OF APPARATUS

The apparatus and methods employed were those used in the similar investigation made with the U.S.A.-27 aerofoil, and described in Aeronautical Research Paper No. 18.

The pressure plotting aerofoil was of brass, arranged with ducts and openings, as in the previous study. The sections were located in the same positions as in the case of the U.S.A.-27 model, but slight differences in the locations of the pressure openings at leading and trailing edges were necessitated by the different curvature of the aerofoil.

Both aerofoils were cut on the Nichols generating machine, as described in Paper No. 18.

RANGE OF INVESTIGATION

Measurements of pressure were made at six sections along the semi span, and at eight points on both upper and lower surface at each section. These measurements were made for the monoplane, and for both planes of three biplane combinations, each with gap equal to chord, and having staggers of zero, $+15^\circ$ and $+30^\circ$, respectively. The range of incidence covered was from -4 to $+18$ degrees. The air speed throughout was 40 f.p.s.

PRESENTATION OF RESULTS

The observed pressures, in terms of ρV^2 pds. per sq. ft. (V in ft. per sec., $\rho = .002373$) are tabulated in Table VII and plotted in Plates 1 to 8. Each plate presents a complete record of the pressure distribution for one angle of incidence for the monoplane and the three biplane combinations.

As in the previous paper, negative pressures are plotted upward and positive pressures downward in these diagrams to indicate more clearly any mutual interference of the surfaces in the biplane combinations.

The observed pressures were assumed to act normally to the aerofoil surfaces at the observation points, but have been plotted normal to the chord to enable the normal force coefficients at the different sections to be determined by the method outlined in Aeronautical Paper No. 18.

The normal force coefficients K_z are tabulated in Table No. III and are plotted on Plates 9 and 11.

From the diagram of Plate 9, the normal coefficients for the complete wings were determined and have been tabulated in Table IV.

DISCUSSION OF RESULTS

The general form of the diagram of pressure distribution at any section for the Gottingen-387 is similar to that for the U.S.A.-27. The pressure distribution diagram for the upper surface of the Gottingen-387 exhibits an abrupt break at the higher incidences, which moves towards the leading edge as the incidence increases. The pressure over the lower surface decreases more or less continuously from leading to trailing edge and is less variable than in the case of the U.S.A.-27.

The curves for the monoplane at 18° incidence on Plate 8 show the distribution of pressure for the Gottingen-387 aerofoil above the critical angle. It will be observed that the suction on the upper surface has greatly decreased, while the pressure on the lower surface has changed but little, with the net result that the wing tips are, at this angle, responsible for the greater part of the lift, the lift of the mid span region being quite low* (see monoplane diagram for 16° incidence on Plate 9).

The diagram for the upper wing in the 30° stagger biplane, Plate 8, apparently shows an intermediate condition, where burbling is just commencing.

These diagrams of Plate 8 appear to indicate that one effect of superimposing of the wings in a biplane is to raise the critical angle.

A further effect, which was also noted in the case of the U.S.A.-27 aerofoil, is that as the stagger is increased in the biplane, the maximum suction at the leading edge of the upper plane is increased. Thus, the load on the front spar of the upper wing of a biplane with positive

*These results are in agreement with those obtained with a biplane of R A F -15 section, see R & M No. 355, Oct. 1917.

stagger will be greater than for no stagger, and greater than for the monoplane.

Also, as noted in the previous investigation, and generally for square tipped aerofoils, a region of high suction develops at the trailing edge of the wing tips at about 8° incidence, on the monoplane and upper wings of the biplanes. On the lower wings, the high suction in this region is partially suppressed by the upper wing, the reduction in the suction increasing with increase in positive stagger.

For incidences above, roughly 5° , the load along the span decreases from mid span to a point about 0.2 chord from wing tip, where a sudden increase occurs due to the region of high suction at the tip just mentioned (see Plates 9 and 11). This increase is most marked for the monoplane and upper wings of biplanes, and decreases with increase in stagger. The increase at the tip is much less marked for the lower wings (due to suppression of the high suction) and also decreases with increase of stagger, so that for the lower wing of the 30° stagger biplane there is a continuous decrease in loading from mid-span to tip at all incidences.

The normal force coefficients for the complete wing, tabulated in Table IV and plotted on Plate 12, show the variation in load in each wing with arrangement of wings and incidence.

For the biplane, composed of either U.S.A.-27 or Gottingen-387 aerofoils, with zero stagger, at 0° incidence, each wing carries substantially the same load, but as the incidence increases, the upper wing takes a greater and greater proportion of the load until, at 18° incidence, it is carrying about 17° more load than the lower wing.

For the 15° stagger biplane, employing U.S.A.-27 aerofoils, the upper wing carries about 23% to 24% more load than the lower wing throughout the whole range of incidence, while with the Gottingen-387 section the excess load taken by the upper wing, as compared with the lower, ranges from 25% at 0° incidence to 28% at 18° incidence.

With the stagger increased to 30° , the upper wing of the U.S.A.-27 biplane takes 48% more load than the lower at 0° , which drops to 38% at 16° incidence, and in the Gottingen 387 biplane, the upper wing takes 64% more load than the lower at 0° and 40% more at 16° incidence.

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LIST OF PLATES

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PRESSURE DISTRIBUTION OVER GOTTINGEN-387 AEROFOIL

TABLE NO. I

ORDINATES OF GOTTINGEN-387 AEROFOIL

(All dimensions expressed in per cent. of chord)

Distance from Leading Edge	Lower Ordinate	Upper Ordinate
0.00	3.61	3.61
1.25	1.35	6.74
2.50	0.81	7.98
5.00	0.36	9.87
7.50	0.18	11.32
10.00	0.13	12.40
15.00	0.00	13.83
20.00	0.08	14.77
30.00	0.22	15.36
40.00	0.38	14.88
50.00	0.54	13.48
60.00	0.54	11.59
70.00	0.54	9.16
80.00	0.50	6.58
90.00	0.27	3.61
100.00	0.00	0.37

Radius of leading edge 3.25

Radius of trailing edge 0.20

TABLE NO. II

POSITIONS OF SECTIONS AND OBSERVATION POINTS

(All dimensions expressed in multiples of the chord)

Section	Distance from Wing Tip	Observation Point	Distance from Leading Edge
A	3.000 Mid-span	1 Upper	.0125
B	2.000	1 Lower	.0400
C	1.000	2 Upper	.0500
D	0.500	2 Lower	.0895
E	0.250	3 Upper	.1200
F	0.083	3 Lower	.1500
		4	.2500
		5	.4000
		6	.5500
		7	.7000
		8	.8870

PRESSURE DISTRIBUTION OVER GOTTINGEN-387 AEROFOIL

TABLE NO. III

NORMAL FORCE COEFFICIENTS K_z AT EACH SECTION

Model 3"×18" Air Speed 40 f.p.s.

MONOPLANE

Angle of Incidence	A	B	Section C	D	E	F
-4	.129	.129	.124	.098	.053	.049
0	.276	.284	.249	.213	.151	.133
2	.355	.347	.333	.293	.222	.165
4	.449	.436	.404	.351	.271	.222
8	.609	.573	.524	.444	.369	.404
12	.746	.706	.640	.564	.502	.551
16	.813	.809	.746	.689	.569	.725
18	.413	.418	.529	.573	.569	.569

BIPLANE—GAP = CHORD—Zero STAGGER

Upper Wing

-4	.102	.102	.089	.076	.062	.036
0	.253	.258	.249	.187	.138	.107
2	.284	.276	.253	.204	.173	.138
4	.369	.369	.347	.276	.222	.218
8	.494	.489	.426	.387	.293	.333
12	.680	.640	.569	.516	.444	.494
16	.773	.755	.680	.600	.546	.666
18	.831	.813	.729	.658	.591	.760

Lower Wing

-4	.120	.116	.111	.102	.080	.036
0	.258	.244	.222	.173	.142	.089
2	.293	.284	.267	.222	.182	.160
4	.364	.342	.307	.280	.253	.218
8	.484	.453	.404	.360	.315	.324
12	.569	.556	.537	.462	.426	.444
16	.666	.649	.626	.565	.498	.565
18	.671	.666	.653	.600	.546	.582

BIPLANE—CAP = CHORD—+15° STAGGER

Upper Wing

Section

Angle of Incidence	A	B	C	D	E	F
-4	.124	.129	.120	.102	.076	.058
0	.275	.275	.236	.200	.147	.124
2	.338	.342	.311	.258	.218	.182
4	.426	.413	.364	.315	.258	.244
8	.591	.551	.472	.413	.369	.387
12	.720	.711	.631	.556	.484	.529
16	.831	.809	.755	.671	.604	.729
18	.889	.840	.791	.685	.653	.635

Lower Wing

-4	.102	.102	.093	.080	.071	.044
0	.213	.218	.209	.165	.142	.084
2	.258	.267	.244	.227	.176	.155
4	.320	.317	.280	.262	.218	.187
8	.453	.440	.409	.373	.315	.267
12	.533	.542	.529	.462	.387	.347
16	.650	.640	.631	.564	.483	.471
18	.711	.666	.671	.618	.524	.533

BIPLANE—CAP = CHORD—+30° STAGGER

Upper Wing

-4	.182	.160	.142	.129	.093	.058
0	.324	.324	.302	.222	.169	.142
2	.409	.395	.364	.302	.209	.178
4	.489	.480	.426	.369	.271	.262
8	.653	.649	.573	.453	.408	.400
12	.818	.760	.685	.578	.520	.569
16	.942	.930	.822	.698	.658	.765
18	.983	.902	.862	.609	.649	.640

Lower Wing

-4	.102	.080	.089	.071	.056	.044
0	.191	.187	.191	.164	.124	.084
2	.240	.231	.235	.182	.164	.124
4	.307	.275	.271	.235	.218	.187
8	.422	.400	.382	.342	.302	.253
12	.538	.511	.494	.431	.373	.300
16	.671	.636	.613	.542	.489	.444
18	.711	.685	.645	.586	.529	.520

PRESSURE DISTRIBUTION OVER GOTTINGER-387 AEROFOIL

TABLE NO. IV

NORMAL FORCE COEFFICIENTS K_z —COMPLETE WINGS

Model 3"×18" Air Speed 40 f.p.s.

MONOPLANE

Angle of Incidence	K_z
-4	.111
0	.244
2	.320
4	.404
8	.533
12	.661
16	.756
18	.481

BIPLANE—GAP = CHORD—NO STAGGER

Angle of Incidence	Upper Wing	Lower Wing	Biplane	$\frac{K_z \text{ Upper Wing}}{K_z \text{ Lower Wing}}$
-4	.092	.107	.100	0.86
0	.226	.213	.220	1.06
2	.253	.256	.255	0.988
4	.334	.316	.325	1.057
8	.442	.415	.429	1.065
12	.594	.533	.564	1.114
16	.705	.619	.662	1.139
18	.767	.650	.709	1.180

BIPLANE—GAP = CHORD—+15° STAGGER

-4	.118	.093	.105	1.269
0	.238	.196	.217	1.214
2	.310	.248	.279	1.250
4	.373	.284	.325	1.313
8	.500	.407	.453	1.227
12	.650	.506	.578	1.285
16	.770	.619	.694	1.248
18	.801	.668	.734	1.200

BIPLANE—GAP = CHORD—+30° STAGGER

-4	.136	.084	.110	1.619
0	.287	.179	.233	1.604
2	.356	.218	.287	1.633
4	.429	.272	.350	1.576
8	.582	.382	.482	1.525
12	.704	.483	.593	1.457
16	.853	.611	.732	1.395
18		.654		

PRESSURE DISTRIBUTION OVER GOTTINGEN-387 AEROFOIL

TABLE V

AERODYNAMIC CHARACTERISTICS (DIRECT FORCE MEASUREMENTS)

Model 3"×18" Duralumin Air Speed 40 f.p.s.

Corrected for Spindle Interference. Not corrected for channel interference

Incidence	Lift Coefficient	Drag Coefficient	Lift/Drag	Centre of Pressure
-6	.0518	.0212	2.45	
-4	.1190	.0166	7.15	66.8
-2	.1878	.0176	10.65	52.1
0	.2604	.0200	13.02	42.6
+2	.3354	.0244	13.77	38.9
4	.4055	.0308	13.16	36.1
6	.4806	.0383	12.55	34.1
8	.5493	.0476	11.54	32.9
10	.6183	.0576	10.75	32.7
12	.6768	.0679	9.96	31.9
14	.7284	.0799	9.11	31.5
16	.7529	.0926	8.14	30.7
18	.4206	.1489	2.82	39.3

PRESSURE DISTRIBUTION OVER GOTTINGEN-387 AEROFOIL

TABLE VI

EFFECT OF STAGGER

Median Section—16° Incidence
Normal Force Coefficient— K_z

MONOPLANE

0.813

BIPLANE—GAP = CHORD

	Upper Wing	Lower Wing
0° Stagger	0.773	.666
15° Stagger	0.831	.650
30° Stagger	0.942	.671

Maximum Observed Suctions on Upper Surface Near Leading Edge (Suction in multiples of ρV^2 pds. per sq. ft.)

MONOPLANE

1.627

BIPLANE—GAP = CHORD

	Upper Wing	Lower Wing
0° Stagger	1.676	1.336
15° Stagger	1.950	1.170
30° Stagger	2.194	0.958

TABLE VII

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

MONOPLANE

SECTION A—UPPER SURFACE								
Incidence degs.	1	2	3	4	5	6	6	8
-4	+.387	+.051	-.242	-.377	-.326	-.257	-.228	-.016
0	+.213	-.204	-.463	-.519	-.412	-.350	-.206	-.024
2	+.004	-.359	-.587	-.585	-.465	-.423	-.155	-.033
4	-.188	-.517	-.711	-.663	-.532	-.470	-.166	-.038
8	-.696	-.909	-.984	-.800	-.716	-.332	-.193	-.040
12	-1.281	-1.307	-1.230	-.997	-.612	-.372	-.195	-.049
16	-1.627	-1.582	-1.361	-1.237	-.550	-.315	-.157	-.083
18	-.501	-.379	-.313	-.315	-.297	-.293	-.293	-.315
SECTION A.—LOWER SURFACE.								
-4	-.470	-.410	-.231	-.069	-.022	+.006	+.031	+.030
0	-.213	-.104	-.058	-.011	+.024	+.040	+.053	+.071
2	-.060	-.009	+.006	+.036	+.055	+.059	+.036	+.073
4	+.075	+.073	+.069	+.078	+.084	+.080	+.078	+.077
8	+.288	+.217	+.180	+.155	+.140	+.120	+.106	+.089
12	+.410	+.317	+.266	+.219	+.186	+.153	+.129	+.093
16	+.470	+.395	+.337	+.270	+.226	+.180	+.140	+.075
18	+.359	+.279	+.226	+.182	+.135	+.084	+.033	-.055
SECTION B.—UPPER SURFACE.								
-4	+.406	+.067	-.297	-.377	-.319	-.248	-.213	-.004
0	+.188	-.195	-.474	-.510	-.401	-.341	-.202	-.016
2	+.029	-.346	-.594	-.585	-.457	-.412	-.144	-.022
4	-.166	-.510	-.718	-.654	-.519	-.463	-.153	-.024
8	-.645	-.869	-.964	-.787	-.705	-.324	-.177	-.029
12	-1.200	-1.248	-1.201	-.971	-.614	-.357	-.177	-.033
16	-1.706	-1.553	-1.347	-1.219	-.539	-.306	-.146	-.068
18	-.497	-.398	-.315	-.317	-.295	-.295	-.288	-.284
SECTION B.—LOWER SURFACE.								
-4	-.173	-.412	-.219	-.073	-.024	+.002	+.029	+.058
0	-.151	-.113	-.064	-.018	+.020	+.036	+.049	+.064
2	-.140	-.020	.000	+.029	+.053	+.055	+.062	+.066
4	-.133	+.064	+.060	+.071	+.079	+.075	+.075	+.073
8	-.135	+.202	+.162	+.144	+.133	+.111	+.097	+.082
12	-.166	+.317	+.257	+.213	+.182	+.146	+.122	+.088
16	-.259	+.390	+.319	+.262	+.215	+.168	+.129	+.069
18	-.275	+.281	+.222	+.177	+.135	+.082	+.075	-.058

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL.

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

MONOPLANE

SECTION C.—UPPER SURFACE.

Incidence degs.	Pressure Opening							
	0	2	3	4	5	6	7	8
-4	+.395	+.060	-.255	-.361	-.317	-.237	-.206	-.002
0	+.199	-.171	-.446	-.482	-.375	-.321	-.186	-.011
2	+.062	-.306	-.552	-.541	-.417	-.384	-.131	-.018
4	-.106	-.454	-.667	-.601	-.470	-.428	-.133	-.020
8	-.532	-.776	-.882	-.714	-.616	-.295	-.160	-.024
12	-1.037	-1.119	-1.097	-.869	-.594	-.326	-.166	-.035
16	-1.513	-1.405	-1.241	-1.081	-.517	-.317	-.151	-.040
18	-.638	-.541	-.423	-.430	-.392	-.401	-.392	-.366

SECTION C.—LOWER SURFACE.

-4	-.587	-.395	-.173	-.071	-.027	+.002	+.028	+.058
0	-.237	-.126	-.069	-.013	+.018	+.033	+.049	+.066
2	-.102	-.044	-.018	+.019	+.040	+.047	+.055	+.066
4	+.036	+.042	+.042	+.062	+.067	+.064	+.066	+.071
8	+.228	+.182	+.144	+.133	+.117	+.098	+.089	+.080
12	+.381	+.288	+.231	+.197	+.166	+.135	+.113	+.084
16	+.459	+.366	+.297	+.246	+.195	+.155	+.124	+.073
18	+.383	+.295	+.231	+.188	+.135	+.086	+.039	-.058

SECTION D.—UPPER SURFACE.

-4	+.383	+.067	-.231	-.339	-.277	-.213	-.195	.000
0	+.239	-.129	-.392	-.434	-.377	-.286	-.177	-.007
2	+.131	-.248	-.481	-.481	-.366	-.333	-.126	-.016
4	-.020	-.375	-.581	-.539	-.415	-.390	-.124	-.024
8	-.361	-.643	-.762	-.632	-.567	-.286	-.153	-.035
12	-.776	-.924	-.946	-.745	-.614	-.306	-.180	-.047
16	-1.197	-1.186	-1.090	-.895	-.481	-.328	-.186	-.055
18	-.758	-.740	-.612	-.614	-.468	-.432	-.383	-.299

SECTION D.—LOWER SURFACE.

-4	-.443	-.401	-.235	-.064	-.022	+.002	+.027	+.055
0	-.275	-.151	-.093	-.037	+.006	+.024	+.040	+.060
2	-.151	-.075	-.040	-.004	+.022	+.035	+.044	+.058
4	-.035	-.006	+.008	+.029	+.044	+.046	+.051	+.060
8	+.166	+.124	+.099	+.091	+.086	+.075	+.071	+.069
12	+.310	+.226	+.180	+.148	+.124	+.095	+.093	+.073
16	+.412	+.310	+.246	+.197	+.160	+.122	+.109	+.078
18	+.359	+.264	+.204	+.160	+.120	+.091	+.049	-.015

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

MONOPLANE

SECTION E.—UPPER SURFACE.

Incidence degs.	1	2	3	4	5	6	7	8
-4	+.403	+.073	-.217	-.299	-.250	-.191	-.155	.000
0	+.264	-.095	-.348	-.372	-.297	-.246	-.137	-.011
2	+.164	-.195	-.423	-.417	-.330	-.279	-.124	-.024
4	+.064	-.293	-.497	-.457	-.366	-.295	-.131	-.038
8	-.219	-.512	-.652	-.541	-.450	-.284	-.175	-.080
12	-.556	-.756	-.809	-.636	-.521	-.301	-.222	-.124
16	-.911	-.977	-.929	-.725	-.483	-.339	-.262	-.142
18	-.703	-.767	-.678	-.721	-.439	-.370	-.326	-.226

SECTION E.—LOWER SURFACE.

-4	-.452	-.403	-.217	-.064	-.027	-.002	+.015	+.046
0	-.313	-.175	-.113	-.051	-.006	+.011	+.024	+.042
2	-.204	-.106	-.067	-.027	+.006	+.018	+.027	+.039
4	-.068	-.046	-.027	+.002	+.022	+.027	+.031	+.039
8	+.093	+.070	+.051	+.053	+.053	+.044	+.044	+.044
12	+.244	+.171	+.124	+.097	+.084	+.066	+.058	+.051
16	+.288	+.199	+.142	+.109	+.079	+.053	+.038	+.019
18	+.313	+.219	+.155	+.115	+.084	+.053	+.033	-.002

SECTION F.—UPPER SURFACE.

-4	+.372	+.062	-.177	-.233	-.202	-.157	-.099	-.035
0	+.255	-.069	-.275	-.290	-.253	-.190	-.142	-.111
2	+.175	-.144	-.330	-.326	-.277	-.208	-.175	-.191
4	+.091	-.222	-.383	-.361	-.295	-.235	-.228	-.242
8	-.113	-.379	-.490	-.437	-.355	-.308	-.415	-.508
12	-.366	-.552	-.565	-.532	-.432	-.423	-.687	-.729
16	-.621	-.720	-.716	-.598	-.494	-.598	-.980	-.886
18	-.503	-.594	-.576	-.510	-.410	-.448	-.700	-.694

SECTION F.—LOWER SURFACE.

-4	-.446	-.357	-.102	-.078	-.027	-.007	+.009	-.024
0	-.304	-.168	-.109	-.075	-.018	-.009	.000	+.058
2	-.228	-.124	-.082	-.066	-.016	-.009	-.004	+.002
4	-.146	-.082	-.055	-.060	-.039	-.009	-.007	-.004
8	+.004	.000	-.007	-.047	.000	-.011	-.033	-.018
12	+.131	+.075	+.033	-.033	+.004	-.018	-.024	-.031
16	+.233	+.137	+.078	-.024	+.004	-.024	-.031	+.009
18	+.202	+.115	+.060	-.024	+.004	-.033	-.042	-.055

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—Zero STAGGER

UPPER WING—SECTION A—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	-.370	+.015	-.288	-.423	-.368	-.295	-.257	-.042
0	+.173	-.226	-.499	-.563	-.459	-.397	-.257	-.062
2	+.016	-.384	-.636	-.649	-.525	-.472	-.206	-.080
4	-.160	-.536	-.760	-.720	-.585	-.536	-.215	-.086
8	-.574	-.878	-1.011	-.860	-.745	-.408	-.255	-.097
12	-1.090	-1.268	-1.281	-1.033	-.878	-.457	-.279	-.109
16	-1.676	-1.644	-1.502	-1.252	-.700	-.474	-.273	-.120
18	-1.986	-1.804	-1.605	-1.418	-.714	-.461	-.259	-.135

UPPER WING—SECTION A—LOWER SURFACE

-4	-.519	-.446	-.264	-.140	-.091	-.062	-.029	+.015
0	-.337	-.208	-.162	-.115	-.075	-.055	-.031	+.009
2	-.211	-.135	-.115	-.089	-.060	-.047	-.029	+.002
4	-.095	-.071	-.073	-.064	-.047	-.040	-.029	.000
8	+.109	+.051	+.011	-.011	-.018	-.024	-.019	-.002
12	+.270	+.157	+.089	+.038	+.011	-.009	-.015	-.007
16	+.386	+.244	+.160	+.084	+.040	+.013	-.002	-.009
18	+.423	+.288	+.197	+.113	+.062	+.022	.000	-.020

UPPER WING—SECTION B—UPPER SURFACE

-4	+.383	+.033	-.288	-.410	-.357	-.281	-.244	-.027
0	+.197	-.202	-.499	-.547	-.443	-.377	-.250	-.044
2	+.040	-.364	-.634	-.636	-.505	-.406	-.195	-.066
4	-.117	-.505	-.758	-.707	-.563	-.523	-.195	-.071
8	-.508	-.833	-.988	-.842	-.723	-.390	-.235	-.084
12	-1.011	-1.205	-1.241	-.995	-.867	-.430	-.259	-.093
16	-1.558	-1.589	-1.471	-1.230	-.687	-.459	-.257	-.106
18	-1.835	-1.764	-1.549	-1.370	-.687	-.432	-.237	-.120

UPPER WING—SECTION B—LOWER SURFACE

-4	-.106	-.423	-.259	-.133	-.080	-.053	-.020	+.024
0	-.082	-.208	-.157	-.106	-.071	-.047	-.024	+.018
2	-.069	-.135	-.113	-.080	-.053	-.038	-.018	+.009
4	-.062	-.073	-.071	-.058	-.040	-.033	-.018	+.009
8	-.062	+.044	+.004	-.013	-.018	-.024	-.018	+.002
12	-.078	+.146	+.078	-.033	+.009	-.009	-.011	+.004
16	-.102	+.237	+.146	+.082	+.038	+.007	-.009	-.015
18	-.155	+.281	+.186	+.109	+.058	+.016	-.002	-.027

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—Zero STAGGER

UPPER WING—SECTION C—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.379	+.031	-.288	-.439	-.337	-.266	-.233	-.022
0	+.199	-.188	-.483	-.521	-.412	-.355	-.222	-.040
2	+.069	-.324	-.592	-.585	-.459	-.412	-.173	-.055
4	-.073	-.510	-.689	-.643	-.510	-.477	-.175	-.062
8	-.432	-.747	-.906	-.760	-.643	-.366	-.206	-.073
12	-.851	-1.064	-1.117	-.891	-.778	-.377	-.226	-.079
16	-1.343	-1.398	-1.327	-1.081	-.621	-.417	-.237	-.095
18	-1.600	-1.560	-1.405	-1.199	-.629	-.417	-.237	-.102

UPPER WING—SECTION C—LOWER SURFACE

-4	-.601	-.403	-.217	-.117	-.078	-.049	-.013	+.029
0	-.346	-.211	-.151	-.089	-.064	-.042	-.016	+.022
2	-.211	-.137	-.111	-.071	+.047	-.038	-.018	+.013
4	-.113	-.082	-.075	-.049	-.042	-.033	-.016	+.011
8	+.093	+.040	+.004	-.004	-.020	-.022	-.016	+.004
12	+.231	+.133	+.069	+.040	+.004	-.011	-.011	-.002
16	+.355	+.231	+.144	+.086	+.031	+.002	-.006	-.011
18	+.392	+.264	+.168	+.106	+.042	+.009	-.006	-.022

UPPER WING—SECTION D—UPPER SURFACE

-4	+.390	+.042	-.259	-.364	-.295	-.233	-.211	-.011
0	+.237	-.151	-.423	-.468	-.355	-.306	-.199	-.024
2	+.117	-.273	-.519	-.521	-.395	-.359	-.153	-.040
4	-.004	-.383	-.612	-.572	-.437	-.417	-.148	-.051
8	-.308	-.641	-.929	-.669	-.548	-.344	-.188	-.071
12	-.687	-.917	-.977	-.782	-.683	-.346	-.224	-.091
16	-1.099	-1.205	-1.146	-.926	-.567	-.392	-.248	-.109
18	-1.270	-1.336	-1.230	-1.015	-.556	-.408	-.259	-.115

UPPER WING—SECTION D—LOWER SURFACE

-4	-.485	-.428	-.237	-.104	-.064	-.040	-.009	+.031
0	-.352	-.217	-.157	-.098	-.057	-.038	-.013	+.022
2	-.293	-.153	-.120	-.078	-.049	-.033	-.013	+.016
4	-.148	-.102	-.086	-.064	-.042	-.031	-.016	+.011
8	+.071	-.002	-.020	-.024	-.022	-.024	-.016	+.004
12	+.182	+.097	+.047	+.015	-.002	-.013	-.011	.000
16	+.306	+.186	+.113	+.060	+.022	.000	-.006	-.004
18	+.352	+.224	+.144	+.082	+.036	+.006	-.004	-.006

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—Zero STAGGER

UPPER WING—SECTION E—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.384	+.049	-.244	-.324	-.270	-.211	-.173	-.015
0	+.257	-.113	-.377	-.401	-.324	-.268	-.153	-.027
2	+.160	-.208	-.450	-.443	-.357	-.304	-.144	-.044
4	+.051	-.317	-.532	-.494	-.401	-.330	-.157	-.066
8	-.202	-.521	-.683	-.581	-.485	-.319	-.204	-.109
12	-.554	-.758	-.838	-.672	-.567	-.339	-.253	-.153
16	-.862	-1.008	-.997	-.782	-.572	-.395	-.308	-.195
18	-1.050	-1.128	-1.061	-.842	-.561	-.426	-.335	-.215

UPPER WING—SECTION E—LOWER SURFACE

-4	-.477	-.412	-.231	-.093	-.062	-.038	-.013	+.024
0	-.359	-.219	-.157	-.095	-.055	-.035	-.015	+.013
2	-.268	-.164	-.124	-.078	-.051	-.035	-.020	+.007
4	-.177	-.115	-.095	-.062	-.044	-.033	-.020	+.002
8	-.011	-.022	-.038	-.033	-.033	-.031	-.024	-.007
12	+.135	+.067	-.022	.000	-.015	-.024	-.022	-.013
16	+.262	+.155	+.084	+.042	+.009	-.013	-.015	-.018
18	+.313	+.191	+.111	+.059	+.018	-.007	-.013	-.020

UPPER WING—SECTION F—UPPER SURFACE

-4	+.359	+.086	-.199	-.250	-.222	-.173	-.113	-.047
0	+.242	-.091	-.304	-.321	-.279	-.199	-.155	-.115
2	+.175	-.162	-.355	-.352	-.304	-.226	-.188	-.186
4	-.082	-.246	-.415	-.395	-.324	-.253	-.231	-.262
8	-.117	-.403	-.528	-.479	-.388	-.321	-.386	-.448
12	-.359	-.576	-.649	-.576	-.468	-.430	-.634	-.629
16	-.616	-.762	-.776	-.667	-.545	-.603	-.889	-.780
18	-.749	-.842	-.836	-.749	-.592	-.711	-.977	-.847

UPPER WING—SECTION F—LOWER SURFACE

-4	-.459	-.357	-.133	-.104	-.066	-.038	-.020	+.009
0	-.344	-.204	-.140	-.104	-.060	-.042	-.027	-.006
2	-.275	-.164	-.120	-.097	-.060	-.044	-.035	-.015
4	-.197	-.126	-.097	-.089	-.055	-.051	-.040	-.024
8	-.071	-.064	-.062	-.082	-.058	-.060	-.053	-.044
12	+.053	+.004	-.020	-.069	-.051	-.060	-.057	-.053
16	+.160	+.071	+.020	-.053	-.042	-.062	-.060	-.062
18	+.213	+.104	+.044	-.049	-.038	-.060	-.058	-.062

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—Zero STAGGER

LOWER WING—SECTION A—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.390	+.055	-.244	-.386	-.339	-.266	-.237	-.009
0	+.224	-.140	-.399	-.470	-.375	-.313	-.226	-.009
2	+.104	-.246	-.470	-.494	-.388	-.337	-.146	-.009
4	-.047	-.365	-.559	-.534	-.421	-.383	-.117	-.007
8	-.419	-.636	-.725	-.605	-.510	-.259	-.126	.000
12	-.873	-.931	-.889	-.698	-.503	-.253	-.122	+.009
16	-1.336	-1.177	-1.008	-.900	-.399	-.242	-.102	+.004
18	-1.531	-1.281	-1.086	-.827	-.390	-.215	-.086	+.004

LOWER WING—SECTION A—LOWER SURFACE

-4	-.483	-.439	-.233	-.071	-.027	+.002	+.027	+.055
0	-.237	-.117	-.064	-.015	+.024	+.044	+.058	+.077
2	-.102	-.029	-.002	+.031	+.053	+.062	+.066	+.077
4	+.031	+.051	+.055	+.073	+.082	+.082	+.082	+.089
8	+.257	+.199	+.171	+.153	+.144	+.126	+.117	+.106
12	+.401	+.315	+.264	+.224	+.195	+.166	+.146	+.122
16	+.479	+.379	+.337	+.286	+.246	+.202	+.171	+.129
18	+.488	+.421	+.364	+.304	+.257	+.213	+.177	+.126

LOWER WING—SECTION B—UPPER SURFACE

-4	+.403	+.071	-.253	-.379	-.326	-.253	-.219	+.009
0	+.197	-.120	-.403	-.454	-.361	-.299	-.217	+.007
2	+.124	-.230	-.474	-.490	-.377	-.328	-.153	+.002
4	-.020	-.355	-.559	-.525	-.408	-.370	-.104	+.004
8	-.388	-.621	-.729	-.596	-.492	-.253	-.111	+.011
12	-.395	-.902	-.884	-.683	-.598	-.239	-.106	+.022
16	-1.294	-1.166	-1.002	-.882	-.434	-.184	-.089	+.020
18	-1.502	-1.259	-1.070	-.819	-.377	-.197	-.073	+.007

LOWER WING—SECTION B—LOWER SURFACE

-4	+.024	-.430	-.222	-.073	-.022	+.007	+.031	+.064
0	+.049	-.124	-.068	-.018	+.029	+.047	+.060	+.080
2	+.060	-.033	-.002	+.031	+.060	+.066	+.073	+.084
4	+.062	+.047	+.053	+.071	+.086	+.084	+.086	+.093
8	+.060	+.191	+.162	+.148	+.142	+.124	+.115	+.109
12	+.020	+.304	+.253	+.219	+.193	+.166	+.144	+.124
16	-.022	+.388	+.328	+.279	+.239	+.199	+.168	+.131
18	-.071	+.403	+.346	+.299	+.255	+.211	+.173	+.124

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—Zero STAGGER

LOWER WING—SECTION C—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.406	+.069	-.253	-.368	-.313	-.244	-.217	+.004
0	+.248	-.111	-.392	-.443	-.348	-.288	-.202	+.004
2	+.135	-.215	-.466	-.472	-.364	-.317	-.135	.000
4	+.009	-.326	-.534	-.501	-.381	-.350	-.102	+.002
8	-.277	-.563	-.694	-.572	-.419	-.273	-.113	+.004
12	-.736	-.640	-.853	-.658	-.532	-.242	-.120	+.007
16	-1.170	-1.183	-.969	-.818	-.392	-.239	-.104	+.009
18	-1.374	-1.186	-1.011	-.895	-.381	-.217	-.089	+.002

LOWER WING—SECTION C—LOWER SURFACE

-4	-.554	-.452	-.179	-.069	-.024	+.002	+.033	+.062
0	-.288	-.144	-.124	-.022	+.018	+.035	+.051	+.071
2	-.133	-.055	-.019	+.022	+.044	+.053	+.062	+.078
4	-.013	+.022	+.035	+.058	+.039	+.071	+.075	+.084
8	+.202	+.164	+.135	+.133	+.122	+.109	+.105	+.099
12	+.355	+.281	+.231	+.202	+.171	+.148	+.131	+.111
16	+.450	+.366	+.304	+.259	+.213	+.177	+.153	+.120
18	+.477	+.397	+.379	+.284	+.231	+.191	+.162	+.120

LOWER WING—SECTION D—UPPER SURFACE

-4	+.406	+.066	-.233	-.346	-.273	-.217	-.204	+.009
0	+.268	-.093	-.355	-.403	-.297	-.257	-.182	+.011
2	+.171	-.181	-.417	-.432	-.268	-.284	-.126	+.002
4	+.051	-.293	-.490	-.463	-.337	-.326	-.099	.000
8	-.231	-.499	-.625	-.521	-.406	-.275	-.109	-.002
12	-.596	-.740	-.760	-.587	-.492	-.226	-.122	-.009
16	-.984	-.971	-.878	-.716	-.359	-.244	-.129	-.011
18	-1.166	-1.070	-.918	-.802	-.354	-.239	-.122	-.013

LOWER WING—SECTION D—LOWER SURFACE

-4	-.461	-.423	-.248	-.064	-.022	+.007	+.029	+.058
0	-.304	-.166	-.104	-.038	+.011	+.029	+.044	+.066
2	-.177	-.086	-.047	-.007	+.027	+.040	+.051	+.066
4	-.064	-.015	+.004	+.031	+.049	+.053	+.060	+.071
8	+.140	+.113	+.095	+.093	+.093	+.084	+.082	+.080
12	+.293	+.222	+.177	+.151	+.125	+.113	+.102	+.091
16	+.406	+.313	+.253	+.206	+.171	+.142	+.124	+.099
18	-.022	+.346	+.282	+.231	+.188	+.155	+.131	+.058

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—Zero STAGGER

LOWER WING—SECTION E—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.401	+.080	-.213	-.304	-.253	-.193	-.157	+.002
0	+.279	-.064	-.319	-.355	-.277	-.228	-.135	.000
2	+.199	-.144	-.381	-.383	-.297	-.244	-.113	-.007
4	+.098	-.233	-.434	-.406	-.317	-.268	-.106	-.020
8	-.140	-.415	-.552	-.459	-.375	-.255	-.124	-.042
12	-.437	-.616	-.672	-.519	-.437	-.233	-.155	-.060
16	-.767	-.813	-.769	-.594	-.388	-.259	-.177	-.069
18	-.931	-.913	-.822	-.645	-.368	-.266	-.184	-.073

LOWER WING—SECTION E—LOWER SURFACE

-4	-.452	-.408	-.215	-.062	-.027	.000	+.015	+.049
0	-.319	-.182	-.117	-.049	-.002	+.015	+.029	+.051
2	-.213	-.109	-.066	-.022	+.011	+.024	+.033	+.051
4	-.109	-.049	-.024	+.004	+.027	+.033	+.040	+.053
8	+.084	+.082	+.033	+.058	+.062	+.055	+.033	+.060
12	+.197	+.175	+.133	+.111	+.095	+.078	+.073	+.066
16	+.361	+.266	+.202	+.160	+.129	+.102	+.091	+.075
18	+.401	+.344	+.231	+.182	+.144	+.115	+.099	+.078

LOWER WING—SECTION F—UPPER SURFACE

-4	+.372	+.060	-.180	-.193	-.213	-.180	-.095	-.035
0	+.262	-.058	-.215	-.257	-.248	-.168	-.122	-.086
2	+.193	-.122	-.301	-.288	-.270	-.184	-.151	-.089
4	+.113	-.186	-.341	-.315	-.290	-.208	-.188	-.191
8	-.075	-.333	-.430	-.366	-.324	-.264	-.295	-.315
12	-.295	-.470	-.512	-.412	-.377	-.328	-.421	-.434
16	-.548	-.623	-.607	-.470	-.434	-.408	-.567	-.505
18	-.661	-.688	-.647	-.492	-.466	-.450	-.634	-.512

LOWER WING—SECTION F—LOWER SURFACE

-4	-.443	-.359	-.100	-.080	-.027	-.009	-.011	+.031
0	-.319	-.182	-.111	-.075	-.015	-.004	+.007	+.020
2	-.231	-.126	-.078	-.064	-.011	-.002	+.004	+.015
4	-.155	-.082	-.053	-.053	-.002	.000	+.004	+.011
8	+.004	+.004	.000	-.035	+.011	+.004	+.002	+.007
12	+.135	+.082	+.048	-.015	+.020	+.002	.000	+.002
16	+.239	+.148	+.091	+.004	+.029	+.004	.000	+.002
18	+.286	+.180	+.113	+.007	+.035	+.007	+.004	+.007

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—+15° STAGGER

UPPER WING—SECTION A—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4]	+.355	-.007	-.306	-.434	-.379	-.304	-.268	-.046
0	+.111	-.286	-.505	-.601	-.483	-.419	-.255	-.077
2	-.064	-.446	-.683	-.680	-.550	-.499	-.213	-.089
4	-.268	-.625	-.622	-.762	-.627	-.556	-.235	-.099
8]	-.762	-1.015	-1.115	-.915	-.813	-.421	-.275	-.113
12	-1.345	-1.429	-1.383	-1.121	-.798	-.488	-.299	-.126
16]	-1.950	-1.835	-1.620	-1.423	-.749	-.499	-.293	-.151
18]	-2.188	-1.968	-1.698	-1.494	-.791	-.465	-.279	-.186

UPPER WING—SECTION A—LOWER SURFACE

-4	-.463	-.361	-.224	-.106	-.069	-.055	-.031	+.004
0	-.226	-.131	-.097	-.071	-.047	-.044	-.033	-.007
2	-.100	-.062	-.053	-.040	-.033	-.040	-.035	-.011
4	+.017	+.007	-.007	-.018	-.015	-.033	-.033	-.018
8	+.222	+.129	+.080	+.035	+.011	-.020	-.029	-.024
12]	+.368	+.235	+.162	+.089	+.040	-.007	-.027	-.033
16	+.448	+.315	+.226	+.133	+.034	+.004	-.029	-.053
18	+.477	+.348	+.253	+.146	+.070	+.002	-.038	-.073

UPPER WING—SECTION B—UPPER SURFACE

-4	+.361	+.002	-.319	-.423	-.359	-.288	-.253	-.033
0	+.133	-.262	-.541	-.576	-.457	-.399	-.248	-.060
2	-.033	-.419	-.672	-.663	-.519	-.477	-.197	-.073
4	-.226	-.585	-.807	-.736	-.592	-.543	-.215	-.083
8	-.698	-.960	-1.113	-.833	-.767	-.401	-.253	-.100
12	-1.252	-1.358	-1.334	-1.070	-.780	-.461	-.277	-.113
16	-1.842	-1.742	-1.551	-1.363	-.714	-.485	-.279	-.144
18	-2.103	-1.890	-1.638	-1.467	-.707	-.446	-.264	-.168

UPPER WING—SECTION B—LOWER SURFACE

-4	-.459	-.352	-.231	-.095	-.055	-.038	-.018	+.022
0	-.226	-.131	-.097	-.060	-.035	-.029	-.015	+.009
2	-.086	-.055	-.044	-.031	-.020	-.024	-.018	+.002
4	+.018	+.004	-.007	-.037	-.009	-.018	-.018	-.002
8	+.215	+.133	+.082	+.049	+.022	-.004	-.013	-.009
12	+.357	+.235	+.155	+.095	+.047	+.004	-.015	-.022
16	+.439	+.315	+.222	+.137	+.073	+.015	-.020	-.044
18	+.463	+.341	+.242	+.151	+.078	+.011	-.031	-.067

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—+15° STAGGER

UPPER WING—SECTION C—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.368	+.015	-.301	-.401	-.341	-.268	-.237	-.029
0	+.151	-.226	-.512	-.541	-.428	-.372	-.224	-.053
2	+.013	-.368	-.625	-.607	-.346	-.434	-.180	-.064
4	-.160	-.514	-.747	-.578	-.534	-.490	-.182	-.071
8	-.572	-.838	-.973	-.802	-.694	-.366	-.219	-.086
12	-1.070	-1.199	-1.212	-.957	-.762	-.415	-.250	-.102
16	-1.598	-1.546	-1.411	-1.188	-.649	-.441	-.259	-.120
18	-1.808	-1.677	-1.469	-1.314	-.660	-.432	-.253	-.129

UPPER WING—SECTION C—LOWER SURFACE

-4	-.565	-.330	-.180	-.086	-.055	-.035	-.011	+.024
0	-.231	-.140	-.089	-.053	-.038	-.029	-.015	+.011
2	-.120	-.075	-.058	-.031	-.024	-.024	-.015	+.007
4	-.015	-.011	-.018	-.007	-.015	-.022	-.018	+.002
8	+.177	+.110	+.064	-.042	+.008	-.013	-.018	-.009
12	+.313	+.208	+.135	+.089	+.033	-.002	-.018	-.018
16	+.412	+.293	+.199	+.133	+.058	+.009	-.018	-.035
18	+.446	+.324	+.226	+.177	+.077	+.009	-.022	-.049

UPPER WING—SECTION D—UPPER SURFACE

-4	+.306	+.027	-.264	-.361	-.293	-.237	-.125	-.015
0	+.148	-.182	-.446	-.459	-.361	-.319	-.202	-.035
2	+.038	-.301	-.539	-.534	-.399	-.372	-.155	-.049
4	-.100	-.434	-.643	-.547	-.452	-.430	-.162	-.062
8	-.417	-.709	-.831	-.694	-.567	-.339	-.204	-.084
12	-.809	-1.008	-1.025	-.824	-.692	-.368	-.239	-.104
16	-1.232	-1.316	-1.216	-1.000	-.563	-.412	-.268	-.124
18	-1.416	-1.431	-1.283	-1.090	-.574	-.426	-.275	-.133

UPPER WING—SECTION D—LOWER SURFACE

-4	-.436	-.381	-.206	-.080	-.047	-.031	-.007	+.027
0	-.279	-.162	-.113	-.064	-.035	-.024	-.011	+.018
2	-.120	-.094	-.071	-.042	-.024	-.020	-.011	+.011
4	-.027	-.044	-.038	-.027	-.018	-.020	-.013	+.004
8	+.122	+.069	+.040	+.020	+.004	-.011	-.013	-.002
12	+.264	+.164	+.104	+.062	+.027	-.002	-.013	-.009
16	+.368	+.246	+.166	+.102	+.049	+.009	-.011	-.020
18	+.403	+.279	+.195	+.122	+.060	+.015	-.011	+.022

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—+15° STAGGER

UPPER WING—SECTION E—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.383	+.042	-.250	-.328	-.270	-.213	-.175	-.018
0	+.231	-.133	-.392	-.408	-.328	-.228	-.153	-.035
2	+.129	-.244	-.481	-.463	-.372	-.313	-.153	-.055
4	+.020	-.346	-.559	-.510	-.415	-.337	-.171	-.078
8	-.268	-.572	-.720	-.603	-.508	-.326	-.222	-.124
12	-.610	-.829	-.891	-.711	-.587	-.361	-.277	-.168
16	-.988	-1.090	-1.048	-.835	-.576	-.428	-.335	-.213
18	-1.181	-1.161	-1.120	-.891	-.574	-.450	-.361	-.228

UPPER WING—SECTION E—LOWER SURFACE

-4	-.430	-.364	-.206	-.071	-.044	-.027	-.007	+.022
0	-.306	-.175	-.122	-.047	-.038	-.027	-.018	+.007
2	-.211	-.122	-.089	-.022	-.027	-.027	-.020	.000
4	-.117	-.071	-.058	-.042	-.024	-.024	-.020	-.004
8	+.057	+.029	+.007	.000	-.007	-.020	-.022	-.015
12	+.202	+.122	-.069	+.040	-.011	-.013	-.022	-.020
16	+.313	+.199	-.124	+.073	+.029	-.013	-.022	-.029
18	+.352	+.231	+.146	+.089	+.040	.000	-.018	-.031

UPPER WING—SECTION F—UPPER SURFACE

-4	+.348	+.031	-.206	-.257	-.226	-.175	-.117	-.055
0	+.226	-.109	-.315	-.324	-.286	-.206	-.166	-.142
2	+.137	-.186	-.372	-.366	-.310	-.235	-.204	-.213
4	+.055	-.266	-.430	-.408	-.333	-.266	-.255	-.304
8	-.168	-.439	-.552	-.503	-.408	-.344	-.421	-.503
12	-.415	-.614	-.674	-.598	-.490	-.463	-.669	-.696
16	-.696	-.809	-.809	-.689	-.574	-.658	-.944	-.867
18	-.636	-.747	-.738	-.636	-.523	-.547	-.767	-.751

UPPER WING—SECTION F—LOWER SURFACE

-4	-.388	-.273	-.155	-.080	-.042	-.029	-.018	+.002
0	-.301	-.173	-.115	-.089	-.040	-.036	-.029	-.011
2	-.233	-.137	-.0975	-.082	-.036	-.042	-.038	-.024
4	-.162	-.102	-.078	-.080	-.038	-.049	-.044	-.035
8	-.020	-.031	-.038	-.071	-.033	-.053	-.053	-.051
12	+.100	+.038	+.002	-.062	-.029	-.062	-.064	-.066
16	+.202	+.100	+.036	-.055	-.027	-.069	-.071	-.078
18	+.248	+.129	+.058	-.053	-.022	-.067	-.069	-.075

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—+15° STAGGER

LOWER WING—SECTION A—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.399	+.062	-.237	-.372	-.324	-.253	-.222	-.007
0	+.255	-.097	-.364	-.439	-.352	-.286	-.233	-.004
2	+.155	-.202	-.441	-.483	-.377	-.319	-.171	-.007
4	+.018	-.315	-.517	-.510	-.399	-.348	-.126	-.009
8	-.304	-.548	-.667	-.572	-.463	-.335	-.124	-.007
12	-.714	-.813	-.825	-.652	-.572	-.255	-.131	+.004
16	-1.170	-1.093	-.977	-.798	-.421	-.270	-.126	+.007
18	-1.407	-1.221	-1.042	-.911	-.434	-.268	-.120	-.002

LOWER WING—SECTION A—LOWER SURFACE

-4	-.481	-.439	-.250	-.080	-.031	-.004	-.020	+.053
0	-.270	-.137	-.084	-.031	+.020	+.038	+.051	+.071
2	-.140	-.053	-.018	+.013	+.044	+.051	+.058	+.073
4	-.004	+.029	+.042	+.055	+.073	+.073	+.073	+.082
8	+.213	+.166	+.148	+.133	+.129	+.115	+.104	+.097
12	+.370	+.286	+.244	+.202	+.182	+.157	+.137	+.115
16	+.461	+.379	+.326	+.264	+.231	+.195	+.164	+.129
18	+.479	+.406	+.352	+.293	+.253	+.211	+.175	+.129

LOWER WING—SECTION B—UPPER SURFACE

-4	+.403	+.075	-.231	-.357	-.301	-.231	-.195	-.009
0	+.270	-.082	-.359	-.421	-.330	-.268	-.219	-.013
2	+.217	-.177	-.474	-.457	-.355	-.297	-.168	+.009
4	+.051	-.290	-.505	-.492	-.379	-.335	-.111	+.007
8	-.268	-.521	-.682	-.559	-.443	-.337	-.109	+.009
12	-.667	-.782	-.811	-.634	-.548	-.239	-.115	+.015
16	-1.108	-1.050	-.940	-.760	-.417	-.253	-.109	+.018
18	-1.347	-1.181	-1.015	-.844	-.421	-.250	-.102	-.011

LOWER WING—SECTION B—LOWER SURFACE

-4	-.466	-.417	-.242	-.073	-.022	+.007	+.031	+.055
0	-.268	-.137	-.078	-.022	+.027	+.042	+.058	+.080
2	-.135	-.053	-.018	+.022	+.051	+.060	+.069	+.082
4	0.000	+.033	+.044	+.064	+.082	+.082	+.084	+.091
8	+.206	+.168	+.146	+.137	+.137	+.120	+.113	+.109
12	+.348	+.284	+.239	+.206	+.186	+.160	+.144	+.124
16	+.346	+.372	+.315	+.268	+.231	+.195	+.166	+.133
18	+.257	+.408	+.348	+.295	+.253	+.211	+.177	+.133

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—+15° STAGGER

LOWER WING—SECTION C—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.406	+.073	-.242	-.352	-.297	-.184	-.199	+.009
0	+.268	-.086	-.368	-.419	-.328	-.266	-.248	+.009
2	+.160	-.186	-.437	-.410	-.348	-.297	-.148	+.002
4	+.049	-.284	-.505	-.488	-.370	-.328	-.109	+.002
8	-.237	-.494	-.643	-.550	-.434	-.330	-.111	+.002
12	-.683	-.734	-.785	-.616	-.532	-.237	-.122	+.004
16	-1.039	-1.004	-.929	-.829	-.430	-.262	-.122	+.007
18	-1.217	-1.108	-.977	-.829	-.370	-.259	-.117	+.004

LOWER WING—SECTION C—LOWER SURFACE

-4	-.576	-.441	-.178	-.071	-.026	+.004	+.031	+.062
0	-.306	-.146	-.082	-.020	+.020	+.035	+.055	+.075
2	-.106	-.071	-.031	+.015	+.042	+.055	+.064	+.080
4	+.007	+.004	+.022	+.051	+.064	+.067	+.073	+.082
8	+.171	+.142	+.122	+.122	+.115	+.104	+.102	+.097
12	+.335	+.262	+.215	+.191	+.164	+.140	+.126	+.113
16	+.426	+.346	+.290	+.248	+.208	+.173	+.146	+.120
18	+.466	+.383	+.324	+.275	+.226	+.188	+.157	+.122

LOWER WING—SECTION D—UPPER SURFACE

-4	+.406	+.069	-.226	-.330	-.259	-.204	-.191	+.011
0	+.275	-.084	-.344	-.392	-.284	-.244	-.177	+.011
2	+.188	-.166	-.403	-.419	-.301	-.273	-.122	+.007
4	+.089	-.259	-.463	-.448	-.324	-.310	-.098	+.002
8	-.173	-.457	-.592	-.503	-.430	-.308	-.106	-.004
12	-.505	-.678	-.725	-.561	-.477	-.226	-.124	-.011
16	-.884	-.907	-.849	-.672	-.403	-.246	-.135	-.015
18	-1.059	-1.004	-.891	-.738	-.368	-.255	-.135	-.015

LOWER WING—SECTION D—LOWER SURFACE

-4	-.457	-.419	-.244	-.062	-.020	+.004	+.029	+.060
0	-.304	-.175	-.109	-.040	+.008	+.027	+.042	+.064
2	-.193	-.193	-.051	-.007	+.024	+.038	+.051	+.066
4	-.073	-.022	000	+.026	+.047	+.053	+.060	+.071
8	+.120	+.100	+.086	+.084	+.086	+.080	+.077	+.078
12	+.282	+.211	+.171	+.146	+.129	+.111	+.100	+.089
16	+.392	+.301	+.246	+.202	+.168	+.140	+.122	+.100
18	+.434	+.339	+.279	+.226	+.184	+.155	+.133	+.102

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—+15° STAGGER

LOWER WING—SECTION E—UPPER SURFACE

Incidence deg.	Pressure Opening							
	2	2	3	4	5	6	7	8
-4	+.339	+.075	-.219	-.301	-.244	-.184	-.148	+.009
0	+.286	-.062	-.317	-.350	-.270	-.222	-.122	+.004
2	+.206	-.140	-.375	-.377	-.288	-.242	-.104	-.002
4	+.120	-.215	-.426	-.399	-.308	-.259	-.095	-.013
8	-.106	-.386	-.532	-.452	-.366	-.253	-.117	-.040
12	-.390	-.583	-.654	-.463	-.430	-.219	-.148	-.060
16	-.700	-.774	-.749	-.579	-.412	-.253	-.175	-.073
18	-.858	-.869	-.796	-.621	-.379	-.264	-.184	-.080

LOWER WING—SECTION E—LOWER SURFACE

-4	-.450	-.410	-.215	-.062	-.024	.000	+.022	+.051
0	-.328	-.191	-.122	-.051	.000	+.015	+.031	+.051
2	-.219	-.113	-.069	-.022	+.011	+.024	+.035	+.053
4	-.078	-.055	-.029	+.002	+.027	+.033	+.040	+.053
8	+.062	+.058	+.047	+.051	+.058	+.051	+.053	+.058
12	+.222	+.162	+.122	+.104	+.093	+.075	+.066	+.064
16	+.339	+.250	+.191	+.151	+.124	+.100	+.069	+.073
18	+.364	+.289	+.219	+.173	+.140	+.109	+.093	+.075

LOWER WING—SECTION F—UPPER SURFACE

-4	+.372	+.064	-.177	-.226	-.199	-.148	-.089	-.027
0	+.268	-.053	-.257	-.277	-.235	-.155	-.111	-.075
2	+.204	-.113	-.297	-.297	-.253	-.168	-.133	-.117
4	+.124	-.180	-.341	-.328	-.268	-.188	-.166	-.168
8	-.055	-.315	-.419	-.384	-.286	-.219	-.228	-.239
12	-.266	-.452	-.503	-.448	-.333	-.266	-.341	-.313
16	-.510	-.576	-.592	-.505	-.392	-.352	-.508	-.415
18	-.627	-.665	-.627	-.514	-.421	-.408	-.612	-.466

LOWER WING—SECTION F—LOWER SURFACE

-4	-.450	-.366	-.102	-.082	-.031	-.007	+.007	+.075
0	-.315	-.180	-.111	-.071	-.013	.000	+.007	+.022
2	-.235	-.126	-.080	-.064	-.011	+.002	+.007	+.020
4	-.160	-.086	-.055	-.053	-.004	.000	+.004	+.013
8	-.067	-.002	-.007	-.035	+.007	.000	.000	+.007
12	+.120	+.071	+.040	-.020	+.013	-.002	-.004	-.002
16	+.226	+.140	+.084	.000	+.024	.000	-.004	.000
18	+.275	+.173	+.106	+.007	+.027	.000	-.002	.000

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—+30° STAGGER

UPPER WING—SECTION A—UPPER SURFACE

Incidence degs.	Pressure Openig							
	1	2	2	4	5	6	7	8
-4	+.344	-.020	-.317	-.441	-.386	-.306	-.275	-.051
0	+.069	-.317	-.576	-.616	-.494	-.428	-.246	-.082
2	-.067	-.485	-.707	-.694	-.563	-.517	-.217	-.098
4	-.477	-.680	-.862	-.785	-.649	-.503	-.246	-.109
8	-.895	-1.104	-1.159	-.960	-.875	-.443	-.297	-.131
12	-1.553	-1.556	-1.327	-1.188	-.745	-.508	-.313	-.146
16	-2.194	-1.952	-1.709	-1.518	-.782	-.508	-.310	-.188
18	-2.374	-2.070	-1.786	-1.757	-.754	-.463	-.299	-.233

UPPER WING—SECTION A—LOWER SURFACE

-4	-.412	-.324	-.177	-.075	-.035	-.022	-.009	+.013
0	-.160	-.078	-.049	-.027	-.004	-.009	-.007	+.004
2	-.018	.000	+.004	+.009	+.018	+.004	-.002	.000
4	+.102	+.071	+.053	+.040	+.031	+.009	-.007	-.009
8	+.304	+.202	+.153	+.104	+.071	+.031	+.002	-.015
12	+.423	+.304	+.233	+.157	+.102	+.044	.000	-.029
16	+.481	+.379	+.297	+.199	+.126	+.053	-.007	-.060
18	+.488	+.399	+.317	+.215	+.131	+.049	-.018	-.086

UPPER WING—SECTION B—UPPER SURFACE

-4	+.352	+.009	-.301	-.421	-.357	-.288	-.250	-.031
0	+.095	-.293	-.563	-.587	-.466	-.412	-.239	-.064
2	-.086	-.461	-.700	-.672	-.532	-.490	-.199	-.077
4	-.317	-.647	-.838	-.749	-.607	-.532	-.219	-.084
8	-.835	-1.050	-1.119	-.913	-.811	-.437	-.268	-.109
12	-1.451	-1.467	-1.396	-1.137	-.720	-.488	-.290	-.126
116	-2.081	-1.870	-1.622	-1.471	-.734	-.528	-.288	-.171
18	-2.338	-1.972	-1.702	-1.378	-.707	-.441	-.275	-.199

UPPER WING—SECTION B—LOWER SURFACE

-4	-.399	-.293	-.188	-.062	-.020	-.007	+.004	+.031
0	-.162	-.086	-.053	-.020	+.004	+.002	+.007	+.015
2	-.133	+.002	+.007	+.018	+.024	+.013	+.009	+.011
4	+.104	+.075	+.055	+.047	+.040	+.020	+.007	+.004
8	+.286	+.195	+.144	+.104	+.073	+.035	+.009	-.007
12	+.417	+.304	+.228	+.160	+.106	+.051	+.011	-.022
16	+.472	+.370	+.284	+.202	+.131	+.055	+.002	-.053
18	+.477	+.379	+.290	+.204	+.171	+.051	-.009	-.073

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—+30° STAGGER

UPPER WING—SECTION C—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.361	-.009	-.308	-.403	-.344	-.270	-.242	-.029
0	+.137	-.239	-.521	-.545	-.430	-.375	-.443	-.055
2	-.036	-.397	-.643	-.616	-.485	-.448	-.180	-.069
4	-.211	-.550	-.767	-.691	-.554	-.490	-.195	-.080
8	-.669	-.906	-1.015	-.829	-.729	-.379	-.239	-.098
12	-1.217	-1.283	-1.254	-1.004	-.716	-.430	-.266	-.115
16	-1.777	-1.651	-1.465	-1.281	-.676	-.459	-.275	-.137
18	-2.023	-1.790	-1.555	-1.385	-.691	-.457	-.282	-.166

UPPER WING—SECTION C—LOWER SURFACE

-4	-.523	-.308	-.155	-.062	-.029	-.011	+.007	+.031
0	-.195	-.111	-.062	-.019	-.002	.000	+.051	+.020
2	-.067	-.029	-.015	+.009	+.013	+.009	+.009	+.016
4	+.064	+.051	+.038	+.042	+.031	+.016	+.011	+.011
8	+.242	+.171	+.120	+.098	+.060	+.029	+.009	.000
12	+.379	+.273	+.199	+.151	+.091	+.044	+.011	-.013
16	+.454	+.348	+.262	+.193	+.113	+.051	+.007	-.036
18	+.474	+.375	+.284	+.208	+.122	+.053	+.002	-.053

UPPER WING—SECTION D—UPPER SURFACE

-4	+.315	+.022	-.273	-.375	-.297	-.242	-.219	-.016
0	+.144	-.184	-.446	-.477	-.359	-.321	-.202	-.040
2	+.020	-.321	-.554	-.545	-.408	-.379	-.157	-.053
4	-.117	-.448	-.652	-.601	-.459	-.437	-.168	-.069
8	-.470	-.745	-.864	-.714	-.590	-.344	-.215	-.093
12	-.909	-1.077	-1.075	-.858	-.678	-.388	-.257	-.120
16	-1.352	-1.387	-1.263	-1.041	-.579	-.434	-.286	-.142
18	-.942	-.918	-.771	-.769	-.559	-.521	-.459	-.350

UPPER WING—SECTION D—LOWER SURFACE

-4	-.432	-.357	-.197	-.062	-.026	-.009	+.006	+.033
0	-.277	-.133	-.086	-.039	-.011	-.004	+.004	+.020
2	-.162	-.062	-.039	-.013	-.004	+.002	+.004	+.016
4	-.059	.000	+.002	+.011	+.016	+.006	+.002	+.009
8	+.104	+.117	+.082	+.062	+.038	+.016	-.004	.000
12	+.211	+.211	+.153	+.106	+.066	+.026	+.002	-.013
16	+.273	+.295	+.217	+.148	+.091	+.040	+.004	-.022
18	+.224	+.242	+.164	+.100	+.033	-.029	-.076	-.136

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—+30° STAGGER

UPPER WING—SECTION E—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-41	+.379	+.042	-.246	-.364	-.268	-.211	-.171	-.018
0	+.264	-.140	-.393	-.408	-.326	-.273	-.155	-.042
2	+.113	-.250	-.479	-.461	-.370	-.310	-.168	-.062
4	-.009	-.359	-.565	-.514	-.421	-.337	-.175	-.084
8	-.306	-.585	-.723	-.603	-.508	-.326	-.231	-.133
12	-.674	-.856	-.900	-.716	-.590	-.372	-.295	-.188
16	-1.082	-1.137	-1.075	-.849	-.581	-.446	-.361	-.234
18	-1.276	-1.245	-1.141	-.913	-.594	-.477	-.386	-.208

UPPER WING—SECTION E—LOWER SURFACE

-4	-.415	-.350	-.195	-.058	-.030	-.011	+.002	+.027
0	-.279	-.155	-.104	-.051	-.018	-.009	-.004	+.009
2	-.175	-.095	-.064	-.031	-.009	-.007	-.007	+.002
4	-.075	-.040	-.031	-.009	.000	-.004	-.009	-.004
8	+.102	+.066	+.038	+.031	+.018	.000	-.011	-.013
12	+.246	-.162	+.104	+.071	+.040	+.009	-.013	-.027
16	+.357	+.242	+.164	+.109	+.060	+.016	-.011	-.033
18	+.319	+.208	+.131	+.080	+.022	-.027	-.016	-.091

UPPER WING—SECTION F—UPPER SURFACE

-4	+.350	+.036	-.196	-.253	-.219	-.160	-.113	-.053
0	+.195	-.089	-.335	-.344	-.301	-.219	-.193	-.177
2	+.135	-.186	-.368	-.361	-.306	-.233	-.211	-.231
4	+.040	-.275	-.430	-.406	-.328	-.268	-.268	-.333
8	-.188	-.452	-.561	-.497	-.403	-.344	-.437	-.548
12	-.457	-.641	-.649	-.607	-.501	-.496	-.709	-.765
16	-.745	-.831	-.822	-.694	-.592	-.696	-.971	-.953
18	-.672	-.760	-.743	-.596	-.536	-.579	-.780	-.820

UPPER WING—SECTION F—LOWEE SURFACE

-4	-.401	-.306	-.117	-.080	-.035	-.018	-.006	+.053
0	-.284	-.157	-.102	-.091	-.027	-.024	-.020	-.013
2	-.202	-.115	-.080	-.086	-.018	-.027	-.027	-.024
4	-.129	-.078	-.060	-.089	-.016	-.038	-.038	-.038
8	+.013	-.006	-.018	-.091	-.011	-.044	-.053	-.062
12	+.135	+.064	+.022	-.053	-.006	-.058	-.071	-.082
16	+.231	+.124	+.060	-.049	.000	-.111	-.082	-.098
18	+.211	+.104	+.040	-.058	-.020	-.086	-.102	-.113

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—+30° STAGGER

LOWER WING—SECTION A—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.412	+.086	-.211	-.299	-.299	-.228	-.195	-.015
0	+.282	-.067	-.328	-.410	-.335	-.266	-.239	-.004
2	+.197	-.151	-.392	-.443	-.355	-.288	-.224	-.004
4	+.082	-.257	-.466	-.483	-.381	-.321	-.166	-.011
8	-.193	-.463	-.610	-.554	-.439	-.395	-.126	-.011
12	-.552	-.703	-.756	-.623	-.530	-.315	-.140	-.009
16	-.958	-.991	-.991	-.898	-.559	-.284	-.146	-.004
18	-1.354	-1.115	-.993	-.822	-.466	-.293	-.140	-.004

LOWER WING—SECTION A—LOWER SURFACE

-4	-.490	-.457	-.257	-.080	-.033	-.009	+.020	+.053
0	-.308	-.166	-.104	-.038	+.016	+.031	+.044	+.066
2	-.173	-.071	-.031	+.004	+.038	+.049	+.058	+.073
4	-.047	+.006	+.027	+.044	+.067	+.069	+.071	+.080
8	-.171	+.144	+.131	+.120	+.120	+.109	+.100	+.095
12	+.335	+.259	+.226	+.188	+.173	+.148	+.129	+.113
16	+.446	+.361	+.308	+.255	+.222	+.186	+.155	+.124
18	+.479	+.399	+.344	+.282	+.244	+.204	+.171	+.129

LOWER WING—SECTION B—UPPER SURFACE

4	+.419	+.097	-.208	-.330	-.279	-.211	-.175	.000
0	+.290	-.055	-.328	-.399	-.317	-.253	-.224	-.033
2	+.204	-.142	-.395	-.434	-.337	-.279	-.211	+.006
4	+.100	-.235	-.459	-.466	-.359	-.306	-.155	+.004
8	-.160	-.437	-.598	-.536	-.417	-.381	-.111	+.002
12	-.528	-.687	-.751	-.614	-.505	-.266	-.126	+.002
16	-.955	-.957	-.915	-.714	-.550	-.275	-.133	+.006
18	-1.197	-1.104	-.993	-.816	-.459	-.286	-.131	+.004

LOWER WING—SECTION B—LOWER SURFACE

-4	-.490	-.443	-.255	-.075	-.022	+.006	+.031	+.064
0	-.304	-.168	-.104	-.029	+.026	+.040	+.055	+.080
2	-.176	-.073	-.036	+.009	+.044	+.055	+.069	+.084
4	-.053	.000	+.022	+.051	+.073	+.075	+.080	+.089
8	+.162	+.137	+.126	+.124	+.126	+.113	+.109	+.104
12	+.328	+.257	+.217	+.193	+.177	+.155	+.137	+.120
16	+.439	+.352	+.297	+.255	+.222	+.186	+.162	+.133
18	+.470	+.390	+.333	+.281	+.244	+.204	+.173	+.135

PRESSURE DISTRIBUTION ON GOTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—+30° STAGGER

LOWER WING—SECTION C—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.412	+.100	-.213	-.321	-.275	-.208	-.175	+.006
0	+.297	-.053	-.333	-.395	-.313	-.250	-.217	+.011
2	+.206	-.140	-.403	-.430	-.333	-.277	-.197	+.006
4	+.102	-.233	-.466	-.463	-.355	-.310	-.146	.000
8	-.155	-.435	-.607	-.536	-.415	-.375	-.111	-.002
12	-.403	-.663	-.749	-.605	-.501	-.266	-.129	-.002
16	-.884	-.913	-.902	-.705	-.559	-.275	-.140	-.002
18	-1.112	-1.066	-.982	-.785	-.472	-.288	-.140	-.004

LOWER WING—SECTION C—LOWER SURFACE

-4	-.550	-.477	-.195	-.069	-.027	+.002	+.033	+.062
0	-.346	-.162	-.091	-.022	+.018	+.036	+.053	+.075
2	-.191	-.093	-.042	+.006	+.036	+.049	+.062	+.080
4	-.069	-.013	-.011	+.042	+.060	+.067	+.073	+.084
8	+.142	+.124	+.109	+.111	+.109	+.098	+.095	+.095
12	+.299	+.235	+.193	+.177	+.155	+.135	+.124	+.109
16	+.423	+.335	+.237	+.238	+.199	+.166	+.146	+.120
18	+.470	+.368	+.310	+.264	+.219	+.140	+.157	+.124

LOWER WING—SECTION D—UPPER SURFACE

-4	+.350	+.080	-.208	-.310	-.246	-.191	-.173	+.011
0	+.242	-.064	-.321	-.375	-.273	-.228	-.193	+.011
2	+.164	-.144	-.381	-.406	-.293	-.259	-.148	+.007
4	+.073	-.231	-.441	-.432	-.313	-.293	-.106	+.002
8	-.153	-.415	-.567	-.492	-.372	-.339	-.104	-.007
12	-.432	-.621	-.694	-.550	-.477	-.239	-.124	-.013
16	-.794	-.853	-.831	-.647	-.485	-.259	-.142	-.022
18	-.975	-.964	-.891	-.714	-.410	-.273	-.104	-.024

LOWER WING—SECTION D—LOWER SURFACE

-4	-.468	-.426	-.295	-.064	-.022	+.002	+.027	+.058
0	-.346	-.193	-.122	-.042	+.009	+.020	+.042	+.067
2	-.242	-.106	-.060	-.013	+.022	+.036	+.049	+.069
4	-.142	-.042	-.016	+.016	+.042	+.047	+.055	+.069
8	+.044	+.086	+.075	+.080	+.084	+.078	+.075	+.078
12	+.184	+.199	+.162	+.142	+.126	+.108	+.100	+.088
16	+.259	+.290	+.231	+.193	+.162	+.135	+.117	+.095
18	+.284	+.328	+.270	+.219	+.182	+.151	+.129	+.100

PRESSURE DISTRIBUTION ON GÖTTINGEN-387 AEROFOIL

Pressures expressed in terms of ρV^2 Air Speed 40 f.p.s.

BIPLANE—+30° STAGGER

LOWER WING—SECTION E—UPPER SURFACE

Incidence degs.	Pressure Opening							
	1	2	3	4	5	6	7	8
-4	+.381	+.084	-.202	-.284	-.226	-.171	-.140	+.006
0	+.295	-.044	-.299	-.332	-.255	-.204	-.129	+.004
2	+.224	-.117	-.350	-.357	-.273	-.224	-.109	.000
4	+.137	-.197	-.408	-.386	-.339	-.248	-.102	-.011
8	-.071	-.364	-.516	-.439	-.352	-.264	-.113	-.038
12	-.337	-.536	-.621	-.492	-.419	-.226	-.151	-.064
16	-.638	-.738	-.731	-.565	-.441	-.253	-.180	-.078
18	-.800	-.831	-.782	-.612	-.403	-.275	-.195	-.084

LOWER WING—SECTION E—LOWER SURFACE

-4	-.450	-.408	-.253	-.062	-.024	.000	+.022	+.051
0	-.333	-.195	-.126	-.051	-.002	+.013	+.029	+.051
2	-.237	-.124	-.078	-.029	+.006	+.020	+.033	+.051
4	-.137	-.064	-.036	-.002	-.022	+.031	+.038	+.051
8	+.038	+.042	+.036	+.044	+.053	+.049	+.051	+.055
12	+.195	+.144	+.109	+.093	+.084	+.071	+.064	+.062
16	+.319	+.233	+.177	+.140	+.115	+.093	+.082	+.069
18	+.372	+.275	+.208	+.166	+.131	+.102	+.091	+.071

LOWER WING—SECTION F—UPPER SURFACE

-4	+.359	+.064	-.164	-.215	-.188	-.137	-.084	-.022
0	+.315	-.042	-.239	-.257	-.215	-.142	-.104	-.067
2	+.208	-.098	-.277	-.277	-.231	-.153	-.124	-.102
4	+.135	-.162	-.321	-.308	-.246	-.168	-.148	-.142
8	-.029	-.295	-.401	-.361	-.269	-.211	-.219	-.215
12	-.235	-.430	-.481	-.428	-.299	-.253	-.299	-.293
16	-.470	-.525	-.572	-.501	-.364	-.330	-.430	-.377
18	-.592	-.647	-.623	-.523	-.406	-.390	-.543	-.441

LOWER WING—SECTION F—LOWER SURFACE

-4	-.439	-.352	-.098	-.080	-.027	-.004	+.011	+.078
0	-.315	-.180	-.113	-.076	-.013	.000	+.009	+.024
2	-.242	-.131	-.082	-.071	-.011	+.002	+.009	+.022
4	-.166	-.091	-.058	-.067	-.004	.000	+.007	+.016
8	-.024	-.013	-.011	-.058	+.004	.000	.000	+.009
12	+.106	+.062	+.036	-.049	+.013	-.002	-.002	+.002
16	+.250	+.126	+.073	-.042	+.018	+.004	-.007	-.002
18	+.255	+.160	+.098	-.033	+.022	+.004	-.007	.000

PLATE I.

PRESSURE DISTRIBUTION OVER GÖTTINGEN-387 AEROFOIL

WITH SQUARE WING TIPS.

NORMAL FORCE DIAGRAMS.

MODEL 3" 18" BRASS. [UNIT OF PRESSURE = ρV^2 POS. PER SQ. FT. AIR SPEED = 40 FT. PER SEC.ANGLE OF INCIDENCE = -4°

MID-SPAN

SECTION A

SECTION B

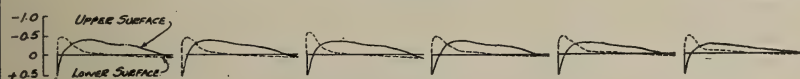
SECTION C

SECTION D

SECTION E

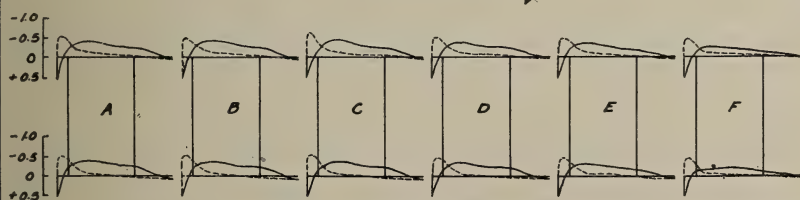
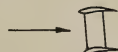
SECTION F

MONOPLANE



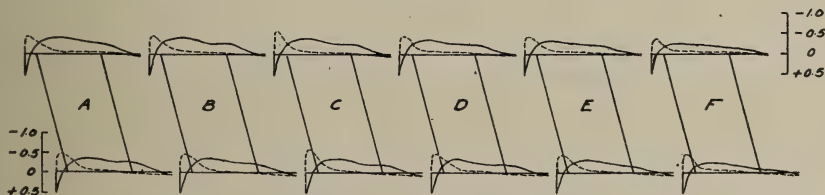
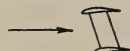
BIPLANE - GAP = CHORD.

NO STAGGER.



BIPLANE - GAP = CHORD.

+15° STAGGER.



BIPLANE - GAP = CHORD.

+30° STAGGER.

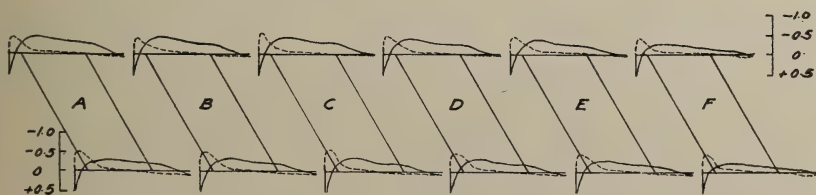
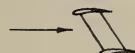


PLATE 2.

PRESSURE DISTRIBUTION OVER GÖTTINGEN-387 AEROFOIL

WITH SQUARE WING TIPS.

NORMAL FORCE DIAGRAMS.

MODEL 3 1/8" BRASS

UNIT OF PRESSURE = ρV^2 POS. PER SQ. FT.

AIR SPEED - 40 FT. PER SEC.

ANGLE OF INCIDENCE = 0°

MID-SPAN

SECTION A

SECTION B

SECTION C

SECTION D

SECTION E

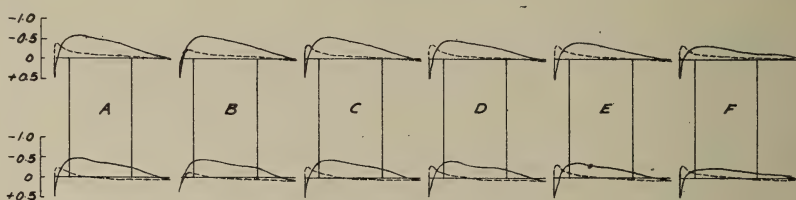
SECTION F

MONOPLANE



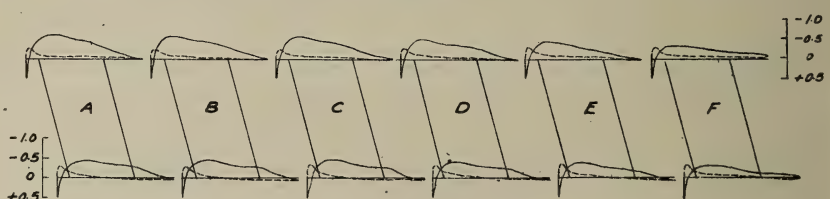
BIPLANE - GAP = CHORD.

NO STAGGER.



BIPLANE - GAP = CHORD.

+15° STAGGER.



BIPLANE - GAP = CHORD

+30° STAGGER.

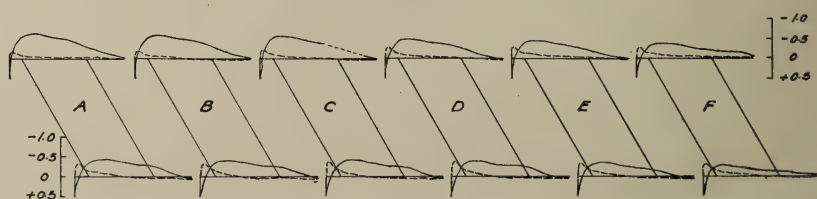


PLATE 3

PRESSURE DISTRIBUTION OVER GÖTTINGEN-387 AEROFOIL

WITH SQUARE WING TIPS.

NORMAL FORCE DIAGRAMS.

MODEL 3 $\frac{1}{8}$ " BRASSUNIT OF PRESSURE $-\rho V^2$ PDS. PER SQ. FT.

AIR SPEED 40 FT. PER SEC.

ANGLE OF INCIDENCE = 2°

MID-SPAN

SECTION A

SECTION B

SECTION C

SECTION D

SECTION E

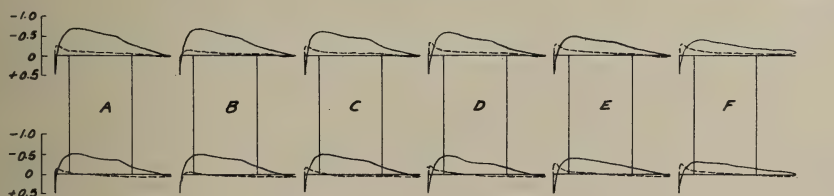
SECTION F

MONOPLANE.



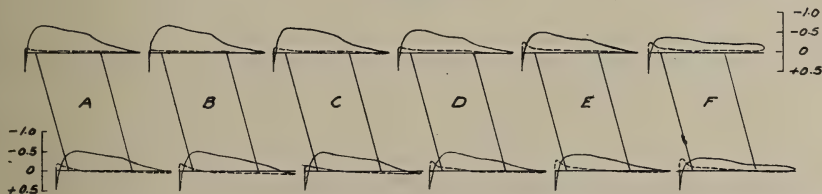
BIPLANE - GAP = CHORD.

NO STAGGER.



BIPLANE - GAP = CHORD.

+15° STAGGER.



BIPLANE - GAP = CHORD.

+30° STAGGER.

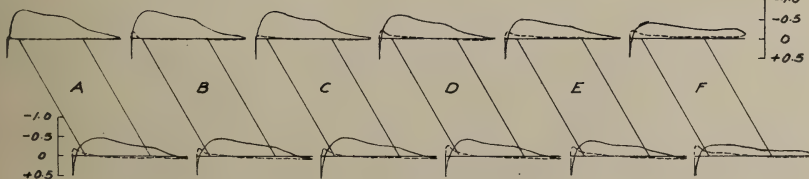


PLATE 4

PRESSURE DISTRIBUTION OVER GÖTTINGEN-387 AEROFOIL

WITH SQUARE WING TIPS.

NORMAL FORCE DIAGRAMS

MODEL 3"x18" BRASS UNIT OF PRESSURE $-\rho V^2$ LBS. PER SQ. FT. AIR SPEED - 40 FT. PER SEC.ANGLE OF INCIDENCE $= 4^\circ$

MID-SPAN

SECTION A.

SECTION B.

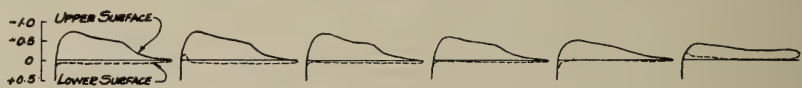
SECTION C.

SECTION D.

SECTION E.

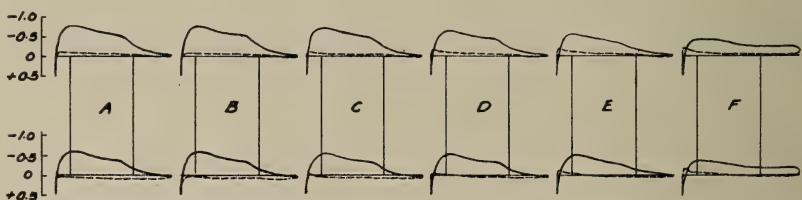
SECTION F.

MONOPLANE.

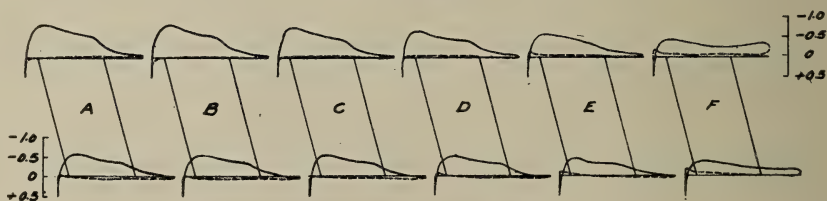


BIPLANE - GAP = CHORD.

NO STAGGER.



BIPLANE - GAP = CHORD.

 $+15^\circ$ STAGGER.

BIPLANE - GAP = CHORD.

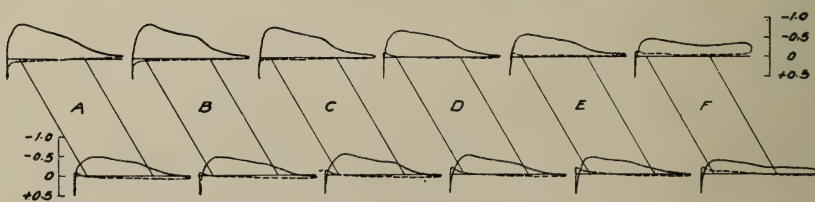
 $+30^\circ$ STAGGER.

PLATE 5.

PRESSURE DISTRIBUTION OVER GÖTTINGEN-387 AEROFOIL.

WITH SQUARE WING TIPS.

NORMAL FORCE DIAGRAMS.

MODEL 3"x18" BRASS

UNIT OF PRESSURE - PV^2 LBS. PER SQ. FT.

AIR SPEED - 40 FT. PER SEC.

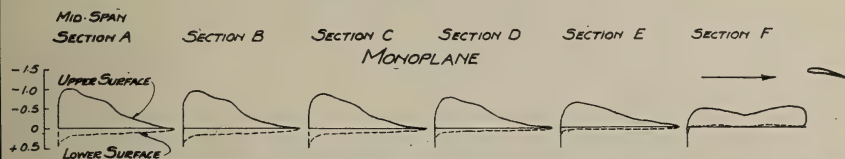
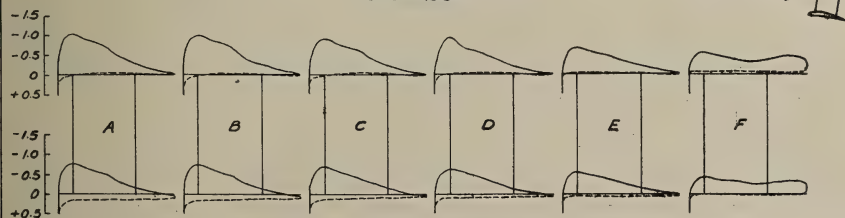
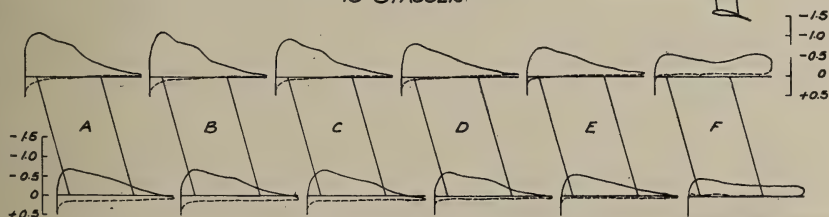
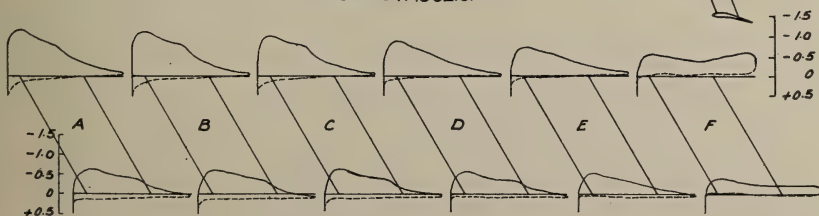
ANGLE OF INCIDENCE = 8° BIPLANE - GAP = CHORD
NO STAGGER.BIPLANE - GAP = CHORD
+ 15° STAGGER.BIPLANE - GAP = CHORD.
+ 30° STAGGER.

PLATE 6. PRESSURE DISTRIBUTION OVER GÖTTINGEN-387 AIRFOIL

WITH SQUARE WING TIPS.

NORMAL FORCE DIAGRAMS.

MODEL 3"x18" BRASS UNIT OF PRESSURE = ρV^2 PDS. PER SQ. FT. AIR SPEED - 40 FT. PER SEC.

ANGLE OF INCIDENCE - 12°

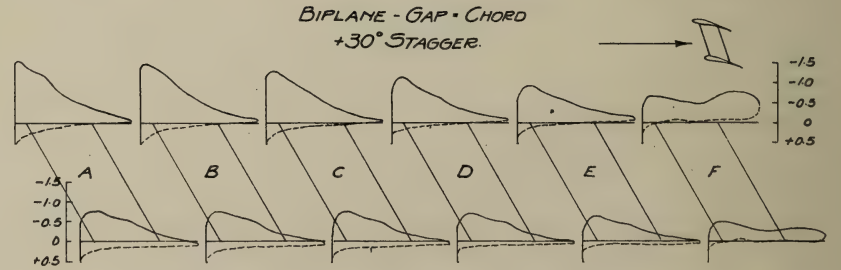
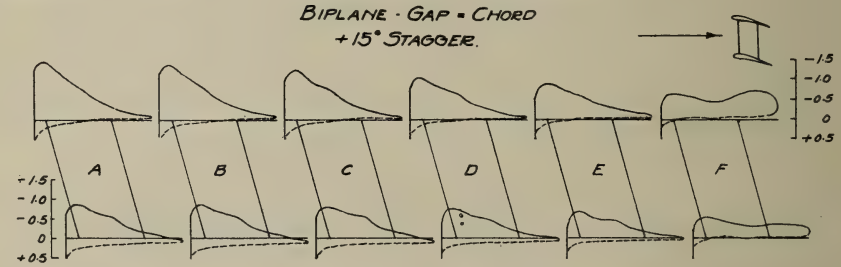
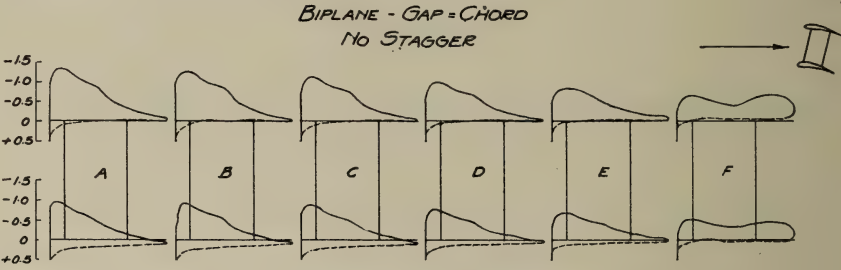
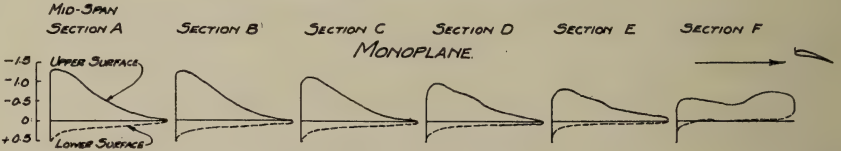


PLATE 7

PRESSURE DISTRIBUTION OVER GÖTTINGEN-387 AEROFOIL

WITH SQUARE WING TIPS.

NORMAL FORCE DIAGRAMS

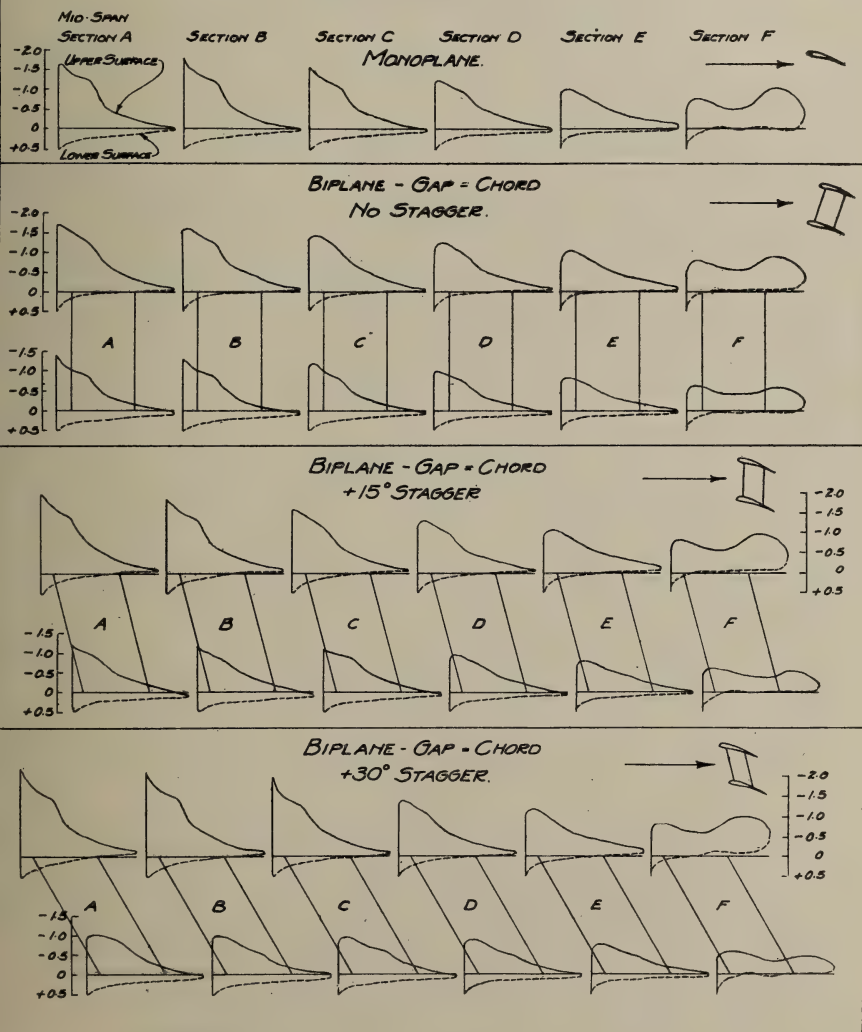
MODEL 3" x 18" BRASS UNIT OF PRESSURE = ρV^2 PDS. PER SQ. FT. AIR SPEED = 40 FT. PER SEC.ANGLE OF INCIDENCE = 10° 

PLATE 8

PRESSURE DISTRIBUTION OVER GÖTTINGEN-387 AEROFOIL

WITH SQUARE WING TIPS

NORMAL FORCE DIAGRAMS

MODEL 3" x 18" BRASS

UNIT OF PRESSURE = ρV^2 LBS. PER SQ. FT.

AIR SPEED - 40 FT. PER SEC.

ANGLE OF INCIDENCE = 18°

MID-SPAN

SECTION A

SECTION B

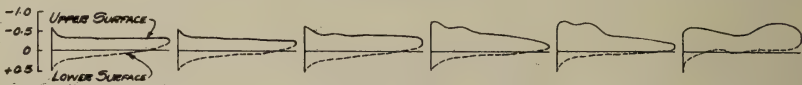
SECTION C

SECTION D

SECTION E

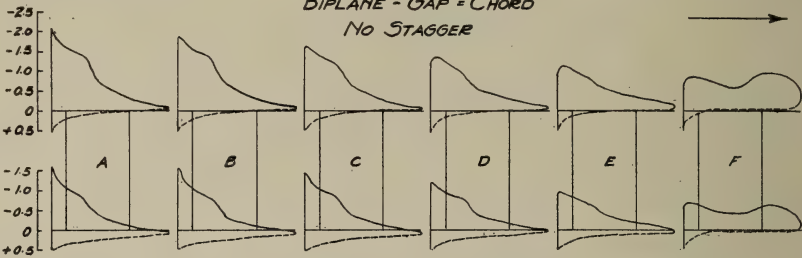
SECTION F

MONOPLANE



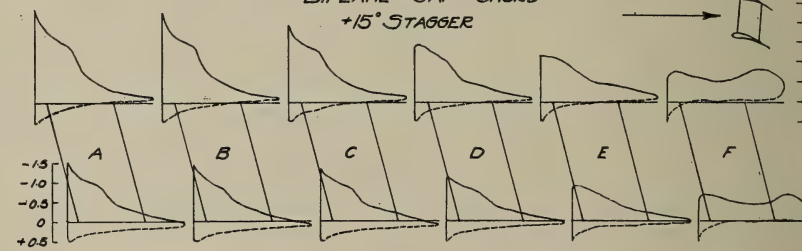
BIPLANE - GAP = CHORD

NO STAGGER



BIPLANE - GAP = CHORD

+15° STAGGER



BIPLANE - GAP = CHORD

+30° STAGGER

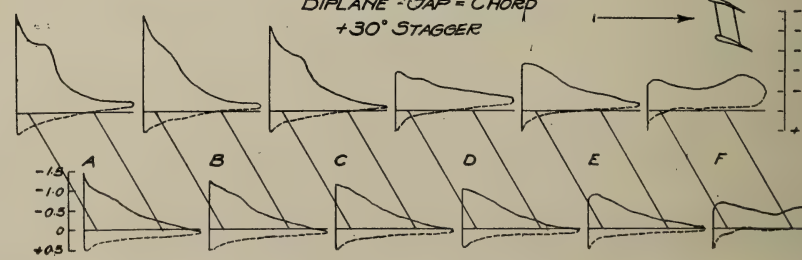


PLATE 9.

PRESSURE DISTRIBUTION OVER GOTTINGEN-387 AEROFOIL
WITH SQUARE WING TIPS.

NORMAL FORCE ALONG SEMI-SPAN

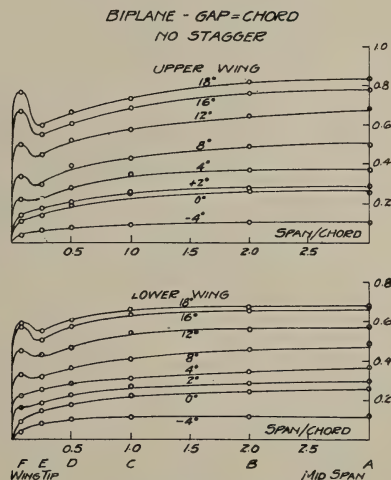
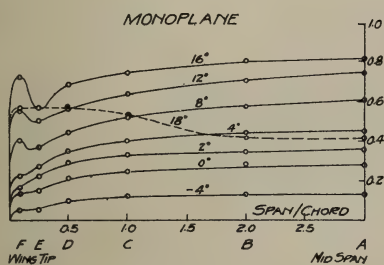
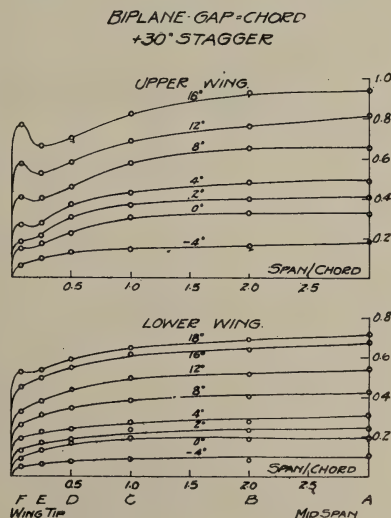
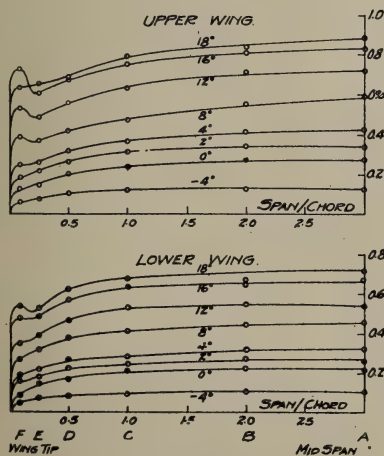
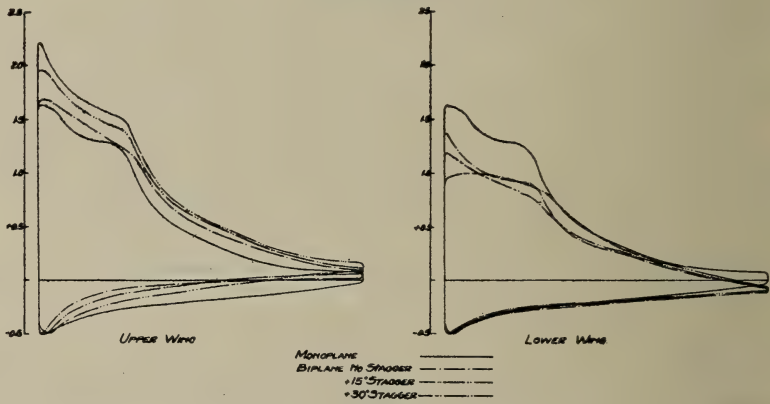
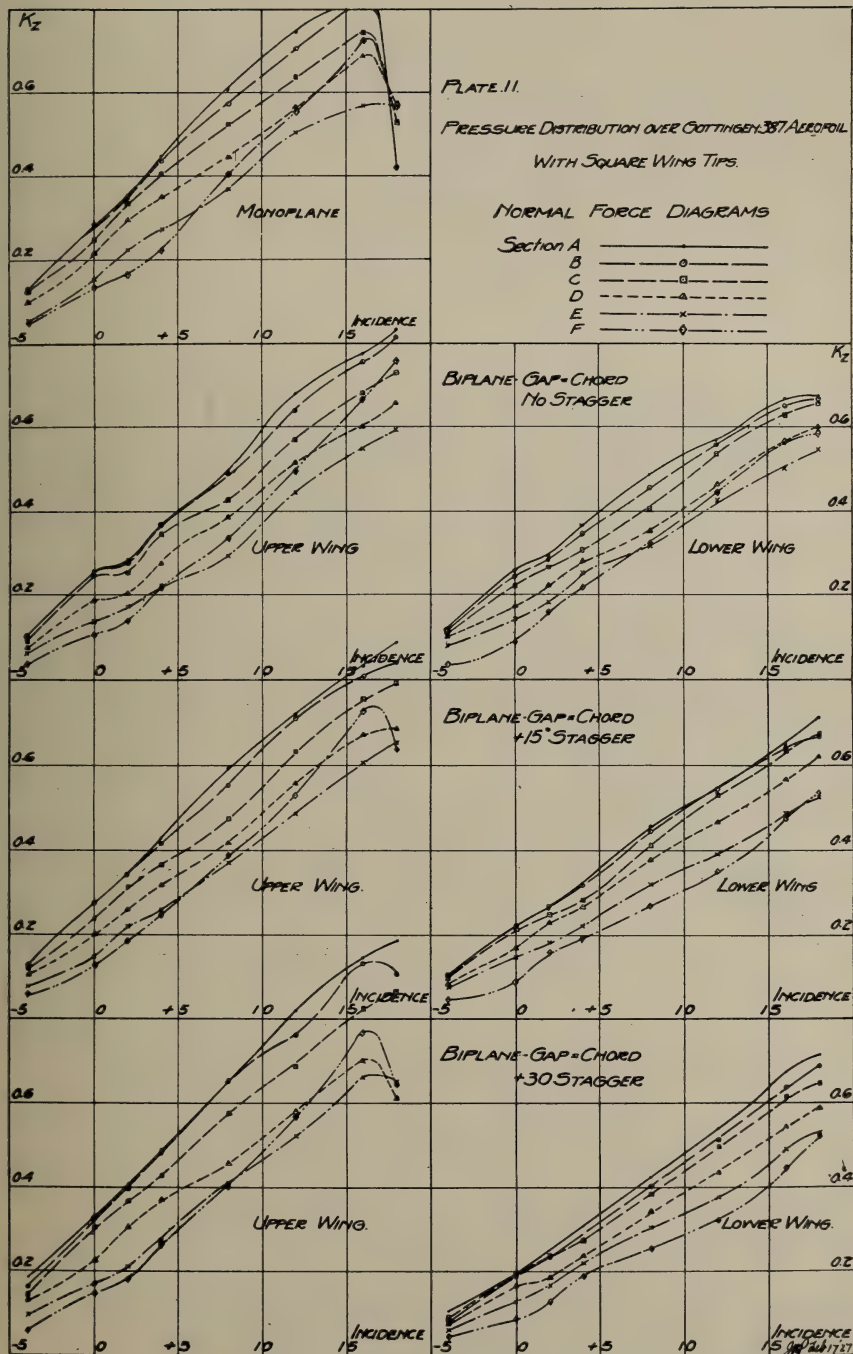
MODEL 3 $\frac{1}{8}$ " BRASS UNIT OF PRESSURE = ρV^2 LBS. PER SQ. FT. AIR SPEED 40 FT. PER SECBIPLANE - GAP = CHORD
+15° STAGGER.

PLATE 10 PRESSURE DISTRIBUTION OVER GÖTTINGEN-387 AIRFOIL
WITH SQUARE WING TIPS.
NORMAL FORCE DIAGRAMS
MODEL 5'x18" BRASS UNIT OF PRESSURE ρV^2 PDS. PER SQ. FT. AIR SPEED 40 FT. PER SEC.
ANGLE OF INCIDENCE -16°
MEDIAN SECTION (A)



SAUNDERS



BIPLANE INVESTIGATION

By J. H. PARKIN, M.E., F.R.Ae.S., J. E. B. SHORTT, B.A.Sc., and
J. G. CADE, B.A.Sc.

SUMMARY

Reason for Investigation.

The following study was made to determine the aerodynamic characteristics of complete biplanes and of the separate wings of biplanes composed of wings of moderate and thick section, and to determine suitable factors for use in designing and checking aeroplanes using such wings.

Range of Investigation.

Three biplanes of unit gap/chord ratio and staggers of 0° , $+15^\circ$ and $+30^\circ$, of each of three aerofoil sections, namely, Aircscrew-4, U.S.A.-27 and Gottingen-387, were tested. The characteristics of the complete biplanes, and of the individual wings were determined at an air speed of 40 f.p.s. over a range of incidence from -6° to about $+20^\circ$. Similar tests were made on a biplane of Aircscrew-4 section, with gap equal to one-half the chord and staggers of -15° , 0° , $+15^\circ$, 30° and 45° . The corresponding monoplanes were also tested. The aerofoils were of duralumin of 3" chord and 18" span.

Comparisons have been made with published results of certain previous biplane investigations.

Conclusions.

The influences of biplaning, stagger and gap for these aerofoils are generally of the same character as for thin aerofoils. The principal effects are on lift and lift/drag, drag and centre of pressure, being little influenced.

The lift of the unstaggered biplane of unit gap/chord ratio at vl-10. was found to be 85% of the monoplane lift, which is in good agreement with the results of previous tests of biplanes of thin and thick section,

The stagger effect on lift is relatively small, the biplane with 15° stagger having 1.8% more lift than the unstaggered biplane, and that

with 30° stagger 6% more; or the lifts are 86.5% and 90% respectively, of the monoplane lift.

The effects of biplaning and stagger on the characteristics of the individual wings were observed to be generally the same, although somewhat more marked than for the thin wing biplanes.

The lift of the upper wing compared with that of the lower wing is as follows:

	Stagger 0°	Stagger 15°	Stagger 30°
Low incidence condition (2°)	1.03	1.23	1.51
High incidence condition (14°) . . .	1.14	1.28	1.41
Average (positive incidences)	1.08	1.26	1.45

The percentage of the total lift of the biplane carried by each wing is:

	Stagger 0°	Stagger 15°	Stagger 30°
Upper Wing	52.	55.5	59.
Lower Wing	48.	44.5	41.

These figures indicate that the upper wing carries more load and the lower wing less load than in the case of biplanes of thin wing section, and hence the load factors for the wings of biplanes of the latter type are not strictly applicable to biplane wings of the type here tested.

Introduction and Reasons for the Investigation.

There is at the present time a tendency to use wings of moderate and thick section in biplane combinations, not only for heavy military machines, but particularly for transport aeroplanes for commercial and civil purposes. While considerable information has been collected and is available relative to the aerodynamic characteristics of biplanes composed of thin planes, very little information¹ has been published concerning the characteristics of biplanes made up of thicker planes.

The present investigation was therefore undertaken, primarily with the object of supplying the Department of National Defence and aircraft constructors in Canada with the aerodynamic characteristics of biplanes composed of U.S.A.-27 and Gottingen-387 aerofoils for use in the design, stressing and checking of aeroplanes for Canadian service. The investigation was also intended to supplement information derived from the pressure distribution studies on these same biplanes described in Aeronautical Research Papers Nos. 18 and 18a. Incidentally, advantage was taken of the opportunity thus afforded to compare the general behaviour of biplanes composed of moderate and thick section wings

¹Refer to list of literature at the end of this paper.

with that of the more usual thin wing biplanes and to derive suitable biplane, stagger and relative wing loading factors for the former type of biplane, for use in aeroplane computations.

The models and holding apparatus employed in the investigation were secured with funds supplied by the Royal Canadian Air Force, through the Associate Air Research Committee of the National Research Council. The work was carried out in the Aerodynamic Laboratory of the Department of Mechanical Engineering, Faculty of Applied Science and Engineering, University of Toronto.

Range of Investigation.

In order to keep the investigation within reasonable limits and secure the results quickly, certain important factors had to be ignored and the work confined to a study of a few typical sections and conventional biplane combinations likely to be most generally useful in practice.

In addition, to the U.S.A.-27 and Gottingen-387 aerofoils, on which information was specifically required and for which pressure distribution studies had already been made, the only other section tested was the Aircsrew-4, a fairly thick aerofoil section (maximum thickness to chord ratio of 0.127) with flat undersurface and good aerodynamic qualities. Other work had also been done on the latter section at Toronto.² The U.S.A.-27, a moderately thick section ($t/c=0.119$) of excellent characteristics, and the Gottingen-387, a thick section ($t/c=0.154$) of very good qualities, are sufficiently well known to require no further comment.

The biplane combinations investigated were limited to three, without decalage, of unit gap/chord ratio, and of three staggers, namely 0° , $+15^\circ$ and $+30^\circ$. The corresponding monoplanes were also tested to afford a basis of comparison.

The main investigation consisted in the determination of the aerodynamic characteristics, lift and drag coefficients, lift/drag and centre of pressure, for the complete biplanes and for the individual wings composing the biplanes over the usual range of incidence from -6° to just beyond the critical angle, at an air speed of 40 feet per second.

Subsequently a similar series of measurements were made on biplanes of Aircsrew-4 section, with gap equal to one-half the chord and staggers of -15° , 0° , $+15^\circ$, 30° and 45° . These more or less extreme conditions were examined to determine the general influence on the characteristics of these biplanes of stagger and reduction in gap.

²See Aero. Res. Papers Nos. 8 and 10. Aerofoil termed NPLAS-4.

Apparatus and Method of Test.

The tests were made in the 4 foot R.A.E. type wind channel of the University of Toronto. Measurements were made on an N.P.L. type balance.

The aerofoils tested were of duralumin, machine cut on the Nichols generating machine, of 3 inch chord and 18 inch span. The ordinates to which the three aerofoils were cut are tabulated in Table No. 1.

The method of supporting the two aerofoils when making the measurements on the complete biplanes differed somewhat from that usually employed with this type of balance. The upper aerofoil of the biplane was supported on a short end spindle carried in an arm projecting from the spindle supporting the lower aerofoil. The wing tips remote from the spindles were tied together by means of a narrow thin brass strip. The arrangement will be apparent from the Photographs Nos. 1 and 2.

This form of support was devised to provide a rigid mounting, interfering as little as possible with the air flow about the aerofoils, and at the same time convenient and flexible for setting and adjusting the aerofoils for the different staggers and gaps.

Spindle interference was determined in the usual way, employing a duplicate dummy set of spindles and guard (see Photographs Nos. 3 and 4).

In making the measurements on the individual aerofoils of each combination, the aerofoil under test was supported in the ordinary way on an end spindle in the chuck of the balance. The second aerofoil was supported independently on an end spindle held in a standard screwed into the turntable in the channel floor surrounding the vertical arm of the balance. (See Photograph No. 5). With this arrangement when the incidence of the test aerofoil is changed by rotating the vertical arm of the balance, the incidence of the other aerofoil of the biplane is changed to correspond by rotating the turntable through an equal angle. Spindle interference was determined using dummy spindles as before. (See Photograph No. 6).

Since the deflections of the two aerofoils with the foregoing set up are not likely equal, a small error may occur, due to the alteration in gap, and probably is the reason for the small differences found in the results for the complete biplane and for the individual wings.

The aerofoils were lined up with the chords parallel to the air stream, using an optical alignment device of great sensitivity. (Refer to Aero Res. Paper No. 16).

The air speed was measured using a side plate and micromanometer of the Chattock type, described in Aeronautical Research Paper No. 16.

Presentation of Results.

The characteristics are presented in the form of the usual absolute coefficients, calculated as follows. Corrections have been applied for the spindle, but no corrections have been made for channel wall interference.

$$\text{Lift Coefficient} - K_L = \frac{L}{\rho A V^2}$$

$$\text{Drag Coefficient} - K_D = \frac{D}{\rho A V^2}$$

where L = lift—pounds

D = drag—pounds

A = aerofoil area—square feet

V = air speed—feet per second

ρ = mass density of air—.002373 slugs per cubic foot

The centres of pressure are expressed in percent of chord from the leading edge in the case of individual wings. For the complete biplanes, instead of calculating the centres of pressure with respect to the geometric mean chord, the centres of pressure are expressed in *percent of chord of the lower wing*.

The centres of pressure were determined using the graphical method devised at Toronto and described in Aeronautical Research Paper No. 15.

The characteristics of the monoplanes and complete biplanes, together with those of the individual wings, are tabulated in Tables Nos. 2-11.

The characteristics of the monoplanes and complete biplanes are plotted on Plates Nos. 1-4, and those of the separate wings on Plates Nos. 5-8. The information relating to one aerofoil is plotted on each plate.

On Plates Nos. 1-4, lift and drag coefficients and centres of pressure are plotted on angle of incidence, at the left, and at the right lift/drag is plotted on lift coefficient, for the monoplane and the three biplanes.

On Plates Nos. 5-8 lift and drag coefficients are plotted on incidence above and centres of pressure on incidence below, for the individual wings, the results for each stagger being, for the sake of clearness, plotted separately. For purposes of comparison the monoplane characteristics have also been plotted on Plates Nos. 5-8.

Discussion of Results.

A. CHARACTERISTICS OF COMPLETE BIPLANES

I. *Effects of Different Variables.*

1. *Biplaning (Zero Stagger).*

- (a) *Gap = Chord.* An examination of Plates Nos. 1-4 indicates that the effects of biplaning on the aerodynamic characteristics are

generally the same for the aerofoils employed in this study as for thin aerofoils. As compared with the monoplane the lift of the biplanes is reduced and the drag increased at all except very small lifts, resulting in a marked reduction in the lift/drag ratio except at small lifts, while travel of the centre of pressure is reduced.³ The critical angle is larger for the biplanes than for the monoplanes. The maximum lift for the biplane of zero stagger is lower than for the monoplane. The minimum drag of the biplane is slightly greater than for the monoplane, in some cases only very slightly.⁴ Beyond the critical the biplane drag is less than that of the monoplane. The maximum L/D of the best biplane is about 10% less than that of the corresponding monoplane.

- (b) $Gap = \frac{1}{2} \text{ Chord}$. Comparatively few figures were obtained in this study on biplanes of gap/chord ratio equal to one-half and these figures, hence, lack the confirmation afforded in the case of the unit gap/chord ratio biplane by the three sets of tests. Hence, the figures for the biplane of this gap/chord ratio are to be regarded as guides showing general effects under these conditions, rather than as absolute figures. The arrangement is an extreme one and unlikely to be used in practice.

The conclusions as to the effect of biplaning to be drawn from the tests with gap equal to one-half chord are generally the same as for the unit gap/chord results. The lift and maximum lift are greatly reduced and the critical angle increased. The drag is increased on the region of minimum drag, but for larger lifts the drag of the biplane is less than that of the monoplane. The C.P. movement is reduced very slightly as compared with the monoplane.

2. Stagger.

- (a) $Gap = \text{Chord}$. Plates 1-4 indicate that positive stagger slightly improves the aerodynamic characteristics of biplanes. Thus, with increase of stagger, while lift and drag are both increased, except at low or negative lifts, the net effect is to increase the L/D at the larger lifts. It will be observed that at low lifts, the curves of L/D on lift coefficient for the biplanes of all staggers are coincident. The gain for 15° stagger is very slight,

³This apparent disagreement with the conclusions of other biplane studies, when the C.P. movement has been found to be increased slightly, is due to the centres of pressure being here referred to the chord of the lower plane, instead of to the geometric mean chord.

⁴Compare R & M, 196, Part II and R & M, 774.

but is appreciable for 30° stagger. The effect of stagger on the movement of the *C.P.* is negligible.

Increasing the positive stagger raises the maximum lift and may increase it above that of the monoplane. Minimum drag is apparently not affected by a variation of positive stagger of the extent here investigated. The maximum L/D is affected to a varying degree for different aerofoils by a change in stagger, but in any case only slightly. Thus, with the Göttingen-387 biplane, the maximum L/D is not affected, while with the U.S.A.-27 the maximum L/D for 30° stagger is about $3\frac{1}{2}\%$ greater than for zero stagger.

- (b) $Gap = \frac{1}{2} \text{ Chord}$. Plate 2 shows that with gap equal to one-half the chord, the general effects of stagger are as pointed out in the foregoing. The effects of an increase in stagger from $+30^\circ$ to $+45^\circ$ are quite marked and about equal to those effected by an increase from 0° to $+30^\circ$. At this small gap increase in stagger reduces the *C.P.* movement, that for a 45° stagger being quite small.

Negative stagger, as shown by the results of the tests on the one combination, -15° stagger, plotted on Plate No. 2, reduces the lift over the whole range and reduces the drag at all angles above that of minimum drag, as compared with biplanes of positive stagger. The curve of L/D plotted on lift coefficient differs but little from that for the biplane of zero stagger, although the maximum L/D is lower.

The curves of lift coefficient plotted on Plate No. 2 are of interest, as indicating that positive stagger reduces the critical angle and increases the maximum lift, while negative stagger increases the angle and reduces the maximum lift, the peaks of the lift coefficient curves exhibiting a general trend downward and to the right.

3. *Gap.*

Information on the effect of variation of gap obtained in this investigation was comparatively limited. However, a comparison of Plates Nos. 1 and 2 will indicate the general effect for biplanes of moderately thick section aerofoils. Reduction in gap is evidently disadvantageous. Lifts are reduced considerably, while the drags are influenced but little, with a consequent appreciable decrease in efficiency, with reduction in gap at all except low lifts. The slope of the lift coefficient curve and the maximum lift are reduced with reduction in gap.

The curves of L/D on K_L for the biplanes of unit gap/chord ratio and for those with gap equal to one-half the chord, are coincident at negative and small positive lifts. The maximum L/D for the gap equal to one-half chord biplane is about 5% less than for the corresponding biplane with gap equal to the chord.

The movement of the *C.P.* with incidence is rather greater in biplanes of small gap than in those of usual gap.

II. *Biplane Factors for Lift and Comparison with Other Results.*

It has been seen that the characteristic of the aerofoil chiefly affected in biplaning is the lift, and hence only biplane factors for lift will be discussed in the present report.

To derive factors enabling the lift of a biplane to be predicted from that of the corresponding monoplane, and in order to better study the effects of biplaning and stagger on the lift, the lifts of the biplanes and monoplanes have been compared on Plate No. 12, using the method of R. & M. No. 589. On this plate for each stagger and gap/chord ratio, the lifts of the biplanes of the three aerofoil sections, have been plotted on the corresponding monoplane lifts.

1. *Biplaning.*

The diagrams for zero stagger plotted at *A* and *D*, Plate No. 12, show the effect of biplaning. It is seen that the points for the three aerofoils tested all lie very close to a single curve in each case. Apparently, then, the biplane effect is the same for these three aerofoils.

If the curves of diagrams *A* and *D* be approximated by radial straight lines, the biplane factors tabulated on Table No. 15 are obtained. The lift of the biplane of unit gap/chord ratio and zero stagger is found to be 85% of the corresponding monoplane lift, and with gap/chord ratio equal to one-half is 76%.

Further, to compare these factors with factors derived for other aerofoils, a comparison has been made with published information pertaining to biplanes composed of wings of other sections.

Tests with a biplane of Bleriot XI bis, an obsolete thin section, at various gaps, described in R. & M. 60, Part III, Report for 1911-12, yielded for the biplane factor at unit gap/chord ratio, for incidences from 6° - 10° , a value of 81 - 82% (page 73).

The average figure as found by Dr. Hunsaker⁵ for an R.A.F.-6 biplane of gap/chord ratio equal to 1.2, under conditions similar to those in this study, was 88%.

⁵The Triplane and Stable Biplane, 1918, J. C. Hunsaker, also "Engineering", Jan. 7-14, 1916.

In R. & M. 196 tests made with a biplane using the R.A.F.-6c aerofoil, are described, and in R. & M. 589 tests with R.A.F.-15 biplanes,⁶ both so-called thin sections, the latter being a well known high speed wing. The results of biplane tests with a heavily cambered high lift section, the R.A.F.-19 are given in R. & M. 648. In all three cases, the conditions of the tests were directly comparable with those of this investigation. The results of these tests for zero stagger have been plotted at *A*, Plate No. 13, and the biplane curve of *D*, Plate No. 12, as found in the present study, drawn thereon. The agreement of these results with the curve is seen to be excellent for the R.A.F.-19 and also for the R.A.F.-6c and R.A.F.-15, except at low and negative lifts.

The conclusion may then reasonably be drawn that for biplanes of zero stagger and unit gap/chord ratio at vl-10, the lift of the biplane is 85% of the lift of the monoplane for all aerofoils, whether thin or thick.

2. Stagger.

The general effect of stagger on lift may be studied by examining all of the diagrams of Plate No. 12. It is seen that for each stagger the points for the three aerofoils lie very close to one curve, indicating that the effects of stagger on lift are practically identical for these aerofoils. The graphs show generally that as the positive stagger is increased the biplane lift approaches that of the monoplane, hence positive stagger is advantageous from a lift standpoint. Also, at small lifts, the biplane lift is seen to be greater than that of the monoplane, the extent of the region over which this is the case increasing with stagger.

Again, to see whether these conclusions are generally applicable, the figures for the three aerofoils previously referred to, namely, the R.A.F.-6c, R.A.F.-15 and R.A.F.-19 at 30° stagger, have been plotted in diagram B, Plate No. 13, and the biplane curve of diagram *F*, Plate No. 12, as found in the present investigation, drawn in. The agreement between the figures for the previous investigations and for the present study is again seen to be good, particularly for the R.A.F.-19, and for the others at all except low and negative lifts. It may then be concluded that the stagger effects are the same for thin or thick aerofoils.

Stagger factors for lift have been derived from the diagrams of Plate No. 12, by tabulating the corresponding lifts of the biplanes of different staggers and plotting the lifts for the biplanes

⁶The corresponding monoplane figures for the R.A.F.-15 are given in R & M, 248.

of 15° and 30° stagger on the lift of the biplane without stagger, in diagram *C* and *D*, Plate No. 13. The points were found to lie on straight lines, which for unit gap/chord ratio (diagram *C*) are practically radial. For the gap/chord ratio equal to one-half, the lines are nearly parallel, or converge at the higher lifts. Fitting radial straight lines in these diagrams the stagger factors for lift, tabulated in Table No. 16, were obtained. Thus, at unit gap/chord ratio the 15° staggered biplane has 1.8% more lift than the unstaggered biplane, and the biplane with 30° stagger 6% more lift. The corresponding figures of 5% and 9.5% respectively, for the biplane with gap/chord ratio equal to one-half are much less definite owing to the approximation introduced in fitting the radial straight lines. The stagger factors have been plotted in diagram *E*, Plate No. 13.

Stagger tests with the Bleriot XI bis section, described in R. & M. 60, part IV, showed an improvement in lift of 5% with a stagger of 40%, roughly 22° (p. 73).

The factors of Tables 15 and 16 and Plate 13 enable the lift of a staggered biplane to be predicted from the test lifts of the corresponding monoplane.

The foregoing biplane and stagger factors may be combined in overall factors permitting the lift of the staggered biplane to be computed directly from that of the monoplane. Thus, for biplanes of unit gap/chord ratio, these overall factors are:

Lift of biplane, stagger $0^\circ = 85\%$ of monoplane lift

Lift of biplane, stagger $15^\circ = 86.5\%$ of monoplane lift

Lift of biplane, stagger $30^\circ = 90\%$ of monoplane lift

An average factor of 86.5% was derived from a series of tests of thin wing biplanes (upper and lower planes being of different sections) of gap/chord ratio equal to 1.067 and stagger 16-2/3% (about 9°) under similar conditions to those of this investigation, which are described in the Bulletin of the Airplane Engineering Department, U.S.A., vol. 1, No. 1, 1918.

B. CHARACTERISTICS OF INDIVIDUAL WINGS.

The lift and drag coefficients, and centres of pressure of the upper and lower planes, together with those of the monoplane, are plotted on angle of incidence for the different biplane combinations on Plates Nos. 5 - 8.

The measurements on the complete biplane differ somewhat in general from the average of the two individual wings, the discrepancy being probably due to deflection of the dummy aerofoil when making measurements on the individual aerofoils.

I. *Effects of Different Variables.*

1. *Biplaning (Zero Stagger).*

- (a) *Gap = Chord.* In a biplane of unit gap chord ratio the upper plane carries the major portion of the lift at positive incidences (see Plates Nos. 5, 7 and 8). The slope of the lift curve plotted on incidence is steeper for the upper plane than for the lower, and the upper plane reaches the critical angle earlier, *i.e.*, the critical angle is smaller for the upper plane. Further, the upper wing, for the sections tested, exhibits generally a more or less abrupt reduction in lift at the critical angle, while the lower wing suffers only a slight drop, or even experiences an increase in lift, so that the lower wing has the same maximum lift as the upper, but at a larger angle of incidence. Hence, beyond the critical angle, the lower instead of the upper plane exerts the greater lift.

As compared with the monoplane, both upper and lower wings have lower lifts at all positive incidence. The critical angles for both wings are larger than for the monoplane and the maximum lift of the upper plane is generally slightly greater than that of the monoplane.

The drag curve of the lower wing is flatter than that of the upper wing, with about the same minimum drag. At small positive incidences the lower wing has the greater drag, but elsewhere is less; very much less at large incidences, than the drag of the upper wing.

The minimum drag of either wing may be equal to or slightly greater than that of the monoplane, but at high lifts the lower plane has less and the upper plane more drag than the monoplane.

The movement of the *C.P.* is practically the same for the two wings at zero stagger, that of the lower wing being generally slightly less than that of the upper for incidences above about 5° . The movements of the *C.P.* for both wings are slightly greater than the movement for the monoplane.

- (b) *Gap = $\frac{1}{2}$ Chord.* At a gap/chord ratio of one-half, and zero stagger, the lifts of the two wings were about the same except at small incidences, and incidences above the critical where the lower wing exerts the greater lift (see Plate No. 6). The drag of the lower is much greater than that of the upper except at negative angles, and the centre of pressure of the lower farther to the rear than that of the upper.

As compared with the monoplane characteristics, the statements made in connection with the unit gap/chord ratio are applicable.

2. *Stagger.*

- (a) $Gap = Chord$. Increase in positive stagger increases the difference in loading on the two wings, the upper exerting a greater and greater proportion of the total lift. This results from the lift of the upper plane increasing while that of the lower decreases. The lift of the upper plane is superior to that of the monoplane for the larger staggers. At the same time, the maximum lift of the upper plane is increased and occurs earlier, the critical angle becoming smaller, while the maximum lift of the lower plane is also increased, becoming eventually greater than that of the upper and occurs at a greater angle than that of the upper. If there is an abrupt drop in the lift curve of the upper plane at the critical angle, there is an abrupt rise in that of the lower plane at a slightly larger incidence. (Compare R. & M. 648).

The drag curves of the two wings are also separated more and more as the stagger increases, through the drag of the lower plane increasing at a faster rate than that of the upper plane. In fact, the drag of the upper plane is but little influenced by stagger variation. At larger staggers the drag of the lower plane is greater than that of the upper except at negative incidences, and its minimum drag is usually very slightly greater.

The drag of the lower plane at large staggers is greater than the monoplane drag, while that of the upper plane is practically equal to the monoplane drag.

As the stagger is increased the centre of pressure of the lower plane moves back toward the trailing edge; while that of the upper remains practically unchanged. At the same time the *C.P.* travel for each plane for a given change of incidence is slightly increased.

- (b) $Gap = \frac{1}{2} Chord$. The effects of stagger are accentuated when the gap is reduced to one-half the chord, the lift of upper and lower wings being about in the ratio of 2:1 for 45° stagger. The drag of the upper wing is practically stationary with increase of stagger until 30° stagger is reached, but between 30° and 45° stagger there is a marked increase. On the other hand the drag of the lower wing increases throughout with

increase of stagger, but the increase for a change in stagger from 30° to 45° is slight.

The centre of pressure of the lower wing moves toward the trailing edge and its travel with incidence increases as the stagger is increased, while the position and travel of the *C.P.* of the upper wing are practically unchanged, so that the centres of pressure of the two wings separate.

At a negative stagger of 15° the upper wing has the greater lift and drag. The change in stagger from 0° to -15° has the same general effects as a reduction in positive stagger, namely, decrease in lift and increase in drag of upper wing, and an increase in lift and decrease in drag of the lower wing, while the centre of pressure of the two wings draw nearer one another, that of the upper wing moving rearward and that of the lower remaining practically stationary.

3. Gap.

The little information available on the effect of variation in gap seems to indicate that reduction in gap reduces the lift of both wings, that of the upper to a greater extent than that of the lower, so that the general tendency is to equalize the lifts of the two wings, except at small incidences. The drag of the upper plane is reduced and that of the lower increased to a rather less extent with gap reduction. The *C.P.* of the upper plane shifts forward slightly and that of the lower plane moves backward, the travel of each, with incidence, remaining unchanged.

II. *Relative Lifts of the Two Wings.*

The lift coefficients of the individual wings are plotted on biplane lift coefficient in diagrams *A*, *B*, *D* and *E*, Plate No. 9. These diagrams enable the lift of each wing to be determined for any biplane lift and show the relative effectiveness of the two wings.

The diagrams also indicate the effect of stagger on the wing loadings, an increase in stagger from 0° to $+15^\circ$ resulting in a large increase in upper wing lift, accompanied by a relatively smaller decrease in lift of lower wing, while an increase from 15° to 30° in stagger carries a small increase in upper wing lift with a large decrease in lower wing lift.

To permit of a comparison being made with biplanes of other sections, the results of the tests on R.A.F.-15 and R.A.F.-19, already referred to, have been plotted in diagrams *C* and *F*, Plate No. 9. The R.A.F.-6c figures, although in general agreement, have not been

plotted, to avoid complexity. The general shapes of the curves and effects of stagger are seen to be similar to those obtained in this investigation.

The ratio of the lift of upper aerofoil to the lift of the lower aerofoil for the combinations here tested have been calculated and are tabulated in Table No. 12 and plotted in diagrams *A* and *D*, Plate No. 10. The ratio is seen to increase with incidence for zero stagger, remains about constant with 15° stagger, and decreases with incidence with 30° stagger. In other words, with zero stagger the upper wing carries an increasing proportion of the load as the incidence is increased, while at 30° stagger its proportion of the load, initially large, decreases with increase in incidence.

The agreement between these diagrams and similar diagrams derived from the pressure distribution measurements on the U.S.A.-27 and Göttingen-387, presented in Aeronautical Research Papers Nos. 18 and 18a, will be found to be quite good.

These diagrams will be of use where it is desired to apportion the load between the two wings with considerable accuracy for different conditions, *i.e.*, high incidence (landing), and low incidence (normal flight), and where the average figures for the whole incidence range, given later, are not sufficiently accurate. Thus, for the three aerofoils tested, the figures are as follows for unit gap/chord ratio:

Ratio of Lift of Upper Wing to Lift of Lower Wing:

	Stagger 0°	Stagger $+15^\circ$	Stagger 30°
Low Incidence— 2°	1.03	1.23	1.51
High Incidence— 14°	1.14	1.28	1.41

The diagrams *B* and *E*, Plate No. 10, plotted for R.A.F.-6c, R.A.F.-15, and R.A.F.-19 biplanes, show variations of the ratios of lifts of the two wings, similar to those found in the present investigation.

Average values of the ratio of lift of upper plane to that of the lower plane for the full range of positive incidences have been derived from Table No. 12 and also from diagrams *A* and *B*, Plate No. 10, and are tabulated in Table No. 14, and plotted in diagram *C*, Plate 10. The straight line drawn on the latter diagram shows the average value of the lift ratio of the wings for the sections here tested, and presents factors to be used in design or in checking the loading on the wings of biplane with gap equal to chord. Values of the ratio taken from the diagram are:

At 0° stagger—	1.08
At $+15^\circ$ stagger—	1.26
At $+30^\circ$ stagger—	1.45

The straight line of diagram *C* has been drawn again in diagram *F*,

on which are also plotted values of the ratio derived from the tests made on the R.A.F.-6c, R.A.F.-15 and R.A.F.-19 biplanes.

The values for the latter sections are seen to differ considerably among themselves and from those of this investigation.

There are also drawn on this diagram lines representing the values of this ratio given in the British Handbook of Strength Calculations Air Publication 970, Second Edition 1924, Figs. 1 and 2, and those of the U.S. Army Air Service, as given in the Handbook of Instruction for Airplane Designers, Third Edition, June 1922, Section II, Part III, Fig. 1. An inspection of these different lines shows that present practice differs considerably from the results of this research, from which it may be concluded that the values of the ratio of upper wing load to lower wing load, commonly used for thin wing biplanes, will require modification for use with biplanes of moderate or thick section.

The fraction of the total lift of the biplane contributed by each wing may be derived from the diagrams of Plate No. 9 or from the figures of Table No. 14. By drawing radial straight lines approximating the curves of Plate No. 9 the fraction may be found directly, but inasmuch as radial straight lines cannot be fitted satisfactorily to many of the curves, particularly those for zero stagger and for gap/chord ratio equal to one-half, more accurate results will be found by calculating the fractions from the figures of Table No. 14. The results, expressed in percent. of total lift of the biplane, are tabulated on Table No. 13 and plotted in diagrams *A* and *C*, Plate No. 11.

The diagram *A* presents factors to be used in design or in checking, for apportioning the loading on each wing of a biplane of unit gap/chord ratio.

The average figures for this case are:

	Stagger—	0°	15°	30°
Lift of Upper Plane, % of total.		52	55.5	59
Lift of Lower Plane, % of total.		48	44.5	41

These factors are compared with similar factors for the R.A.F.-6c, R.A.F.-15 and R.A.F.-19 biplanes in diagram *B*, Plate No. 11. The latter figures differ considerably for the different aerofoils and from the results of the present investigation.

Comparison is made in diagram *D*, Plate No. 11, with figures derived from the values of the ratio of lift of upper wing to lift of lower wing (refer diagram *F*, Plate No. 10) as used in the British and American Air Services. The actual difference in loading of the two wings is seen to be greater for the biplanes here tested, than as used in the design of aircraft for these services. Again, the conclusion to be drawn is that in biplanes of moderate or thick section wings the difference in loading of the two wings is greater than in thin wing biplanes.

CONCLUSIONS

The effects of biplaning, stagger and gap on the aerodynamic characteristics are generally the same for aerofoils of moderate and thick section, as for thin aerofoils. Biplaning, reduction in gap and negative stagger reduce the lift and lift/drag of the complete biplane, while positive stagger increases the lift but has little effect on the lift/drag. The drag and movement of the centre of pressure are but little influenced by these factors, the minimum drag remaining practically unchanged.

The lift of the unit gap/chord ratio, zero stagger biplane at v_{l-10} is 85% of the monoplane lift.

The influence of stagger on biplane lift is slight, the biplane of 15° stagger having only 1.8% more lift, and the biplane of 30° stagger 6% more lift than the unstaggered biplane, or, as compared with the monoplane, the lifts are 86.5% and 90%, respectively.

The lifts of the individual wings of the biplane are reduced and the critical angle increased, as compared with the monoplane, the variation being relatively less for the upper than the lower in each case. Positive stagger increases the upper wing loading and decreases that of the lower wing, while reduction in gap reduces the lift of both wings, as compared with the unit gap/chord arrangement; but that of the upper relatively more than that of the lower, thus tending to equalize the loading on the two wings. The drag of the upper wing is practically uninfluenced by stagger for moderate staggers, while that of the lower is increased, the minimum drag, however, remaining about constant. The drag of both wings increases with reduction in gap. These factors have only slight effect on centre of pressure movement of either wing, although the position of the *C.P.* of the lower wing is moved rearward with increase in stagger and reduction in gap.

The ratio of lift of upper wing to lift of lower wing for these biplanes is generally somewhat greater than for those composed of thin wings. The ratio varies with incidence and stagger, its values at unit gap/chord ratio being:

	Stagger 0°	Stagger 15°	Stagger 30°
Low Incidence (2°)	1.03	1.23	1.51
High Incidence (14°)	1.14	1.28	1.41
Average (positive incidences)....	1.08	1.26	1.45

The corresponding percentage of the total biplane lift borne by each wing are:

	Stagger 0°	Stagger 15°	Stagger 30°
Upper Wing	52	55.5	59
Lower Wing	48	44.5	41

LIST OF TABLES

1. Ordinates of Aerofoil Sections.
2. Lift and Drag Coefficients, Complete Biplanes, Airscrew-4, Gap = Chord, Gap = $\frac{1}{2}$ Chord.
3. Lift/Drag and Centre of Pressure, Complete Biplanes, Airscrew-4, Gap = Chord, Gap = $\frac{1}{2}$ Chord.
4. Lift and Drag Coefficients, Upper Plane, Airscrew-4, Gap = Chord, Gap = $\frac{1}{2}$ Chord.
5. Lift/Drag and Centre of Pressure, Upper Plane, Airscrew-4, Gap = Chord, Gap = $\frac{1}{2}$ Chord.
6. Lift and Drag Coefficients, Lower Plane, Airscrew-4, Gap = Chord, Gap = $\frac{1}{2}$ Chord.
7. Lift/Drag and Centre of Pressure, Lower Plane, Airscrew-4, Gap = Chord, Gap = $\frac{1}{2}$ Chord.
8. Lift and Drag Coefficients, Complete Biplanes and Individual Wings, U.S.A.-27.
9. Lift/Drag and Centre of Pressure, Complete Biplanes and Individual Wings, U.S.A.-27.
10. Lift and Drag Coefficients, Complete Biplanes and Individual Wings, Gottingen-387.
11. Lift/Drag and Centre of Pressure, Complete Biplanes and Individual Wings, Gottingen-387.
12. Ratio of Lift of Upper Wing to Lift of Lower Wing, all Aerofoils.
13. Relative Lifts of Upper and Lower Wings.
14. Average Ratio of Lift of Upper to Lift of Lower Wing.
15. Average of Biplane Corrections on Lift.
16. Average Stagger Corrections on Lift.

BIPLANE INVESTIGATION

TABLE NO. 1.

ORDINATES OF AEROFOIL SECTIONS						
All dimensions expressed in percent of chord						
Distance from Leading Edge	Airscrew-4		U.S.A.-27		Gottingen-387	
	Lower Ordinate	Upper Ordinate	Lower Ordinate	Upper Ordinate	Lower Ordinate	Upper Ordinate
0.00	2.00	2.00	1.77	1.77	3.61	3.61
1.25			0.50	3.80	1.35	6.74
2.50			0.33	5.07	0.81	7.98
5.00	7.94	0.00	0.17	6.93	0.36	9.87
7.50			0.10	8.22	0.18	11.32
10.00	10.20	0.00	0.00	9.17	0.13	12.40
15.00			0.10	10.50	0.00	13.83
20.00	12.18	0.00	0.35	11.33	0.08	14.77
30.00	12.70	0.00	0.95	11.90	0.22	15.36
40.00	12.44	0.00	1.17	11.57	0.38	14.88
50.00	11.51	0.00	0.90	10.77	0.54	13.48
60.00	10.20	0.00	0.25	9.52	0.54	11.59
70.00	8.60	0.00	0.10	8.00	0.54	9.16
77.00			0.00			
80.00	6.68	0.00	0.05	6.03	0.50	6.58
90.00	4.23	0.00	0.15	3.65	0.27	3.61
100.00	0.40	0.40	0.67	0.67	0.00	0.37
Radius L.E.	2.00		1.05		3.25	
Radius T.E.	0.80		0.21		0.20	

TABLE No. 2.

Aerofoil — Airscrew 4 Complete Biplane Air Speed 40 f.p.s.

LIFT COEFFICIENT

Incidence degrees	Mono- plane	Biplane — Gap = Chord				Biplane — Gap = $\frac{1}{2}$ Chord			
		Stagger				Stagger			
		0	+15	+30	-15	0	+15	+30	+45
-6	-.0844	-0387	-0356	-0269	-.0301	-.0154	-.0023	.0210	0.111
-4	+.0379	+0492	+0492	+0621	+.0360	+.0494	+.0345	.0822	.0702
-2	.1315	1172	1212	1282	.0981	.1090	.1222	.1374	.1347
0	.2062	1833	1898	1956	.1474	.1604	.1799	.1963	.1971
2	.2825	2414	2453	2561	.1976	.2161	.2354	.2505	.2600
4	.3521	3078	3059	3212	.2427	.2753	.2873	.3012	.3178
6	.4205	3672	3673	3850	.2939	.3274	.3379	.3603	.3816
8	.4910	4208	4215	4412	.3517*	.3757	.3869	.4052	.4345
10	.5613	4788	4792	5083	.4015	.4229	.4350	.4565	.4903
12	.6175	5288	5365	5622	.4395	.4645	.4780	.5052	.5475
14	.6687	5829	5962	6178	.4805	.5018	.5194	.5560	.5961
15								.5732	.4956
16	.4373	6341	6354	6578	.5203	.5402	.5532	.4486	
17		5544	5254	5609		.4581	.4402		
18	.4273	5440			.4548				
20	.4231								

*Unsteady above 6° incidence.

DRAG COEFFICIENT

-6	.0357	0314	0315	0289	.0312	.0291	.0260	.0240	.0257
-4	.0219	0230	0231	0225	.0233	.0238	.0217	.0203	.0215
-2	.0182	0194	0198	0191	.0199	.0202	.0195	.0189	.0193
0	.0177	0192	0197	0196	.0192	.0195	.0195	.0196	.0198
2	.0197	0210	0209	0211	.0207	.0210	.0220	.0236	.0230
4	.0237	0251	0254	0270	.0234	.0258	.0260	.0283	.0291
6	.0310	0322	0328	0342	.0288	.0320	.0320	.0342	.0364
8	.0377	0393	0405	0422	.0346	.0383	.0390	.0420	.0444
10	.0478	0497	0488	0530	.0425	.0457	.0470	.0510	.0543
12	.0592	0592	0597	0639	.0486	.0545	.0548	.0607	.0663
14	.0683	0696	0727	0766	.0573	.0648	.0642	.0726	.0872
15								.0784	.1216
16	.1187	0819	0834	0899	.0678	.0786	.0732	.1110	
17		1152	1090	1197		.0947	.0992		
18	.1407	1133			.0920				
20	.1594								

TABLE No. 3.

Aerofoil — Aircscrew-4 Complete Biplane Air Speed 40 f.p.s.

LIFT/DRAG

Inci- dence degrees	Mono- plane	Biplane — Gap = Chord			Biplane — Gap = $\frac{1}{2}$ Chord				
		Stagger			Stagger				
		0	+15°	+30°	-15°	0°	+15°	30°	45°
-6	-2.36	-1.23	-1.13	-0.93	-0.96	-0.53	-0.08	0.87	0.43
-4	1.73	+2.14	+2.14	+2.76	+1.55	+2.08	+2.97	4.05	3.26
-2	7.24	6.04	6.11	6.71	4.93	5.39	6.43	7.27	6.97
0	11.68	9.57	9.66	9.97	7.68	8.22	9.22	10.00	9.96
2	14.32	11.51	11.73	12.12	9.54	10.30	10.70	10.60	11.30
4	14.86	12.26	11.91	11.90	10.39	10.67	11.05	10.62	10.91
6	13.53	11.40	11.19	11.25	10.20	10.22	10.55	10.51	10.49
8	13.03	10.70	10.43	10.47	10.17	9.81	9.90	9.66	9.78
10	11.73	9.62	9.83	9.60	9.45	9.26	9.25	8.95	9.02
12	10.43	8.92	8.98	8.80	8.85	8.51	8.73	8.33	8.25
14	9.78	8.37	8.21	8.05	8.39	7.74	8.10	7.66	6.84
15								7.31	4.07
16	3.69	7.74	7.85	7.30	7.68	6.88	7.55	4.04	
17		4.81	4.83	4.69		4.84	4.44		
18	3.04	4.80			4.94				
20	2.65								

CENTRE OF PRESSURE

-6	1.1								
-4		85.1	67.2	42.0		96.4	71.6	47.1	19.4
-2	60.4	54.9	40.7	15.0	71.5	57.0	45.8	29.8	8.7
0	45.6	43.7	28.9	9.2	56.9	44.5	36.2	22.7	4.8
2	40.3	39.0	25.9	6.9	49.9	41.2	32.1	18.8	4.1
4	37.4	36.8	23.0	4.0	45.1	38.1	29.6	17.5	3.4
6	36.2	35.5	22.5	4.0	42.6	36.4	27.8	17.1	4.0
8	34.3	35.5	22.0	3.9	42.1	35.4	27.1	16.3	3.6
10	33.2	35.4	22.1	5.0	41.1	34.8	26.6	16.4	3.7
12	32.8	35.6	22.7	5.8	40.3	33.8	26.5	17.0	4.3
14	31.6	36.0	23.4	6.5	40.1	33.2	26.5	17.5	5.1
15								17.4	14.0
16	37.9	36.6	24.0	7.4	40.0	33.2	26.6	23.0	
17		37.8	21.8	10.3		32.4	24.2		
18	39.8	38.6			42.8				
20	40.6								

TABLE NO. 4.

Aerofoil — Aircrew-4

Upper Plane

Air Speed 40 f.p.s.

LIFT COEFFICIENT

Incidence degrees	Mono- plane	Biplane — Gap = Chord			Biplane — Gap = $\frac{1}{2}$ Chord				
		Stagger			Stagger				
		0°	+15°	+30°	-15°	0°	+15°	30°	45°
-6	-.0844	-.0662	-.0513	-.0380	-.1272	-.1205	-.0536*	-.0336	.0202
-4	+.0379	+.0270	+.0491	+.0741	-.0447	-.0332	+.0418*	+.0580	.1228
-2	.1315	.1126	.1389	.1573	.0386	+.0497	.1197*	.1437	.2176
0	.2062	.1775	.2087	.2392	.0914	.1011	.1906*	.2221	.2892
2	.2825	.2312	.2784	.3110	.1389	.1484†	.2517*	.2817	.3632
4	.3521	.3005	.3427	.3814	.1788†	.2579†	.3117	.3430	.4323
6	.4205	.3618	.4153	.4658	.2557†	.3126†	.3720	.3975	.5034
8	.4910	.4235	.4727	.5329	.3327†	.3713	.4210	.4586	.5742
10	.5613	.4788	.5380	.5938	.3783†	.4250	.4807	.5156	.6429
12	.6175	.5455	.5959	.6610	.4253	.4640	.5245	.5667	.7110
14	.6687	.6035	.6660	.7228	.4726	.5123	.5700	.6166	.7610
15				.5082					.4068
16	.4373	.6559	.7171		.5137†	.5497	.6060	.6586	
17		.4513	.4226				.2337		
18	.4273				.3036	.5703		.2183	
19						.1499			

DRAG COEFFICIENT

-6	.0357	.0317	.0333	.0304	.0386	.0379	.0350	.0331	.0292
-4	.0219	.0237	.0231	.0215	.0287	.0258	.0263	.0261	.0204
-2	.0182	.0192	.0179	.0182	.0218	.0197	.0209	.0204	.0170
0	.0177	.0185	.0180	.0175	.0177	.0157	.0183	.0181	.0173
2	.0197	.0194	.0191	.0194	.0164	.0132	.0185	.0180	.0204
4	.0237	.0236	.0231	.0242	.0169	.0186	.0216	.0199	.0240
6	.0310	.0302	.0296	.0308	.0241	.0211	.0244	.0222	.0287
8	.0377	.0377	.0374	.0390	.0303	.0260	.0292	.0279	.0362
10	.0478	.0476	.0459	.0493	.0360	.0324	.0336	.0340	.0434
12	.0592	.0607	.0587	.0601	.0459	.0388	.0409	.0410	.0539
14	.0683	.0752	.0723	.0732	.0586	.0474	.0508	.0493	.0643
15				.1114					.1018
16	.1187	.0898	.0861		.0652	.0551	.0560	.0544	
17		.1023	.1131				.0660		
18	.1407				.0743	.0644		.0726	
19						.0551			

*Tends to two curves.

†Uncertain.

TABLE NO. 5.

Aerofoil — Airscrew-4 Upper Plane Air Speed 40 f.p.s.

LIFT/DRAG

Inci- dence degrees	Mono- plane	Biplane — Gap = Chord				Biplane — Gap = $\frac{1}{2}$ Chord			
		Stagger				Stagger			
		0°	+15°	+30°	-15°	0°	+15°	30°	45°
-6	-2.36	-2.08	-1.54	-1.25	-3.29	-3.18	-1.52	-1.01	0.69
-4	+1.73	+1.14	+2.12	+3.44	-1.56	-1.29	+1.59	+2.22	5.90
-2	7.24	5.86	7.75	8.65	+1.77	+2.52	5.72	7.05	12.79
0	11.68	9.60	11.59	13.65	5.16	6.44	10.40	12.27	16.71
2	14.32	11.91	14.58	16.00	8.46	11.25	13.60	15.65	17.80
4	14.86	12.72	14.83	15.76	10.59	13.86	14.40	17.23	18.00
6	13.53	11.98	14.05	15.10	10.60	14.82	15.23	17.90	17.53
8	13.03	11.22	12.64	13.66	10.96	14.28	14.40	16.42	15.85
10	11.73	10.05	11.73	12.05	10.50	13.11	14.30	15.19	14.81
12	10.43	9.00	10.17	11.00	9.28	11.95	12.81	13.82	13.19
14	9.78	8.02	9.21	9.87	8.06	10.80	11.21	12.50	11.83
15				4.56					4.00
16	3.69	7.30	8.33		7.87	9.97	10.81	12.10	
17		4.41	3.74				3.54		
18	3.04				4.08	8.86		3.01	
19						2.71			

CENTRE OF PRESSURE

-6	1.1								
-4				83.1			78.5	66.2	60.0
-2	60.4	61.5	55.8	52.9		69.2	49.2	45.6	47.1
0	45.6	45.9	44.6	43.0	60.0	43.9	39.5	38.2	39.9
2	40.3	39.6	39.4	38.1	46.5	35.0	37.1	34.2	35.8
4	37.4	36.1	36.0	35.2	38.6	34.0	34.5	32.3	34.1
6	36.2	34.0	34.0	32.7	35.4	32.0	33.4	30.2	32.0
8	34.3	31.8	32.2	32.2	34.2	30.1	29.2	29.0	30.6
10	33.2	30.8	31.1	30.9	32.3	29.0	28.3	27.1	29.8
12	32.8	29.4	30.0	30.0	30.8	27.6	27.2	26.6	28.6
14	31.6	28.7	29.2	29.1	29.5	26.4	26.9	25.7	28.1
15				32.6					39.5
16	37.9	28.0	28.4		28.5	26.0	25.5	30.2	
17		23.0	22.3				18.2		
18	39.8				31.6	25.6		20.4	
19						23.8			

TABLE NO. 6.

Aerofoil — Airscrew-4				Lower Plane		Air Speed 40 f.p.s.			
LIFT COEFFICIENT									
Inci- dence degrees	Mono- plane	Biplane — Gap = Chord			Biplane — Gap = ½ Chord				
		Stagger			Stagger				
		0°	15°	30°	— 15°	0°	+ 15°	30°	45°
— 6	— .0844	— .0216	— .0019	— .0142	.0753	.0959	.0766	.0539	.0019
— 4	+ .0379	+ .0591	+ .0535	+ .0404	.1295	.1313	.1134	.0827	.0203
— 2	.1315	.1214	.1144	.0928	.1755	.1726	.1478	.1143	.0485
0	.2062	.1798	.1685	.1561	.2308	.2111	.1900	.1567	.0971
2	.2825	.2339	.2215	.2019	.2867	.2654	.2166	.1938	.1435
4	.3521	.2931	.2750	.2624	.3362	.2847	.2597	.2352	.1928
6	.4205	.3425	.3300	.3119	.3840	.3287	.3136	.2829	.2401
8	.4910	.3927	.3778	.3596	.4310	.3692	.3517	.3283	.2846
10	.5613	.4433	.4292	.4116	.4601	.4167	.4012	.3689	.3328
12	.6175	.4917	.4775	.4599	.4916	.4657	.4348	.4101	.3710
14	.6687	.5279	.5232	.5105	.5295	.5062	.4813	.4535	.4179
15									
16	.4373	.5678	.5644	.5461	.5656	.5407	.5445	.6398	.5841
17					.4816				
18	.4273	.6542	.6726	.6986		.5790	.5995	.6460	.6250
19		.5448	.7080						
20	.4231		.7070	.7354		.7146	.7065	.6597	.6496
21									
22				.7672		.7367	.7179		
23									
24				.5780			.7528		
26							.7674		
DRAG COEFFICIENT									
— 6	.0357	.0301	.0271	.0258	.0287	.0174	.0162	.0143	.0199
— 4	.0219	.0214	.0212	.0206	.0206	.0158	.0158	.0145	.0199
— 2	.0182	.0188	.0182	.0183	.0237	.0163	.0178	.0166	.0211
0	.0177	.0185	.0187	.0192	.0257	.0197	.0217	.0198	.0229
2	.0197	.0209	.0218	.0220	.0307	.0261	.0261	.0258	.0262
4	.0237	.0250	.0263	.0266	.0360	.0312	.0328	.0310	.0329
6	.0310	.0312	.0329	.0351	.0398	.0385	.0406	.0409	.0419
8	.0377	.0370	.0420	.0445	.0468	.0464	.0516	.0515	.0528
10	.0478	.0444	.0499	.0544	.0523	.0589	.0612	.0631	.0645
12	.0592	.0531	.0588	.0671	.0579	.0646	.0734	.0761	.0805
14	.0683	.0617	.0710	.0782	.0659	.0779	.0860	.0902	.0952
15									
16	.1187	.0727	.0825	.0915	.0773	.0852	.0980	.1443	.1552
17					.1007				
18	.1407	.1036	.1140	.1277		.1015	.1659	.1731	.1813
19		.1418	.1277						
20	.1594		.1368	.1494		.1686	.1897	.2010	.2043
21									
22				.1691		.1930	.2177		
23							.2350		
24				.2021			.2678		

TABLE No. 8.

Aerofoil U.A.S.-27

Biplane—Gap = Chord

Air Speed 40 f.p.s.

LIFT COEFFICIENT

Incidence	Mono-plane	Complete Biplane			Upper Plane			Lower Plane		
		0°	Stagger 15°	30°	0°	Stagger 15°	30°	0°	Stagger 15°	30°
-6	-.0903	-.0520	-.0428	-.0427	-.0601	-.0044	-.0379	-.0640	-.0100	-.0171
-4	+.0184	+.0279	+.0349	+.0338	+.0237	+.0389	+.0553	+.0171	+.0462	+.0322
-2	.1173	.1047	.1109	.1121	.1015	.1239	.1440	.0895	.1068	.0888
0	.1949	.1675	.1732	.1842	.1666	.1927	.2280	.1576	.1623	.1486
+2	.2701	.2360	.2343	.2512	.2355	.2723	.2945	.2229	.2202	.2042
4	.3407	.2923	.2998	.3185	.2961	.3420	.3654	.2736	.2701	.2571
6	.4082	.3518	.3516	.3760	.3573	.3996	.4350	.3333	.3242	.3104
8	.4796	.4085	.4136	.4391	.4183	.4642	.5064	.3819	.3665	.3589
10	.5446	.4583	.4680	.4948	.4834	.5277	.5759	.4374	.4239	.4119
12	.6044	.5177	.5259	.5577	.5392	.5904	.6430	.4877	.4728	.4638
14	.6563	.5679	.5817	.6215	.6044	.6604	.7070	.5340	.5185	.5146
16	.6941	.6146	.6200	.6656	.6474	.7084	.7518	.5802	.5600	.5632
18	.6784	.6530	.6677	.7006	.6869	.7424	.7704	.6052	.5990	.6126
20	.6583	.6596	.6781	.7122	.7140	.7298	.7287	.6125	.6297	.6648
22		.5118	.6493	.6887		.3647		.6025	.6306	

DRAG COEFFICIENT

-6	.0354	.0313	.0301	.0296	.0310	.0305	.0294	.0341	.0270	.0258
-4	.0211	.0205	.0199	.0204	.0211	.0202	.0183	.0216	.0192	.0186
-2	.0160	.0169	.0162	.0166	.0161	.0162	.0145	.0173	.0151	.0156
0	.0153	.0160	.0164	.0171	.0150	.0151	.0140	.0164	.0154	.0166
+2	.0174	.0188	.0185	.0191	.0168	.0180	.0171	.0183	.0188	.0197
4	.0223	.0225	.0236	.0247	.0211	.0230	.0216	.0236	.0232	.0261
6	.0287	.0300	.0295	.0307	.0277	.0284	.0275	.0293	.0317	.0339
8	.0369	.0376	.0365	.0395	.0360	.0369	.0356	.0358	.0398	.0421
10	.0453	.0457	.0466	.0499	.0460	.0443	.0445	.0445	.0480	.0523
12	.0553	.0560	.0559	.0616	.0577	.0567	.0562	.0518	.0579	.0649
14	.0657	.0661	.0672	.0724	.0716	.0679	.0659	.0599	.0697	.0771
16	.0758	.0772	.0781	.0846	.0842	.0826	.0782	.0706	.0789	.0892
18	.0876	.0872	.0895	.0971	.0975	.0922	.0904	.0775	.0879	.1024
20	.1054	.0982	.1012	.1122	.1084	.1048	.1079	.0914	.1006	.1180
22		.1299	.1177	.1348	.1411			.1250	.1210	

TABLE No. 9.

Aerofoil U.S.A.-27

Biplane — Gap = Chord

Air Speed 40 f.p.s.

LIFT/DRAG

Incidence	Mono-plane	Complete Biplane			Upper Plane			Lower Plane		
		0°	Stagger 15°	30°	0°	Stagger 15°	30°	0°	Stagger 15°	30°
-6	-2.55	-1.66	-1.42	-1.44	-1.94	-1.42	-1.29	-1.88	-0.37	-0.66
-4	+0.87	+1.36	+1.75	+1.62	+1.12	+1.92	+3.02	+0.79	+2.40	+1.73
-2	7.34	6.19	6.84	6.75	6.30	7.64	9.93	5.17	7.06	5.69
0	12.75	10.48	10.58	10.78	11.10	12.78	16.30	9.60	10.54	8.95
+2	15.53	12.55	12.68	13.14	14.00	15.15	17.21	12.19	11.71	10.38
4	15.29	12.99	12.70	12.90	14.00	14.85	16.90	11.60	10.71	9.85
6	14.24	11.72	11.90	12.22	12.90	14.08	15.81	11.31	10.22	9.15
8	13.00	10.86	11.31	11.11	11.61	12.59	14.22	10.66	9.20	8.52
10	12.04	10.03	10.02	9.90	10.50	11.90	12.95	9.82	8.88	7.87
12	10.94	9.25	9.41	9.05	9.33	10.43	11.43	9.42	8.17	7.14
14	10.00	8.59	8.65	8.59	8.44	9.73	10.73	8.90	7.44	6.67
16	9.16	7.96	7.94	7.87	7.69	8.57	9.61	8.22	7.10	6.31
18	7.75	7.49	7.46	7.22	7.05	8.04	8.57	7.81	6.81	5.98
20	6.24	6.72	6.69	6.34	6.58	6.96	6.76	6.70	6.25	5.63
22		3.95	5.51	5.10	2.58			4.81	5.20	

CENTRE OF PRESSURE

-6	0.00									
-4			99.0							
-2	62.6	57.6	42.7	15.0	72.3	61.4	60.0	77.4	68.9	81.5
0	47.0	44.3	30.0	7.3	53.6	48.3	47.6	54.6	55.7	58.0
+2	40.6	39.8	24.2	6.0	45.2	41.4	41.3	45.1	45.8	47.6
4	36.5	36.9	23.0	3.7	41.1	37.2	37.5	39.7	42.0	41.8
6	34.6	36.3	22.4	3.7	37.2	35.8	34.9	37.0	38.4	39.0
8	32.6	34.8	22.2	2.9	35.3	33.2	33.3	34.1	35.7	37.1
10	31.7	34.8	21.3	4.0	33.4	31.8	32.2	33.6	35.4	35.0
12	31.2	35.2	22.1	4.1	31.9	31.1	30.9	32.4	34.0	33.2
14	30.6	35.9	23.6	6.0	30.9	30.0	30.6	31.6	33.0	33.2
16	29.5	36.4	24.1	6.9	30.2	29.4	30.1	31.0	33.0	32.6
18	29.7	37.5	24.7	8.9	29.5	29.0	29.2	30.5	32.0	31.5
20	30.6	38.4	25.1	10.1	28.7	28.1	29.0	30.1	31.3	31.2
22		41.2	26.2	10.8	26.1			31.8	31.2	

TABLE No. 10.

Aerofoil Gottingen-387

Biplane — Gap = Chord

Air Speed 40 f.p.s.

LIFT COEFFICIENT

Incidence	Mono-plane	Complete Biplane			Upper Plane			Lower Plane		
		0°	15°	30°	0°	15°	30°	0°	15°	30°
-6	.0518	.0545	.0639	.0626	.0525	.0733	.0895	.0675	.0534	.0448
-4	.1190	.1112	.1153	.1232	.1044	.1302	.1541	.1156	.1048	.0917
-2	.1878	.1672	.1724	.1776	.1602	.1905	.2248	.1708	.1540	.1400
0	.2604	.2246	.2316	.2424	.2209	.2599	.2926	.2246	.2052	.1877
+2	.3354	.2879	.2937	.3017	.2876	.3333	.3696	.2785	.2729	.2380
4	.4055	.3480	.3591	.3682	.3518	.3983	.4451	.3335	.3197	.2896
6	.4806	.4097	.4201	.4264	.4179	.4646	.5156	.3880	.3662	.3426
8	.5493	.4738	.4795	.4925	.4867	.5347	.5872	.4412	.4224	.3976
10	.6183	.5380	.5375	.5552	.5420	.5947	.6638	.4949	.4655	.4464
12	.6768	.5865	.5921	.6178	.6092	.6677	.7352	.5451	.5166	.5005
14	.7284	.6390	.6381	.6690	.6651	.7305	.7872	.5814	.5621	.5482
16	.7529	.6815	.6891	.7148	.7183	.7715	.8388	.6208	.6074	.5940
18	.4206	.7102	.7251	.7607	.7657	.7849	.8700	.6466	.6450	.6382
20		.5519	.5476	.5947	.3248	.3345	.3937	.6563	.7828	.6853
22								.7790	.7974	.8158

DRAG COEFFICIENT

Incidence	Mono-plane	0°	15°	30°	0°	15°	30°	0°	15°	30°
-6	.0212	.0212	.0193	.0194	.0205	.0210	.0187	.0185	.0197	.0190
-4	.0166	.0184	.0171	.0174	.0192	.0190	.0167	.0171	.0164	.0175
-2	.0176	.0181	.0180	.0181	.0196	.0197	.0187	.0184	.0183	.0174
0	.0200	.0209	.0205	.0219	.0216	.0211	.0199	.0216	.0225	.0222
+2	.0244	.0249	.0256	.0265	.0251	.0256	.0249	.0264	.0274	.0261
4	.0308	.0313	.0324	.0336	.0315	.0310	.0305	.0318	.0339	.0346
6	.0383	.0390	.0396	.0416	.0393	.0400	.0381	.0390	.0431	.0440
8	.0476	.0482	.0487	.0518	.0487	.0482	.0478	.0463	.0523	.0551
10	.0576	.0589	.0594	.0632	.0593	.0579	.0590	.0550	.0612	.0660
12	.0679	.0675	.0696	.0750	.0740	.0733	.0706	.0633	.0718	.0774
14	.0799	.0814	.0816	.0871	.0855	.0857	.0846	.0716	.0822	.0912
16	.0926	.0942	.0936	.1015	.1013	.0969	.0986	.0809	.0945	.1047
18	.1489	.1055	.1087	.1168	.1180	.1107	.1133	.0911	.1070	.1194
20		.1364	.1439	.1635	.1356	.1384	.1534	.1013	.1636	.1344
22								.1557	.1766	.1918

TABLE No. 11.

Aerofoil — Gottingen-387

Biplane — Gap = Chord

Air Speed 40 f.p.s.

LIFT/DRAG

Incidence	Mono-plane	Complete Biplane			Upper Plane			Lower Plane		
		0°	15°	30°	0°	15°	30°	0°	15°	30°
-6	2.45	2.57	3.31	3.23	2.56	3.49	4.79	3.65	2.71	2.36
-4	7.15	6.05	6.74	7.08	7.50	6.85	9.24	6.75	6.39	5.24
-2	10.65	9.24	9.58	9.81	8.18	9.67	12.00	9.28	8.41	8.04
0	13.02	10.74	11.29	11.07	10.21	12.31	14.70	10.40	9.11	8.45
+2	13.77	11.55	11.47	11.38	11.46	13.00	14.83	10.55	9.96	9.12
4	13.16	11.12	11.10	10.97	11.18	12.85	14.60	10.49	9.41	8.36
6	12.55	10.50	10.61	10.26	10.63	11.60	13.52	9.95	8.50	7.79
8	11.54	9.82	9.85	9.51	10.00	11.09	12.29	9.53	8.07	7.21
10	10.75	9.14	9.05	8.80	9.14	10.27	11.24	8.99	7.61	6.77
12	9.96	8.69	8.50	8.24	8.23	9.11	10.41	8.61	7.19	6.47
14	9.11	7.85	7.81	7.68	7.78	8.51	9.30	8.12	6.84	6.01
16	8.14	7.24	7.36	7.04	7.09	7.96	8.50	7.67	6.42	5.68
18	2.82	6.74	6.69	6.51	6.49	7.09	7.67	7.10	6.03	5.34
20		4.04	3.81	3.64	2.40	2.42	2.56	6.49	4.78	5.10
22								5.00	4.51	4.26

CENTRE OF PRESSURE

Incidence	Mono-plane	0°	15°	30°	0°	15°	30°	0°	15°	30°
-6							87.0			
-4	66.8	56.5	39.9		67.7	63.6	60.1	73.6	76.1	86.0
-2	52.1	45.0	30.1		56.0	49.9	48.1	55.8	60.3	62.0
0	42.6	40.0	24.5	18.0	46.4	43.3	41.8	47.5	48.1	52.2
+2	38.9	37.4	22.6	14.0	41.4	38.6	38.5	42.2	42.1	45.4
4	36.1	35.6	22.0	10.4	38.0	36.0	35.8	38.8	38.9	41.8
6	34.1	35.2	21.6	9.6	35.6	34.0	34.0	36.6	36.5	39.3
8	32.9	35.0	21.1	9.3	33.5	32.7	32.5	35.0	35.2	36.2
10	32.7	35.2	21.9	9.3	32.6	31.6	31.8	33.7	33.5	34.8
12	31.9	35.6	22.7	9.5	31.2	30.9	31.0	32.9	32.9	33.9
14	31.5	36.1	23.2	10.3	30.8	30.1	30.0	32.0	32.2	33.4
16	30.7	37.0	24.0	11.7	30.0	29.1	29.0	31.3	31.2	32.3
18	39.3	37.4	25.0	12.4	29.2	28.1	29.1	30.9	31.1	31.9
20		38.4	24.0	18.1	34.3	34.9	35.3	31.0	31.9	30.5
22								31.5	31.5	32.3

TABLE No. 12.

RATIO OF LIFT OF UPPER WING TO LIFT OF LOWER WING

Airscrew-4

Incidence	Gap = Chord Stagger			-15°	Gap = ½ Chord Stagger			45°
	0°	15°	30°		0°	+15°	30°	
-6	3.03	27.00	2.68	-1.69	-1.23	-0.70	-0.62	10.6
-4	0.46	0.92	1.84	-0.35	-0.25	0.37	0.70	6.04
-2	0.93	1.21	1.70	0.22	0.29	0.81	1.26	4.49
0	0.99	1.24	1.53	0.40	0.48	1.00	1.42	2.88
2	0.99	1.23	1.54	0.48	0.56	1.16	1.45	2.53
4	1.02	1.25	1.45	0.53	0.90	1.20	1.46	2.24
6	1.06	1.23	1.49	0.67	0.95	1.19	1.41	2.10
8	1.08	1.25	1.48	0.77	1.00	1.20	1.40	2.02
10	1.08	1.25	1.44	0.82	1.02	1.20	1.40	1.93
12	1.11	1.25	1.44	0.83	1.00	1.21	1.38	1.92
14	1.14	1.27	1.41	0.89	1.01	1.18	1.36	1.82
16	1.16	1.27		0.91	1.02	1.12	1.03	
18					0.99		0.34	

Incidence degrees	U.S.A.-27 Aerofoil Stagger			Gottingen-387 Aerofoil Stagger		
	0°	15°	30°	0°	15°	30°
-6	0.94	0.44	2.20	0.78	1.37	2.00
-4	1.39	1.27	1.72	0.90	1.24	1.68
-2	1.13	1.16	1.62	0.94	1.24	1.61
0	1.06	1.19	1.53	0.98	1.27	1.56
+2	1.06	1.24	1.44	1.03	1.22	1.55
4	1.08	1.27	1.42	1.03	1.24	1.54
6	1.07	1.23	1.40	1.08	1.27	1.50
8	1.09	1.27	1.41	1.10	1.27	1.48
10	1.10	1.24	1.40	1.10	1.28	1.49
12	1.10	1.25	1.39	1.12	1.29	1.47
14	1.13	1.27	1.36	1.14	1.30	1.44
16	1.11	1.23	1.34	1.16	1.27	1.41
18	1.13	1.24	1.26	1.18	1.22	1.36
20	1.16	1.16	1.10	0.49	0.43	0.55

TABLE No. 13.

RELATIVE LIFTS OF THE UPPER AND LOWER WINGS

Plotted on Diagram A and C, Plate 11

Derived from Table 14 or Diagrams of Plate 9.

Aerofoil Section	Percent. of Total Lift of Biplane									
	Upper Wing Stagger					Lower Wing Stagger				
	-15°	0°	+15°	+30°	+45°	-15°	0°	+15°	+30°	+45°
	Gap = Chord									
Airscrew-4		0.515	0.555	0.595			0.485	0.445	0.405	
U.S.A.-27		0.522	0.555	0.583			0.478	0.445	0.417	
Gottingen-387		0.522	0.560	0.597			0.478	0.440	0.403	
	Gap = $\frac{1}{2}$ Chord									
Airscrew-4	0.425	0.490	0.540	0.585	0.687	0.575	0.510	0.460	0.415	0.313

TABLE No. 14.

AVERAGE RATIO OF LIFT OF UPPER WING TO LIFT OF LOWER WING

Plotted in Diagram C, Plate 10

Derived from Table 12 or Diagrams A, B, D, E, Plate 10

(May also be derived with less accuracy from Table 13)

Aerofoil	Stagger				
	-15°	0°	+15°	30°	45°
Airscrew-4 Gap = Chord.....		1.06	1.25	1.47	
U.S.A.-27 Gap = Chord.....		1.09	1.25	1.40	
Gottingen-387 Gap = Chord.....		1.09	1.27	1.48	
Airscrew-4 Gap = $\frac{1}{2}$ Chord.....	0.74	0.96	1.18	1.41	2.19

TABLE NO. 15.

AVERAGE BIPLANE CORRECTIONS ON LIFT

Derived from Diagrams A and D, Plate 12.

Gap = Chord

Biplane Lift = 0.85 of Monoplane Lift

Gap = $\frac{1}{2}$ Chord

Biplane Lift = 0.76 of Monoplane Lift

TABLE NO. 16.

AVERAGE STAGGER CORRECTIONS ON LIFT

Derived from Diagrams C and D of Plate 13.

(May also be found using Diagrams of Plate 12.)

Gap = Chord

Lift of Biplane of $+15^\circ$ Stagger = 1.018 Lift of Biplane of Zero StaggerLift of Biplane of $+30^\circ$ Stagger = 1.060 Lift of Biplane of Zero StaggerGap = $\frac{1}{2}$ ChordLift of Biplane of $+15^\circ$ Stagger = 1.050 Lift of Biplane of Zero StaggerLift of Biplane of $+30^\circ$ Stagger = 1.095 Lift of Biplane of Zero Stagger

LITERATURE

Reports of British Advisory Committee for Aeronautics

- R. & M. 60, Report for 1911-12, p. 59, parts III and IV.
Experiments on Models of Aeroplane Wings (Bleriot XI bis Aerofoils)
Bairstow and Jones.
- R. & M. 196, Report for 1915-16, p. 99.
Experiments on Models of Biplane Wings (R.A.F. 6c Aerofoils), Pannell
and others.
- R. & M. 366, Report for 1917-18, p. 184.
Biplane Effect on R.A.F.-15 Wing Section, Bryant and Jones.
- R. & M. 450, Report for 1918-19, Vol. 1, p. 366.
An Empirical Method for the Prediction of Wing Characteristics from
Model Tests, Relf.
- R. & M. 589, Report for 1918-19, Vol. 1, p. 438.
The Contribution of the Separate Planes to the Forces on a Biplane
(R.A.F.-15 Section), Cowley and Levy.
- R. & M. 648, Report for 1920-21, Vol. 1, p. 268.
Model Tests of R.A.F.-19 Wing Section as a Biplane, Carroll and
Bradfield.
- R. & M. 774, Report for 1921-22, Vol. I, p. 80.
Biplane Investigation with R.A.F.-15 Section, Cowley and Lock.
- R. & M. 857, Report for 1922-23, Vol. I, p. 159.
Biplane Investigation with R.A.F.-15 Section (Part II) Cowley and Jones.
- R. & M. 872, Report for 1923-24, Vol. I, p. 148.
Biplane Investigation with R.A.F.-15 Section (Part III) Tests at Various
Staggers and Gap/Chord Ratios, Cowley, Jones and Skan.

Reports of American National Advisory Committee

- Report No. 77—1920, Parker Variable Camber Wing—Parker.
- Report No. 151—1922, General Biplane Theory—Munk.
- Report No. 256—1927, The Air Forces on a Systematic Series of Biplane and Triplane
Cellule Models—Munk.
- Tech. Note No. 70—1921, The Effect of Staggering a Biplane—Norton.

French

- The Resistance of the Air and Aviation—Eiffel trans. Hunsaker, Chapt. II, sect. 4,
Part II, p. 62 and Supplement, Chapt. I, sect. 5, p. 159 (thin plates and
planes).
- Nouvelles Recherches sur la Resistance de l'Air et l'Aviation—Eiffel, 1920, Chapt. V,
Part II, p. 174 (Dorand, thin wings).
- Résumé des Principaux Travaux exécutés pendant la Guerre-Eiffel, 1919, Chapt. I,
Part II, Sect. 1 and 2, (various thin sections).

Miscellaneous

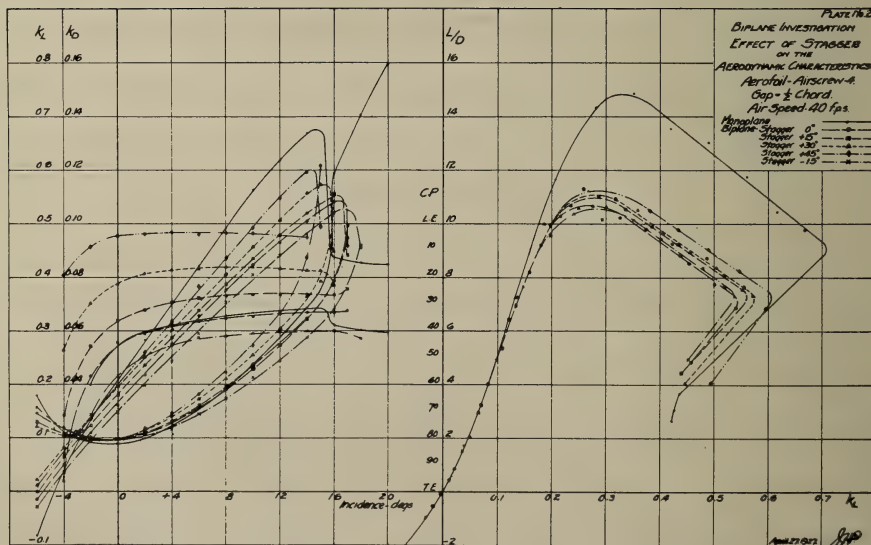
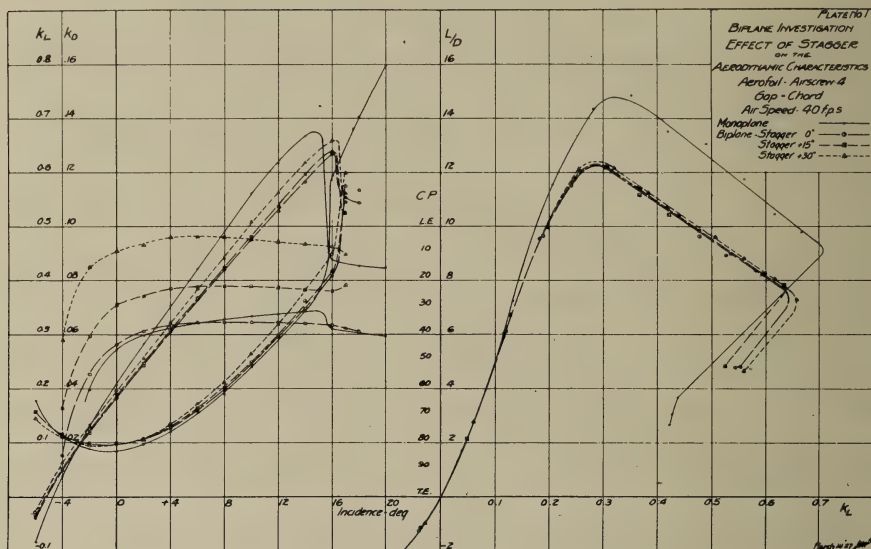
- The Triplane and Stable Biplane (R.A.F.-6 and Curtiss aerofoil) J. C. Hunsaker, 1918.
Biplane Wing Combinations (various thin American aerofoils) Bulletin of Airplane
Engineering Department U.S.A., Vol. I, No. 1, June, 1918, p. 136

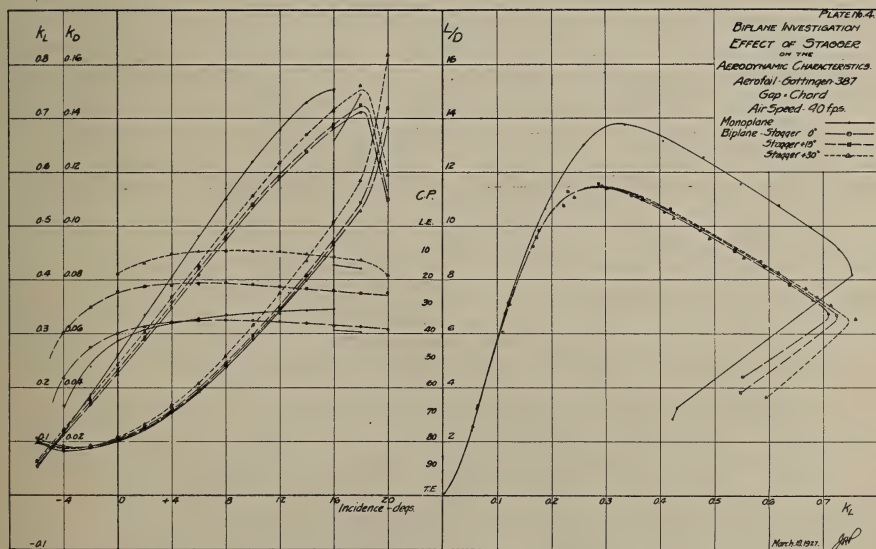
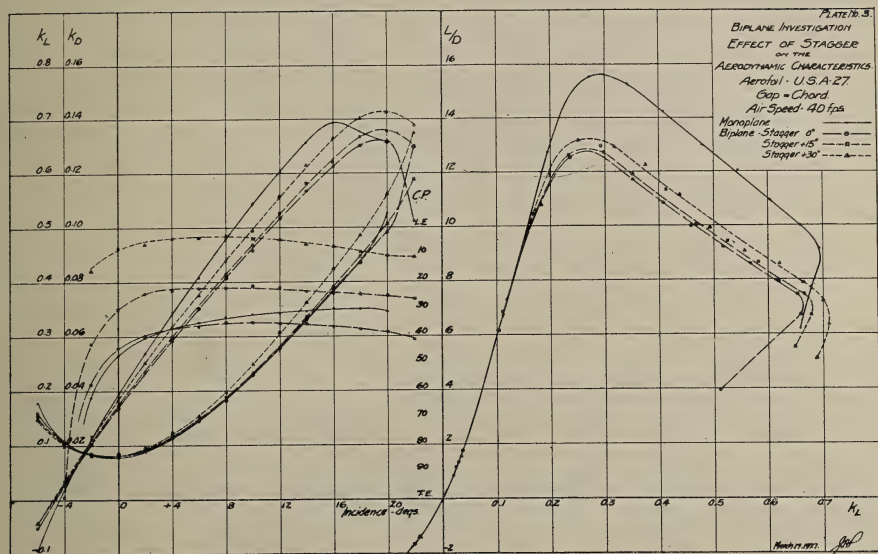
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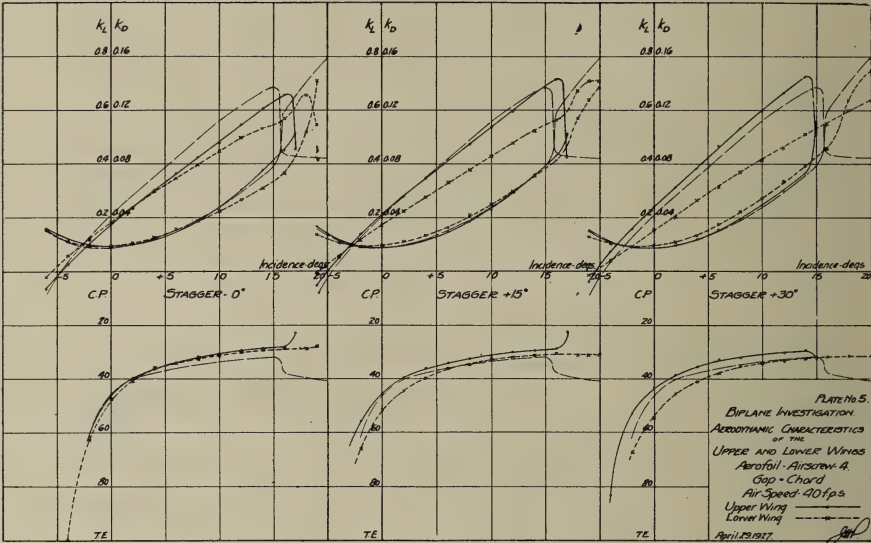
1. Effect of Stagger on the Aerodynamic Characteristics, Airscrew-4, Gap = Chord.
2. Effect of Stagger on the Aerodynamic Characteristics, Airscrew-4, Gap = $\frac{1}{2}$ Chord.
3. Effect of Stagger on the Aerodynamic Characteristics, U.S.A.-27, Gap = Chord.
4. Effect of Stagger on the Aerodynamic Characteristics, Gottingen-387, Gap = Chord.
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6. Aerodynamic Characteristics of Upper and Lower Wings, Airscrew-4, Gap = $\frac{1}{2}$ Chord.
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9. Wing Lift Coefficients *vs.* Biplane Lift Coefficients.
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13. Biplane and Stagger Effects on Lift.

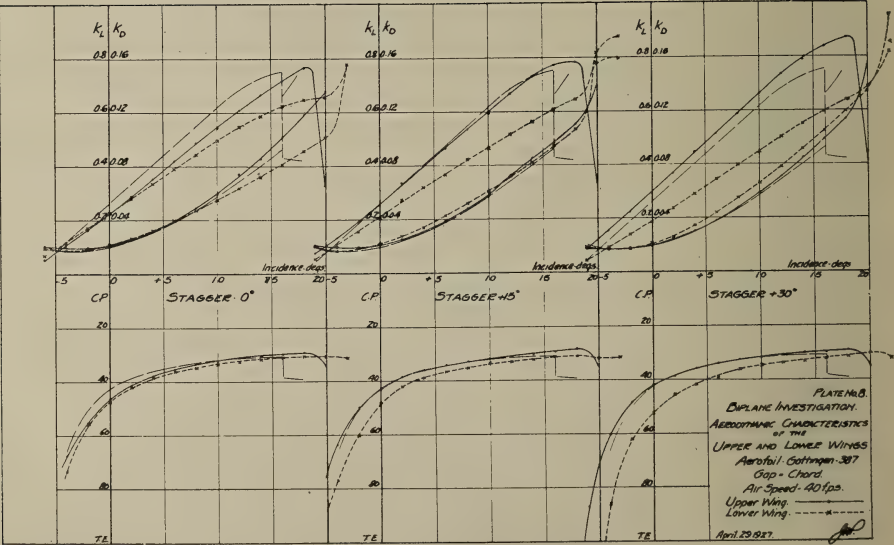
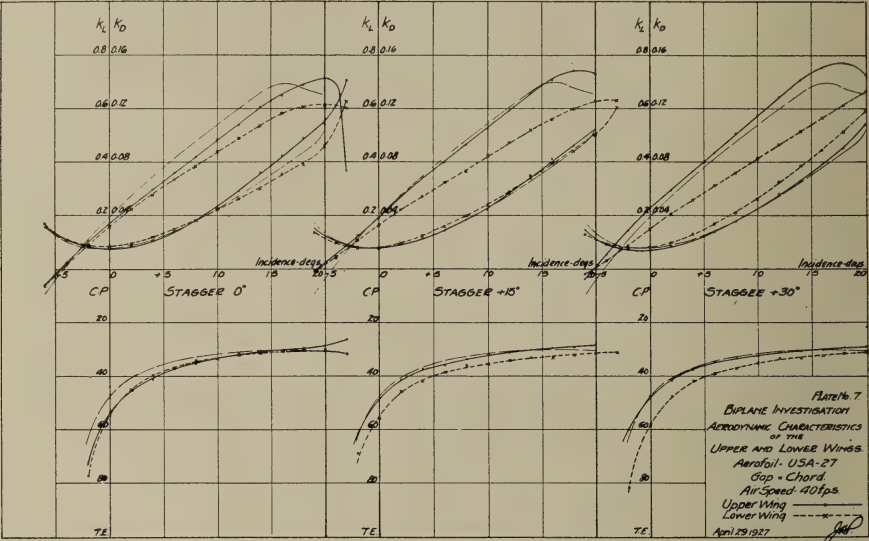
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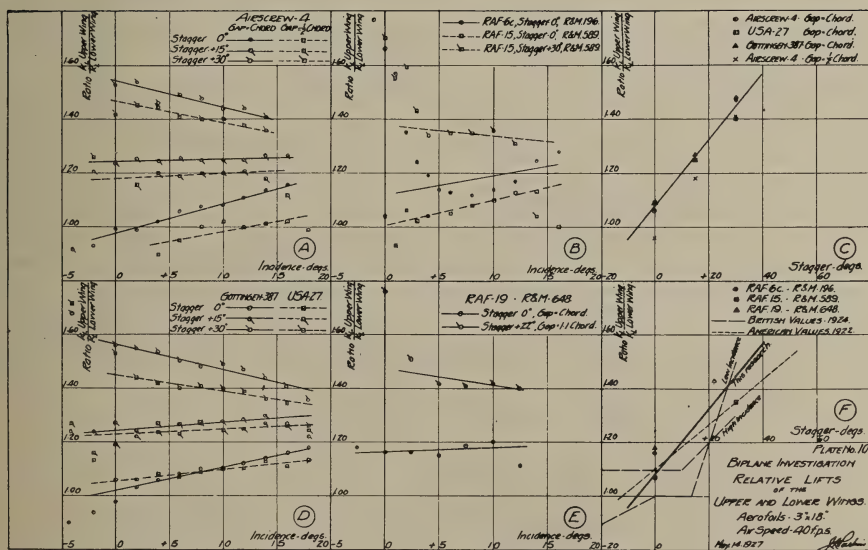
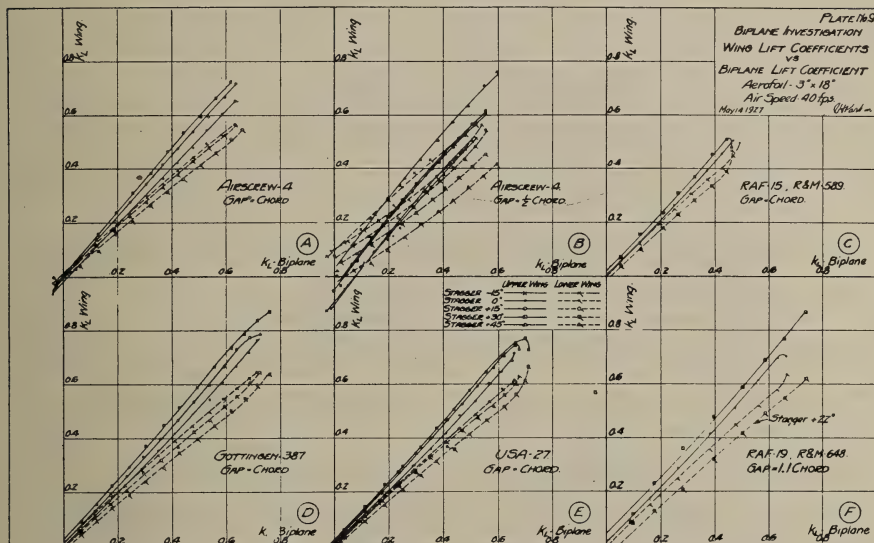
1. Front View of Set Up for Tests of Complete Biplane.
2. Side View of Set up for Tests of Complete Biplane.
3. Front View of Set Up for Determination of Spindle Effect, Complete Biplane.
4. Side View of Set Up for Determination of Spindle Effect, Complete Biplane.
5. Front View of Set Up for Tests of Individual Wings.
6. Front View of Set Up for Determination of Spindle Effect, Individual Wings.

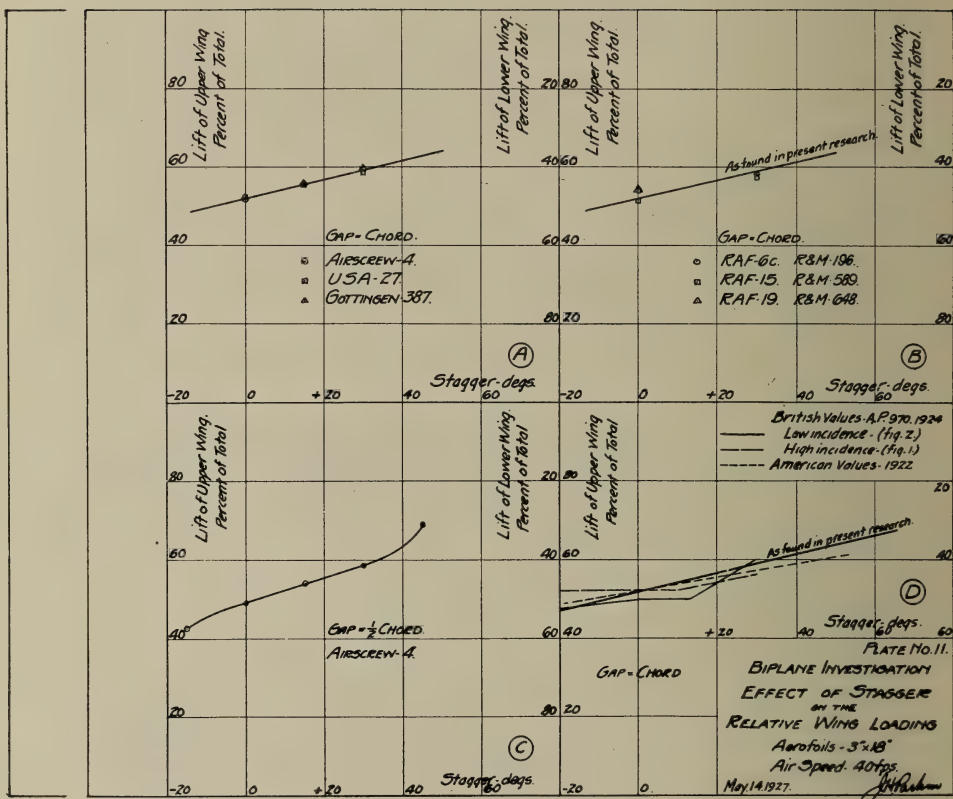


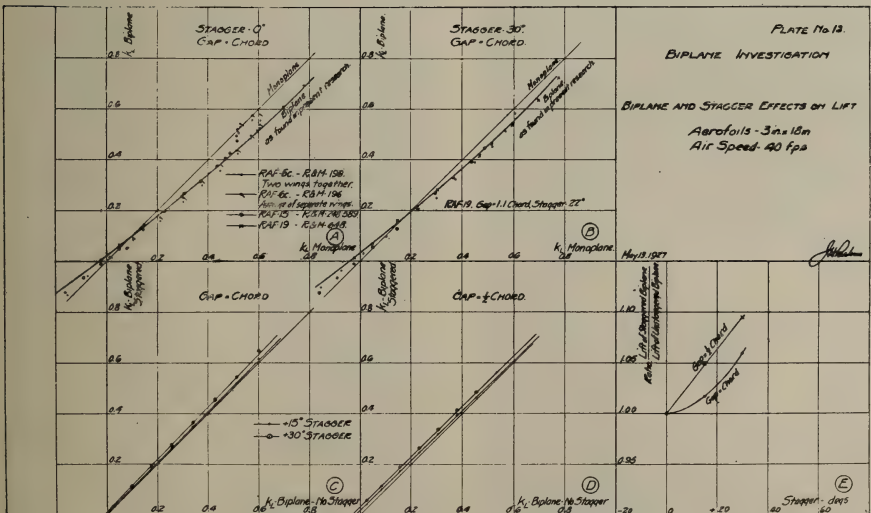
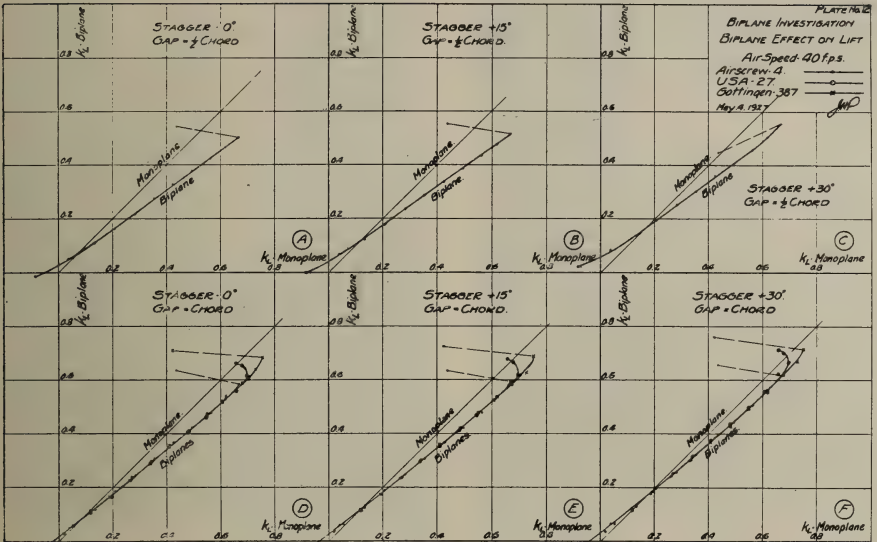


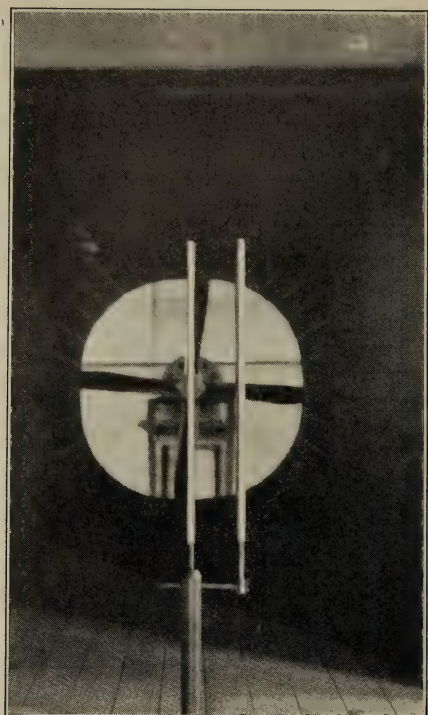








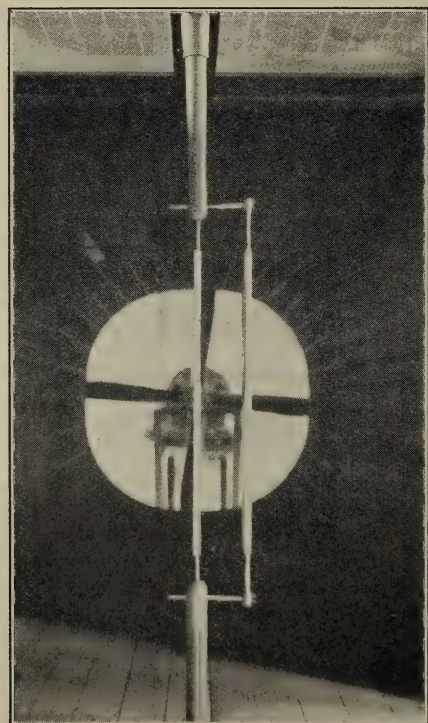




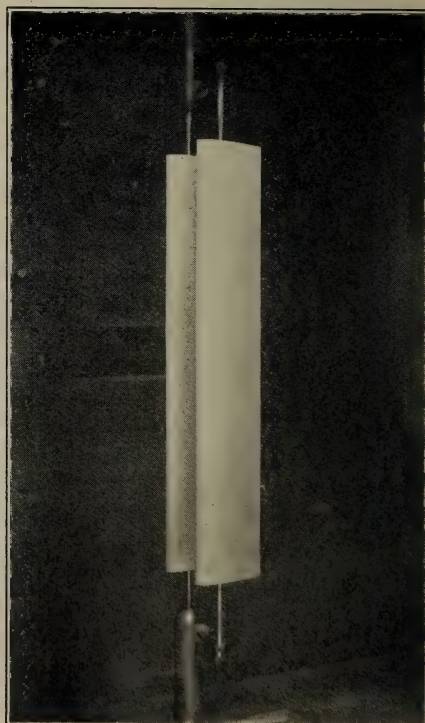
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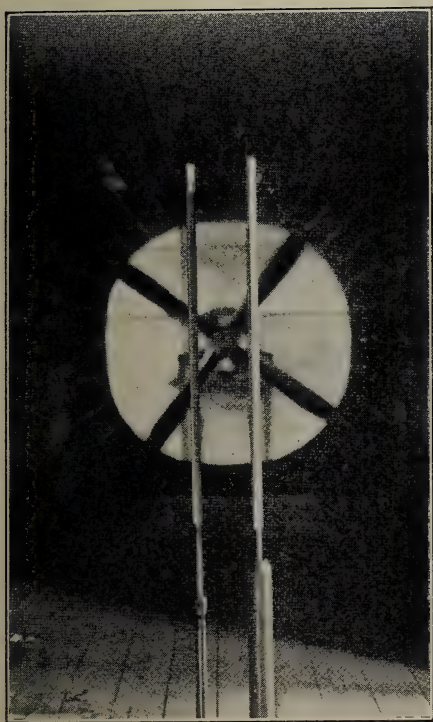
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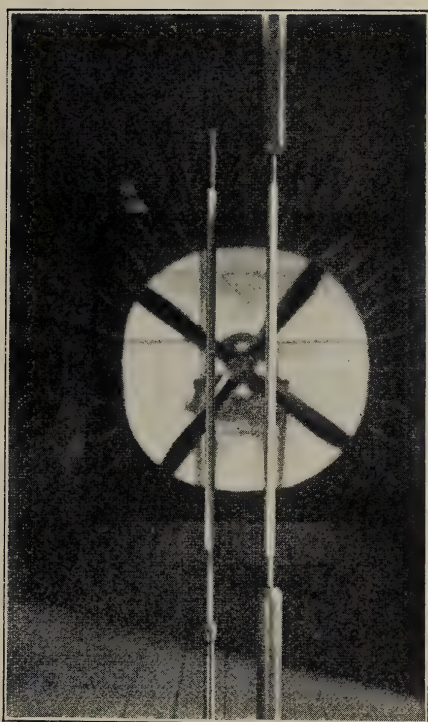
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THE BEHAVIOUR OF CELLULOSE ON HEATING

By J. W. BAIN, *Professor of Chemical Engineering*

Reprinted from Journal of the Society of Chemical Industry, Vol. 46,
p. 193T.

When wood is destructively distilled a large portion of the acetic acid is given off in the neighbourhood of the temperature of exothermic decomposition, *i.e.*, at about 280° , and it has been shown¹ that with wet wood 8 to 10% of the acetic acid may be driven off before actual decomposition commences. Since at the higher temperatures which are reached in wood distillation tar is the chief product, it appeared desirable with a view of increasing the yield of acetic acid to conduct the decomposition at as low a temperature as possible. A series of distillations of wood *in vacuo* carried out in these laboratories in 1913 and 1914 showed that there is a small but definite increase in the yield of acetic acid, which is in agreement with the results obtained by Klason² in the same year. This suggested a further investigation into the behaviour of cellulose when subjected to heat at temperatures well below the point of exothermic decomposition.

WITH J. E. T. MUSGRAVE.

Surgical cotton (100 g.) was heated in an electrical tube furnace at 210 — 220° for 832 hrs. In the first 24 hrs. 15 c.c. of distillate were collected, and the volume of gas given off did not reach 150 c.c. till 72 hrs. had elapsed. After 96 hrs. the gas was evolved at the rate of about 150 c.c. per day, and thereafter the yields of liquid and gas per day were very uniform. Towards the end the volume of the gas gradually diminished and after 34 days it became negligible. The liquid proved to be chiefly water with small quantities of oils and tar during the middle period of the run; the total acid was 0.075 g. expressed as acetic acid. The total volume of the gas was 2350 c.c., and the fractions varied in composition from 38% CO_2 , 18% CO in the first 300 c.c., to 24% CO_2 , 8% CO in the last 300 c.c.

A second run was made at 255 — 235° , using 100 g. of cotton. The evolution of gas was much more rapid, 600 c.c. being collected in the first 2 hrs. The flow of gas slackened after the third day, and after 376 hrs.

¹Hawley and Palmer, U.S. Dept. Agr., Bull. 129.

²Z. angew. Chem., 1909, 25, 1205; 1910, 27, 1252. J. pr. Chem., 1914, 90, 413.

no more was given off. The total volume was 4100 c.c., and the composition was very similar to that of the first experiment. The liquid distillate became steadily darker in colour, and the total acid was 0.405 g. expressed as acetic acid.

It is evident from these experiments that cellulose at temperatures of 210° and 265° undergoes a slow progressive decomposition, yielding chiefly carbon dioxide and monoxide, but the cessation of the evolution of gas cannot be accounted for. The residues in the furnace were dark brown, and the fibres quite brittle; there was no appearance which might suggest a reason for the interruption of the decomposition. Apparently the long heating of cellulose within the range of temperature stated above results in the formation of a residue which is quite stable.

WITH G. F. KAY, G. M. CHUTE, AND S. A. ROWLAND.

Up to 1918 it had been assumed that wood or cellulose decomposed directly into acetic acid, methyl alcohol, and a number of other substances, including gases and charcoal; at least there is no evidence extant to show that any other view was held. The discovery by Pictet and Sarasin³ in that year of a crystalline substance, lævoglucosan, in the distillate obtained by heating cellulose *in vacuo* quite unexpectedly revealed the existence of an intermediate product. While preparing some lævoglucosan from cellulose according to the directions of Pictet and Sarasin, a difficulty was encountered which suggested the following experiment. A quantity of purified cotton was rapidly heated under 15 mm. vacuum until a small quantity of distillate had come over; the distillation was then interrupted, and after cooling the cotton was extracted with water. On concentration no lævoglucosan could be detected in this extract, but there were obviously other solids present. The desirability of examining the non-volatile products resulting from the heating of cellulose was at once apparent, and the investigations which will now be described have this object in view.

Purification of the cotton.—The raw material used was surgical cotton of the best quality, which, however, yielded a considerable amount of water-soluble substances. It was accordingly extracted with hot distilled water continuously for seven days in an extraction apparatus constructed of copper on the Soxhlet principle. The cotton was placed in a bag in a small copper tank, silver-plated inside, to which was attached a glass condenser; steam entered the cylinder from a large flask of boiling water, and at intervals the condensate was returned to the latter by a siphon. Towards the end of the period of extraction the water in the

³Helv. Chim. Acta, 1918, 1, 87.

flask supplying the steam became very dark in colour, and some solids separated.

To determine when the extraction was complete samples of water bathing the cotton were taken through the siphon from time to time, and the concentration of the total solids was found. The removal of the soluble substances was slow, and not until the seventh day did the concentration of the extraction water become constant with 14 mg. per litre of total solids. Since Wislicenus and Gierisch⁴ have shown that cellulose has a small but definite solubility, the purification was considered to be satisfactory after seven days' extraction, and all the cotton used in the investigation was subjected to this treatment.

Heating the cotton.—Considerable difficulty was experienced in the early stages in heating the cotton uniformly to the desired temperature. After experimenting with various stationary ovens in which no satisfactory uniformity of temperature could be obtained, a cylinder rotating in an air bath was devised. This was found to be an improvement, and a larger apparatus electrically heated was built; the shaft which carried the rotating cylinder was hollow, and thus permitted the measurement of the temperature at the centre of the mass of cotton in addition to that of the air bath on the outside. After using this apparatus for some time, an electrically-heated and automatically-controlled furnace made by Leeds and Northrup was placed at our disposal; this proved to be more accurate and more readily operated, and has accordingly been used ever since. The temperature of the furnace is indicated by a thermocouple placed at the centre and near the bottom, and an automatic regulator slowly raises the temperature to the desired point; in these experiments 3 hrs. were required to raise the furnace to 200°. To explore the uniformity of temperature in the furnace, a series of readings were made with mercurial thermometers at various points; these showed that there was a maximum variation of 2°.

The purified cotton was rammed as firmly as possible (200 g. approximately) into a metal container having a diameter about $1\frac{1}{2}$ in. less than that of the furnace. A cap with connexions for the collection of volatile substances was screwed on to the container and made air-tight. To ensure the exclusion of air, the screw connexions were luted with a litharge cement, which proved on experiment satisfactory at the maximum temperatures used. As an evidence of the uniformity of the temperature attained, the appearance of the heated cotton may be cited; it was brownish in colour, and of the same shade throughout the entire mass. During earlier experiments with other methods of heating the variation in the colour was quite noticeable.

⁴Kolloid-Z., 1924, 34, 176.

A number of preliminary experiments were carried out at 180°, 200°, 220° and 230°; the differences were not marked. It was then decided to select 200° as the standard temperature to which the cotton would be heated in this series. Similarly there was chosen a purely arbitrary heating period, *viz.*, 3 hrs. from room temperature to 200° and 3 hrs. at 200°.

Extraction.—The extraction of the heated cotton was carried out by boiling with distilled water in large glass beakers; about 6 litres of water were used per charge of 200 g. of cotton. A series of experiments were made to determine the most advantageous method of extraction, and as a result of these it was decided to maintain the cotton in boiling water for two periods, each of 2 hrs.; the quantity of extract obtained by further treatment was found to be negligible. The extract, which was of a straw colour, was concentrated in a large evaporating dish, the colour turning to dark brown. During the concentration fine, dark brown fibres separated, and when the volume had reached about 200 c.c. the liquid was filtered. After further evaporation to about 40 c.c. on the water bath the syrup was kept in a desiccator over calcium chloride until the loss in weight was less than 1 mg. per day, which usually required about 2 weeks.

The extract was always slightly acid before evaporation commenced, and obviously nearly all the volatile acids would be removed during the concentration. An experiment to determine the amount of the acids thus lost was made as follows: to the extract was added sodium carbonate to slight alkalinity, and the solution was evaporated to dryness. Distillation with syrupy phosphoric acid yielded several fractions boiling from 95° to 110°; on concentration by freezing, formic acid boiling at 104.5° was obtained. The density of the solution was found to be 1.125, which corresponds to approximately 52% formic acid; the amount was 0.8 g., which was obtained from 200 g. of cotton. The amount of formic acid produced was, therefore, approximately 0.2% of the dry cotton taken; a small quantity of acetic acid was also present, but its weight was not determined. The average yield of the syrupy extract on a series of five charges was 2.38 g. per 100 g. of dry cotton.

Repeated heating.—Preliminary work had shown that the heated and extracted cotton on reheating yielded a fresh quantity of extract, and this behaviour was investigated quantitatively. After having been extracted following the first heating, the cotton was thoroughly dried, and was then packed into the metal cylinder as already described. It was then heated and extracted in precisely the same manner as a fresh charge; the cotton fibres became more brittle during the heating, but with care could be handled for further treatment. A third heating and extraction on the

same sample was carried out, yielding more extract, but the cotton was reduced almost to a powder, and no further treatment was attempted.

The following table shows the results of the experiments, using five charges for the first heating, and two charges each for the second and third. The metal cylinder was tightly packed in each case, and for the second and third heatings the weight of dried cotton was much greater than the first owing to the disintegration of the fibres. This extra weight was made up by combining the first two charges after the first heating.

Charge.	First heat.				Total	Remarks.
	1st.	2nd.	3rd.	4th.		
1	1.98	0.26	0.17	—	2.41	First period, 3 hrs.; 2nd and 3rd, 2 hrs. each.
2	1.85	0.37	0.23	0.08	2.53	Four periods, 2 hrs. each.
3	—	—	—	—	2.43	Three periods, 2 hrs. each.
4	1.9	0.59	—	—	2.49	Two periods, 2 hrs. each.
5	1.83	0.25	—	—	2.08	Two periods, 1½ hrs. each.
Second heat.						
1	1.94	0.33	—	—	2.27	Two periods, 2 hrs. each.
2	—	—	—	—	2.3	Two periods, 2 hrs. each.
Third heat.						
1	1.55	0.51	—	—	2.06	Two periods, 2 hrs. each.
2	—	—	—	—	2.17	Two periods, 2 hrs. each.

The figures given in the vertical columns refer to grams of dry extract per 100 g. of dried cotton; the word "period" refers to the time of boiling in each case.

Treatment of the syrup.—To the syrup, which was acid to litmus, was added an excess of barium carbonate, and evaporation to dryness was carried out on the water bath; the dried mass was extracted with 90% alcohol in accordance with the procedure suggested by van der Haar.⁵ The alcoholic extract was concentrated, decolorised with charcoal in aqueous solution, again concentrated, and once more treated with 95% alcohol. Even this treatment did not suffice for the removal of the soluble barium salts which frequently appeared at later stages. The product was a slightly yellow syrup (C) which only lost water very slowly in the desiccator. A sample dried to constant weight at 10 mm. lost 44.8% of its weight.

This syrup (C) yielded the following results: The usual colour reactions failed to show the presence of pentoses or methylpentoses.

Through the kindness of the Department of Zymology, University of Toronto, a fermentation was carried out with *Saccharomyces cerevisia*, which showed a definite evolution of carbon dioxide; a blank and a solu-

⁵"Anleit. Nach. Trenn. Monosacch.", Berlin, 1920, 305.

tion of dextrose were treated at the same time. The presence of a hexose was indicated.

On treatment with phenylhydrazine an osazone melting at 210° in Roth's apparatus was obtained; the melting point was unchanged by admixture with phenylglucosazone. The *p*-nitrophenylosazone was prepared and had m.p. 255° . Mannose is probably absent, since no hydrazone could be obtained. Neither *a*-methylphenylosazone nor *o*-nitrophenylhydrazone could be prepared and identified, so that lævulose is either absent or is present only in small quantity.

When the examination had reached this stage it was discovered unexpectedly that the syrup *C* was partially soluble in absolute alcohol; the insoluble portion is designated as *E* and the soluble as *F*. The latter gave a marked reaction with Fehling's solution, and was further examined for fructose. 2.5 g. of *F*—in the form of syrup, dried by keeping for several days in a desiccator—was treated with slaked lime, and, after keeping, the insoluble residue was filtered off; suspended in water, and treated with a current of carbon dioxide. After removing the calcium carbonate and concentrating the filtrate, a very small residue was obtained which could not be identified as lævulose.

A portion of *F* was tested anew with *o*-nitrophenylhydrazine; a precipitate formed which, on examination under the microscope, showed several small needle-shaped, orange-coloured crystals similar in appearance to the hydrazone prepared simultaneously from lævulose. An attempt to prepare a quantity of these crystals sufficiently pure to serve for a melting point determination was unsuccessful. The conclusion is accordingly reached that lævulose may be present, but only in very small quantity.

The examination of portion *F* optically has not yet been possible owing to the difficulty experienced in decolorising sufficiently to permit readings to be made in the polarimeter tube.

Portion *E*, insoluble in absolute alcohol, has been investigated, but without any definite results; the fact that it is small in amount has added to the difficulty of making satisfactory tests.

The barium carbonate which had been used to neutralise the crude syrup was extracted with water, and the filtrate was decolorised and concentrated to a thick syrup. No crystals formed after keeping for some time; accordingly syrupy phosphoric acid was added, and the solution was distilled with steam. The total acidity was equivalent to 1.75 c.c. of *N*-acid. Formic acid was determined in this distillate by Jones' method,⁶ and found to be equivalent to 1.0 c.c. of *N*-acid. Consequently there is the equivalent of 0.75 c.c. of *N*-acid of some other acid or acids—probably

⁶Amer. Chem. J., 17, 539.

acetic—present in the extract. The figures just given were obtained from crude syrups derived from 1000 g. of dry cotton heated.

Weight balance.—Several runs were made to discover whether the weights of the various products of the heating checked satisfactorily.

Purified cotton.			
Wt. before heating.	Wt. after heating.	Wt. of liquid distillate.	Vol. of gas collected.
g.	g.	g.	litre.
203.2	200.1	2.8	1.0
223.5	219.6	3.9	1.0
204.2	199.5	3.7	1.5

The liquid distillate was water with very slight acidity, and the gas was air with small quantities of carbon dioxide and monoxide. At a temperature of 200°, therefore, cotton gives off scarcely any volatile decomposition products, and the substances formed can only be discovered by extraction with a solvent.

Department of Applied Chemistry,
University of Toronto.

COEFFICIENTS FOR VENTURI METERS AND THE EFFECT OF AN OBSTRUCTION IN THE THROAT

R. W. ANGUS, *Professor of Mechanical Engineering,*
and J. S. E. MACALLISTER, *Demonstrator in Hydraulics*

The Venturi meter has come to be so well known that there is no need to discuss the ordinary use of it in any detail. Many investigations have also been made to establish the coefficient for the meter, and these have shown that this coefficient remains remarkably constant for throat velocities between about 10 and 40 ft. per second, and as the meter is rarely used outside this wide range, it may be stated with some degree of accuracy that the coefficient for a given meter is constant.

The coefficient increases with the size of the meter, and varies slightly with the ratio of the entry and throat diameters, and for definite information on this point the reader is referred to the published papers in various journals.

The present article has to do with the case of a Venturi meter working under unusual conditions, and shows the variation of the coefficient in such a case.

The meter referred to had a tube made by the Simplex Meter Co., the tube having an entry diameter of 6 in. and a throat diameter of 3 in., as indicated on the photograph in Fig. 1. It was made with bell and spigot ends for connection with a leaded joint to a 6 in. water main, and was used to measure the domestic supply sold by one municipality to another. The meter was installed and connected up with the recorder in the usual way and for some time gave complete satisfaction.

In course of time, however, the recorder showed a large increase in the water passing the meter, and as it was in a more or less isolated spot, this increase was not observed immediately. When it was discovered an investigation was made, and it suggested that there was an error in the meter reading. After the recorder had been carefully checked up and found correct, the meter tube was dug up and found to contain the piece of wood shown in the photograph.

That the wood was there by accident was proved beyond doubt, and it had apparently broken off the end of a longer piece which had been used as a pry in placing the water main, and having been left in the pipe it eventually reached the meter. However, the water account had to be adjusted, and with this in view the author was retained and he had the tube set up in the Hydraulic Laboratory of the University

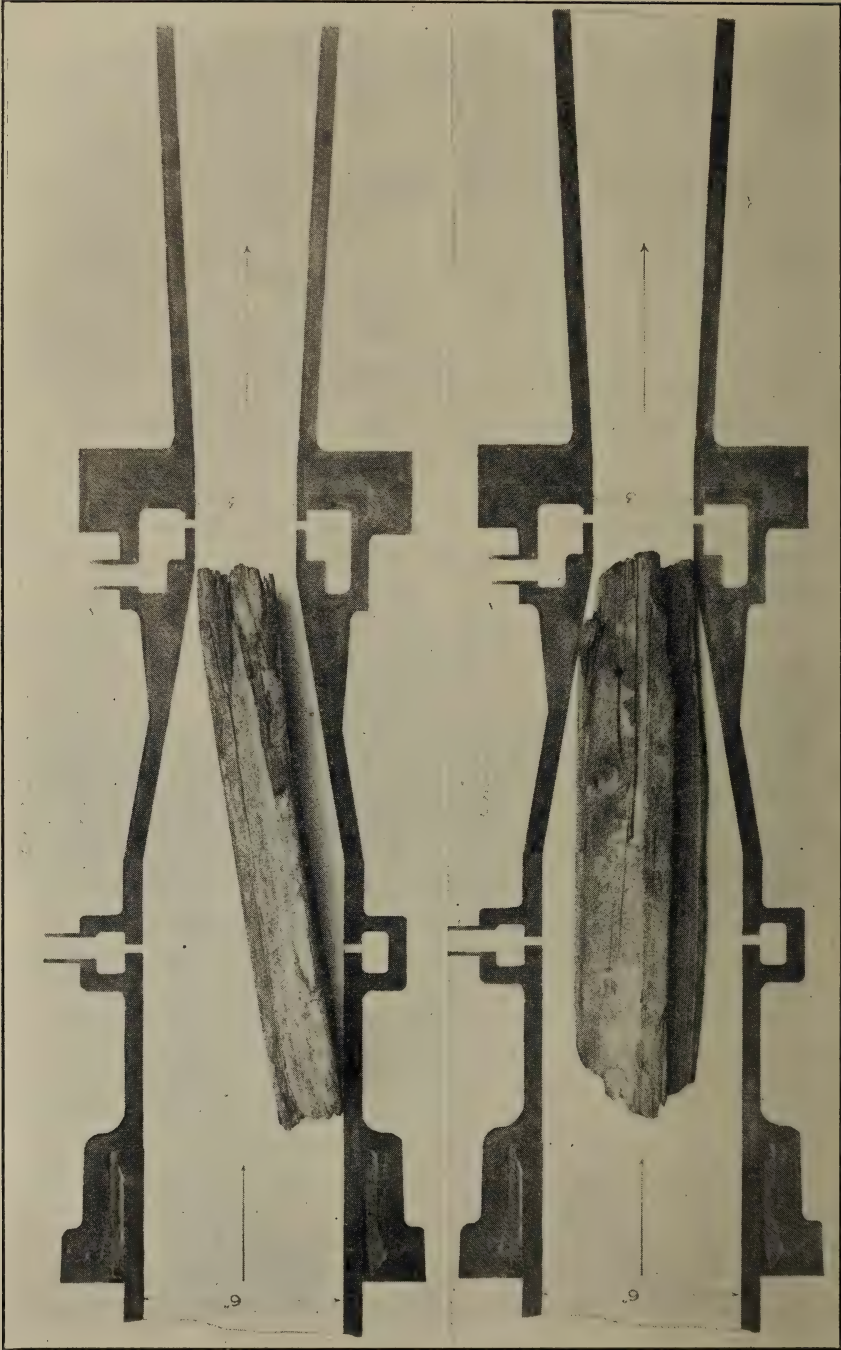


Fig. 1

of Toronto and calibrated with the piece of wood in place, and also with the throat clear.

The equation used in investigating the meter was

$$Q = \frac{cA_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{2gH}$$

where c is the coefficient for the meter

A_1 the area in square feet of the entry end of the meter.

A_2 the area, in square feet, of the throat.

H the drop in pressure in feet of water column, between the entry end and the throat of the meter.

Q is the discharge in cubic feet per second.

Throughout this investigation A_2 has been taken as the area of the 3 in. throat, (although the block of wood actually obstructed part of this throat and reduced its area,) so that for this meter the above formula reduces to

$$Q = .407c\sqrt{H}.$$

In addition to solving the problem directly under consideration, it was thought desirable to see what the coefficient of the meter would be under such an unusual condition. The tube was set up in a vertical position, to suit the conditions of the laboratory, and water was supplied from a pump through a 6 in. horizontal pipe having a straight length of about 20 feet. At the end of this there was a short radius screwed elbow having a vertical downward outlet, and into this a piece of 6 in. steel pipe was screwed, the lower end of which was leaded into the bell end of the meter; the distance from the face of the elbow to the end of the meter was only 11 inches.

The discharge was measured over a 90° V -notch weir, placed in the end of a tank 3 ft. wide, 7 ft. long and 15 in. deep below the vertex of the weir. The meter discharged into the tank at a point 10½ in. from one end, and after leaving the meter the water passed through a series of baffles, leaving these at a point about 5 ft. up from the weir, the water surface below the baffles and in front of the weir being smooth and quiet. The head on the weir was measured by hook gauge, and the drop across the meter by means of a differential mercury manometer.

The discharge over the weir was computed by the well known formula $Q = ah^{2.5}$ where h is the head on the weir in feet and a was taken from Barr's experiments reported in "Engineering" April 8 and 15, 1910.

Two sets of tests were run, the first with the throat clear, as it would ordinarily be, and at throat velocities varying from 13.1 to 38.5 ft. per second, which correspond to discharges of .64 and 1.89 cubic ft. per

second, respectively. In the second set of tests the wood was put into the meter exactly as it was found when the meter was dug up, and in this case the discharges varied from 0.38 to 1.08 cubic ft. per second, some of the tests being run with the wood left free, to take its own place, while in others a wire was put through the wood and the latter pulled down into the throat with considerable force, but both methods of operation gave the same results.

The results of the two sets of tests are set out in the tables herewith, and are also plotted on the curve sheets, Figs. 2, 3 and 4. In examining these it should be borne in mind that no extraordinary means were taken to secure extreme accuracy, that is, the weir coefficients for the

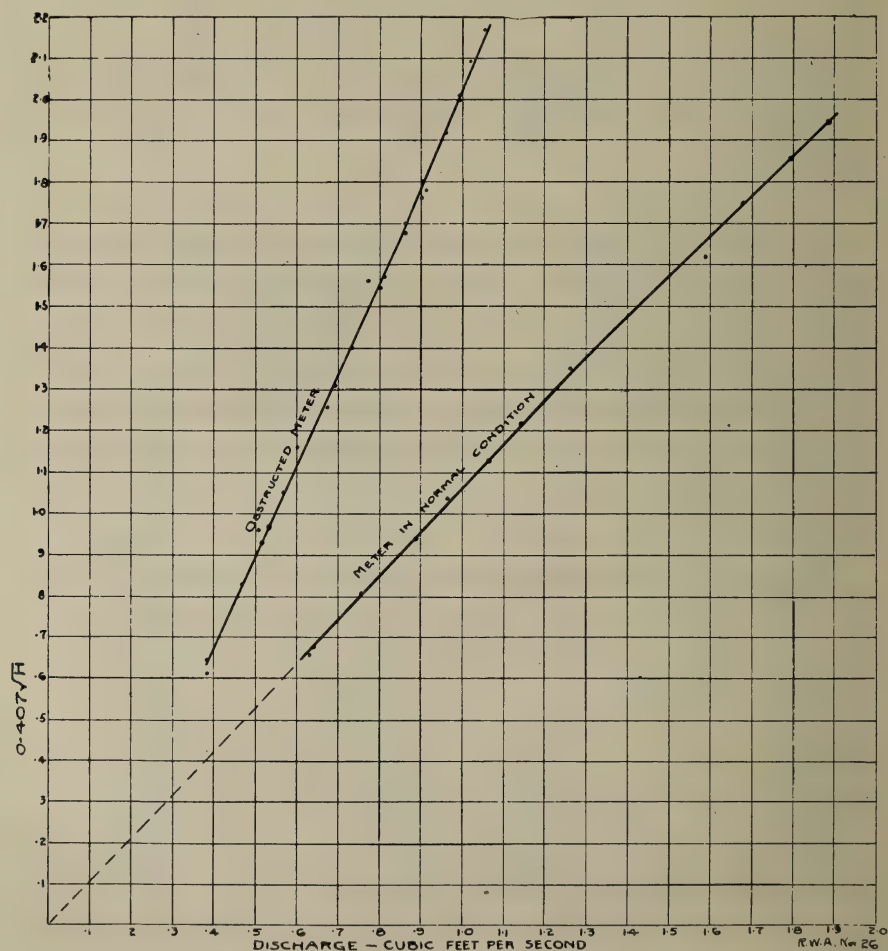


Fig. 2

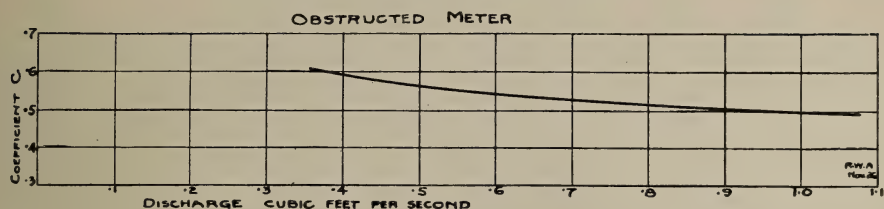


Fig. 3

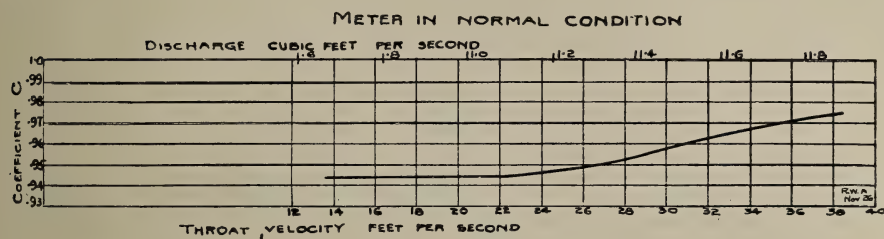


Fig. 4

particular set up were not determined by experiment, but the results are very close, as it was possible to measure the head on the weir to .01 in. and that on the meter to .05 in. of mercury.

An examination of the curves on Fig. 2 shows that the results are in agreement with one another and define smooth curves in both cases, these curves differing little from straight lines. With the unobstructed throat the line would pass through the origin, and this indicates that there is interference, at the lower discharges, from the elbow which is so close to the entry end of the meter, because experiments generally indicate that the coefficient decreases at the lower discharges. The curve of the obstructed throat shows that the coefficient increases as the discharge decreases, which is the reverse of the former case.

The curve in Fig. 3 gives the values of the obstructed meter coefficients and shows a variation from .49 to .61, or about 25 per cent. over the range of calibration, and one is surprised to find such a small variation in this case. Had it been used at the more usual discharge of .7 c.f.s. or over, the variation would not have exceeded 10 per cent. The coefficients for the meter in normal conditions, shown in Fig. 4, are low at the lower throat velocities, and one would not expect them to be constant for the lower part of the range. At the higher throat velocities they are quite nearly correct, indicating that the proximity of the elbow has caused no serious trouble at these velocities. The results on the unobstructed meter must be regarded as good approximations since the discharge could not be weighed.

The authors are indebted to Messers Thomas and Staunton, New Toronto, Ontario, for placing the meter at their disposal.

TABLE I
METER IN NORMAL CONDITION

Test No.	H ft. water	$.407\sqrt{H}$	Q c.f.s.	Coefficient C	Throat Velocity ft. sec.
1	2.58	.654	.634	.965	12.91
2	2.73	.673	.645	.959	13.14
3	3.89	.803	.759	.945	15.46
4	5.29	.936	.890	.951	18.13
5	6.42	1.033	.969	.938	19.73
6	7.68	1.128	1.067	.946	21.73
7	8.90	1.214	1.143	.942	23.28
8	10.98	1.348	1.267	.940	25.80
9	15.78	1.617	1.591	.984	32.40
10	18.36	1.744	1.682	.964	34.26
11	20.70	1.852	1.797	.970	36.60
12	22.78	1.943	1.890	.973	38.50

TABLE II
METER WITH OBSTRUCTED THROAT

Test No.	H ft. water	$.407\sqrt{H}$	Q c.f.s.	Coefficient C
1	2.20	.604	.380	.630
2	2.52	.646	.386	.597
3	4.11	.825	.471	.571
4	5.55	.959	.511	.533
5	5.19	.927	.516	.557
6	5.64	.967	.535	.553
7	6.54	1.041	.568	.545
8	8.15	1.160	.602	.519
9	9.47	1.252	.673	.537
10	10.23	1.302	.693	.532
11	10.86	1.341	.699	.521
12	11.78	1.400	.734	.524
13	14.65	1.560	.774	.496
14	14.32	1.540	.802	.520
15	14.76	1.563	.811	.519
16	17.41	1.700	.863	.507
17	18.79	1.760	.903	.513
18	19.56	1.800	.903	.503
19	19.03	1.776	.913	.514
20	22.19	1.920	.962	.501
21	24.28	2.010	.994	.494
22	24.00	1.995	1.001	.501
23	26.28	2.090	1.022	.490
24	28.25	2.163	1.053	.487
25	29.44	2.210	1.082	.490

THE STANDARD SHORT TUBE

AN INVESTIGATION OF THE PRESSURES AND SHAPE OF THE JET IN THE TUBE

ROBERT W. ANGUS, *Professor of Mechanical Engineering*

The standard short tube may be briefly described as one of the possible end connections between a pipe and a tank or reservoir from which it is drawing water. In ordinary work a pipe taking water from a reservoir may have its connection made with the tank in one of the following ways: (a) the end of the pipe may be cut off normal to the axis and the pipe may be so placed that its inner end is flush with the inside of the reservoir; (b) the inner end of the pipe may spread out into a bellmouth, which is set flush with the inner reservoir wall; and (c) the end of the pipe may project some distance into the reservoir and may have either of the two forms of ends mentioned above. These several cases are illustrated in Fig. 1 at a, b, c and d, respectively.

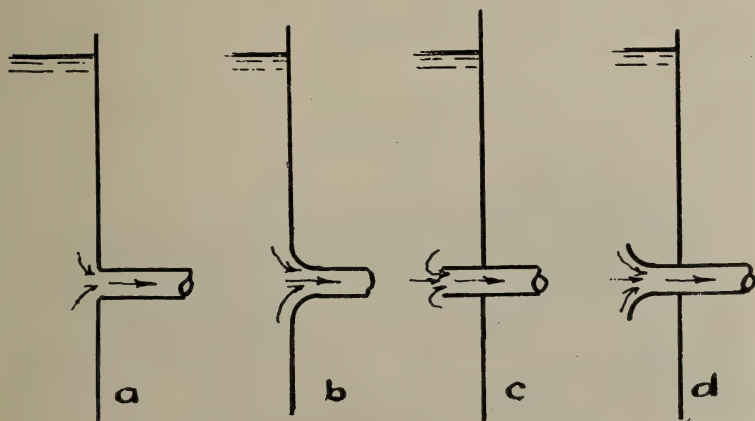


Fig. 1

The pipe laid in the first method is said to have a standard end, and contraction always occurs inside the pipe when water flows, this contraction being followed by expansion again till the water fills the pipe. That part of the pipe near the end, in which this contraction and expansion occurs, is called the Standard Short Tube, and its minimum length is from three to four times the diameter of the pipe. Some

time ago the writer investigated experimentally the conditions existing in this tube and the results of these experiments are given herewith.

The tube used had a diameter of 2 in. and a length of 8 in. and was screwed into the vertical flat end of a cylindrical drum 18 in. diameter, the connection being so made that the inside of the head was a true plane, Fig. 5, this surface being machined to make it smooth. The tube discharged into the atmosphere, the water then passing down into an orifice tank where it was measured. The orifice tank had a large cross sectional area so as to avoid velocity of approach, and the head on the orifice was measured by hook gauge. The coefficients for the orifice used were obtained by direct measurements of the discharge in earlier experiments.

The pressure on the drum supplying the tube was maintained constant at 28 ft., this pressure being read from a very accurately calibrated gauge. Usually it was difficult to get the outer end of the tube to fill with water at the beginning of each days' running, as the water had a marked tendency to spring clear of the outer end, but the latter could be filled by holding the hand over the end of the tube so as to produce full flow at low pressures, after which the pressure was gradually raised till the desired 28 ft. was reached. Once the tube filled, the water had very little tendency to spring clear of the discharge end again.

Outside of determining the discharge coefficient for the tube the

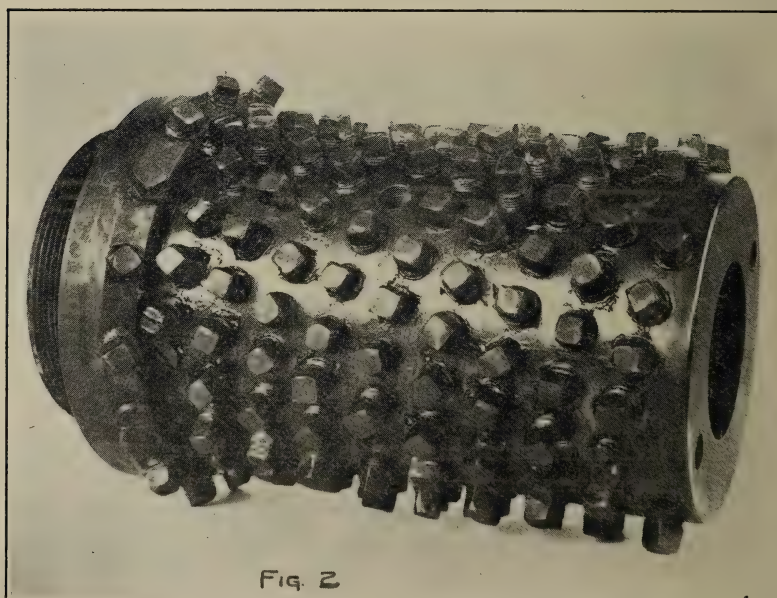


Fig. 2

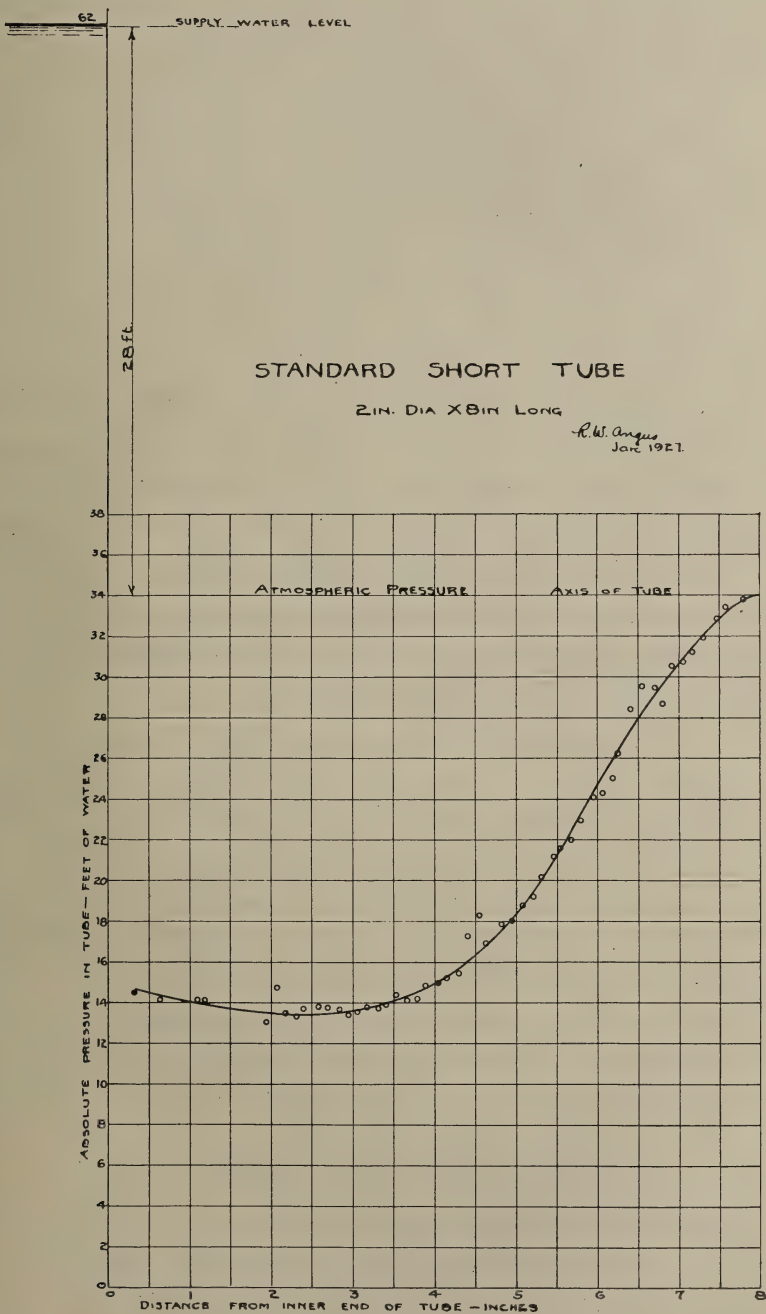


Fig. 3

real purpose of the experiment was to measure the pressures at different points along its length, and afterward to use these pressures to compute the shape of the stream in the tube. To measure the pressures the tube was constructed as shown in Figs. 2 and 5, and measurements were made at four points on each of fifty-one planes along the axis of the tube, these planes being one-eighth of an inch apart in most cases, although at certain points such close spacing was not practicable; their exact location corresponded to the pressure measurements shown on Fig. 3. At each plane where a measurement was taken, four holes, one thirty-second of an inch in diameter and 90 degrees apart, were drilled through the wall of the tube, and these holes were enlarged in the outside of the shell so as to receive one-eighth inch pipe fittings, Fig. 2, a few of the holes being shown on Fig. 5. The average of the pressures at each of these holes was taken as the pressure at each plane.

All of the pressures inside the tube were below the atmosphere and were read off a mercury manometer. The results of the experiment are given in Fig. 3, the manometer readings being adjusted to correspond to the barometric pressure 34 ft. of water column. It will be seen that, with very few exceptions, the points lie on a very smooth curve and that the minimum pressure, or highest vacuum, is at a point 2.5 in. from the inner end of the tube, the pressure then again rising as the discharge end of the tube is approached.

THEORY OF THE STANDARD SHORT TUBE

Let Fig. 4 represent a standard short tube discharging into the air, with axis horizontal, attached to the vertical side of a large tank. Imagine three cross sections of the stream approaching and in the tube, the first a large one in the tank, and so chosen that the paths of all particles of water approaching the tube cut it normally; this section, No. 1, will be hemispherical and is assumed so far back that the velocity through it is negligible.

Let the second and third sections be planes normal to the axis of the tube, the second one being somewhere along the tube and the third one at the discharge end of it. These three sections are shown on the figure. The following notation will be used:

Q	= discharge in cubic feet per second
A_2	= area of section 2 in square feet
A_3	= area of the tube in square feet
h	= height of water, in feet, above the axis of the tube
v_3	= velocity of water at section 3, feet per second
c_d	= coefficient of discharge for the tube
c_v	= coefficient of velocity

- c_r = coefficient of resistance for the tube
 h_{r1-2} = loss of head in friction up to section 2, in feet
 h_{r1-3} = loss of head in the tube due to friction, in feet
 c_c = coefficient of contraction, or A_2/A_3
 h_p = pressure head in feet $= p/62.4$ at a given section
 h_v = velocity head in feet $= v^2/2g$ at a given section
 p = pressure at any point, in pounds per square foot.

Since the water leaves the tube in a parallel stream there is no contraction beyond the end, and the coefficient of velocity is equal to the coefficient of discharge.

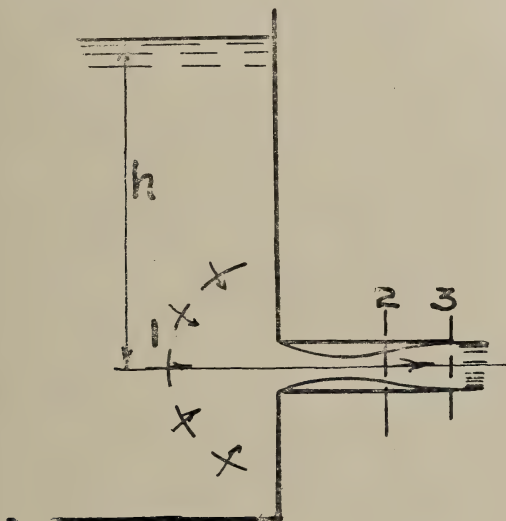


Fig. 4

From the energy equation

$$h = h_{p2} + h_{v2} + h_{r1-3} = h_{v3} + h_{r1-3}$$

$$\text{Also } v_3 = c_d \sqrt{2gh} = c_v \sqrt{2gh}$$

In this particular experiment h was kept constant throughout at 23 feet, and c_d was found to be .81

$$\therefore v_3 = .81 \sqrt{2g \cdot 23} \text{ or}$$

$$h_{v3} = \frac{v_3^2}{2g} = .656 \cdot 23 = 18.4 \text{ ft.}$$

Hence $h_{r1-3} = h - h_{v3} = 9.6$ ft. which was the loss of head in the entire tube.

The experiment was used to measure the values of h_{p2} along the tube, these values being plotted on the curve Fig. 3, and these outline a very regular and smooth curve. From these pressures the shape of the stream may be determined with fair accuracy in the following way.

The middle part of the energy equation contains three terms h_{p_2} , h_{v_2} and $h_{r_{1-2}}$ of which only h_{p_2} was determined by the experiment, leaving h_{v_2} and $h_{r_{1-2}}$ unknown, and as the shape of the stream is being sought, the value of h_{v_2} is to be determined. The limiting values of h_{v_2} in any case may be found by assigning limiting values to $h_{r_{1-2}}$, since this loss must lie between zero and $h_{r_{1-3}}$; if, therefore, these two limiting values are substituted the limiting values of h_{v_2} , and hence of the diameter of the stream, are known.

This may best be illustrated by an actual case. For example, consider the section $5\frac{1}{2}$ in. from the inner end of the tube at which point the measured pressure head was 21.3 ft. absolute, or there was a vacuum of 12.7 ft., since the entire plotting is referred to a 34 ft. barometer.

Case (1) $h_{r_{1-2}} = 0$

Then $h = h_{p_2} + h_{v_2}$

or $28 = -12.7 + h_{v_2}$ giving $h_{v_2} = 40.7$ ft.

But $h_{v_3} = 18.4$ ft.

Hence $\frac{h_{v_2}}{h_{v_3}} = \left(\frac{v_2}{v_3}\right)^2 = \left(\frac{A_3}{A_2}\right)^2 = \frac{40.7}{18.4}$ or $\frac{A_3}{A_2} = 1.487$

or $d_3 = \frac{d_2}{1.219} = \frac{2}{1.219} = 1.64$ in.

Case (2) $h_{r_{1-2}} = h_{r_{1-3}}$

Then $h_{p_2} + h_{v_2} = h_{v_3}$

or $-12.7 + h_{v_2} = 18.4$ or $h_{v_2} = 31.1$ ft.

Hence $\frac{h_{v_2}}{h_{v_3}} = \frac{31.1}{18.4} = 1.69$ or $\frac{A_3}{A_2} = 1.30$.

From which $d_3 = 1.75$ in.

Using the above method the two limiting diameters have been computed for each one-half inch along the axis of the tube and the results are given in the accompanying table and are plotted on Fig. 5. It will be noticed that the two diameters computed in this way do not differ very greatly, and by careful adjustment the true shape of the jet may be very closely determined, for it is evident that for points near the discharge end of the tube, case (2) most nearly represents the results.

Between the tank and the vena contracta the losses would approach those in a sharp edged orifice, and for such an orifice the coefficient of velocity at this total head of 48.5 ft. may be taken roughly as 0.98, so that for it the velocity head at the vena contracta will be

$$h_v = (.98)^2 h = .96 \times 48.5 = 46.6 \text{ ft.}$$

The loss of head between the tank and vena contracta would therefore be $48.5 - 46.6 = 1.9$ ft., where the orifice was discharging freely into the air.

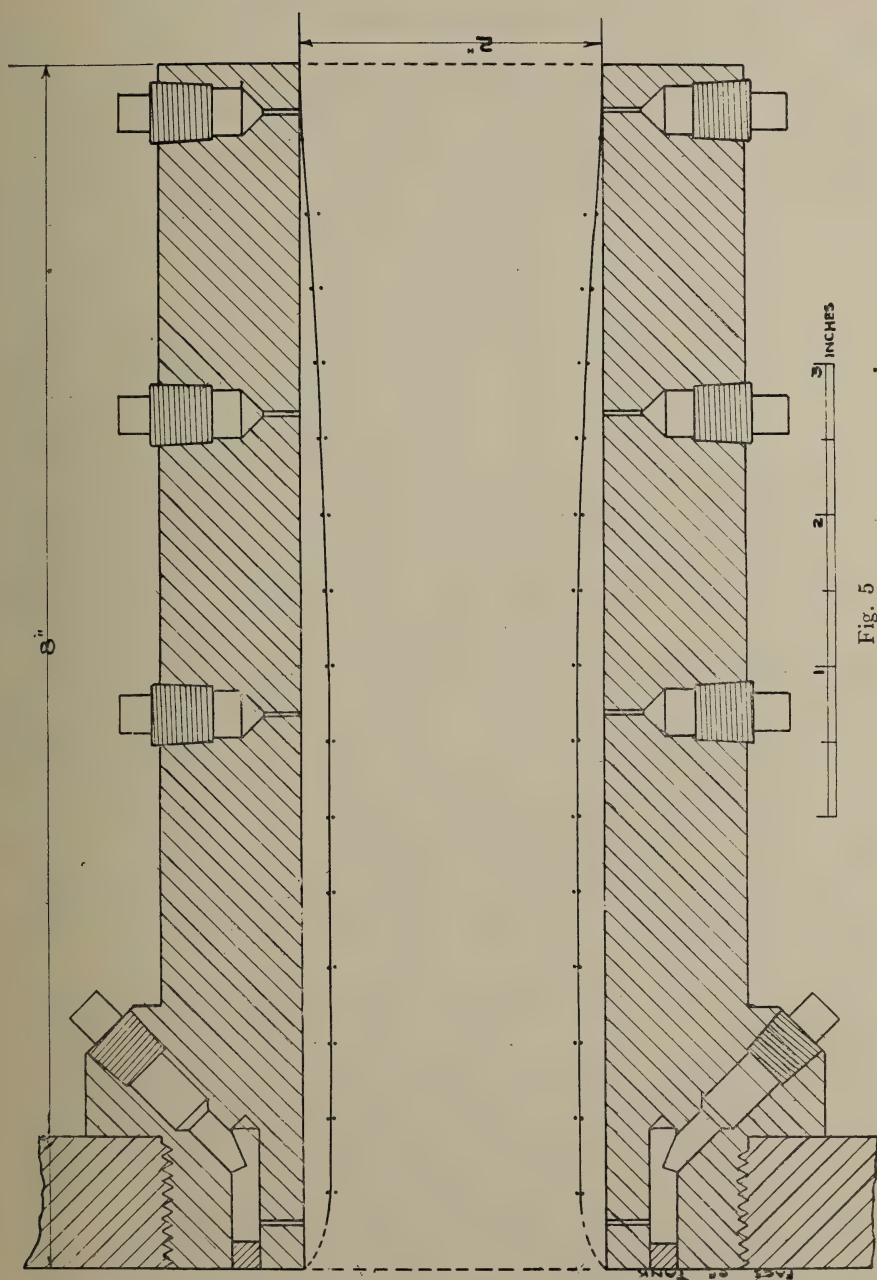


Fig. 5

CALCULATION OF THE SHAPE OF THE JET INSIDE THE TUBE

Head on Tube 28 ft.

Velocity Head at discharge end is $h_{v_3} = \frac{v_3^2}{2g} = 18.4$ ft.

Distance from inner end of tube in.	Absolute pressure h_{p_2} ft.	Assuming no loss from section 1 to 2 or $h_{r_{1-2}} = 0$				Assuming no loss from section 2 to 3 or $h_{r_{1-2}} = h_{r_{1-3}}$			
		h_{v_2} ft.	$\frac{A_3}{A_2}$	$\frac{d_3}{d_2}$	d_2 in.	h_{v_2} ft.	$\frac{A_3}{A_2}$	$\frac{d_3}{d_2}$	d_2 in.
.5	14.5	47.5	1.606	1.267	1.58	37.9	1.421	1.192	1.68
1.0	14.1	47.9	1.612	1.270	1.57	38.3	1.442	1.200	1.67
1.5	13.8	48.2	1.619	1.273	1.57	38.6	1.449	1.204	1.66
2.0	13.6	48.4	1.622	1.274	1.57	38.8	1.453	1.205	1.66
2.5	13.5	48.5	1.632	1.279	1.56	38.9	1.454	1.206	1.66
3.0	13.6	48.4	1.622	1.274	1.57	38.8	1.453	1.205	1.66
3.5	14.0	48.0	1.615	1.271	1.57	38.4	1.446	1.202	1.66
4.0	14.9	47.1	1.600	1.265	1.58	37.5	1.428	1.195	1.67
4.5	16.5	45.5	1.573	1.254	1.59	35.9	1.396	1.181	1.69
5.0	18.5	43.5	1.536	1.239	1.61	33.9	1.356	1.164	1.72
5.5	21.3	40.7	1.487	1.219	1.64	31.1	1.300	1.140	1.75
6.0	24.5	37.5	1.428	1.195	1.67	27.9	1.231	1.109	1.80
6.5	28.0	34.0	1.360	1.166	1.71	24.4	1.151	1.073	1.86
7.0	30.9	31.1	1.300	1.140	1.75	21.5	1.082	1.040	1.92
7.5	32.9	29.1	1.257	1.121	1.78	19.5	1.029	1.015	1.97

On the other hand the loss due to the enlargement from the vena contracta to the end may be taken at $0.08 h_{v_2}$ as given in King's *Handbook of Hydraulics*, p. 181 (first edition), which would indicate a loss between the vena contracta and the discharge end of the tube of

$$.08 \times 48.5 = 3.9 \text{ ft. for case (1)}$$

and

$$.08 \times 38.9 = 3.1 \text{ ft. for case (2).}$$

As the total loss in the tube is 9.6 ft. the result of these two calculations for $h_{r_{1-3}}$ is too low, the first giving $1.9 + 3.9 = 5.8$ ft., while the latter gives $1.9 + 3.1 = 5.0$ ft.

It would, however, appear that the greater part of the loss in the tube is between the vena contracta and the discharge end, and the diameter at the vena contracta has been taken as nearest that found by using the small value of $h_{r_{1-2}}$, and an effort has been made to adjust the diameter of the stream in accordance with the above discussion. In Fig. 5, however, all points shown in the table are plotted and these very clearly outline the curve. It is quite evident that the assumption $h_{r_{1-2}}$ equal zero is inaccurate for points near the discharge end.

The mean curve in Fig. 5 is quite definite in size and outline. The stream appears to be parallel for three inches of its length, which suggests that the tube might have been effective if it had been shorter, but a 2 in. by 6 in. tube did not work satisfactorily under the head of 28 ft.

HEAT TRANSFER IN HOT BLAST HEATERS

By ROBERT W. ANGUS, *Professor of Mechanical Engineering*

Heated air is used for many purposes, such as drying materials, heating buildings, etc., and many methods have been devised for producing the desired temperature of the air. One of the very common devices used is the hot blast heater, made up of cast iron sections, or of pipes, or of combinations of the two, so arranged that steam is supplied to the inside of the heater while the air to be warmed is blown past the steam heated sections and picks up heat in its travel.

The change in temperature of the air will depend on the size and design of the heater, the temperature of the steam, and the velocity of the air passing through the heater, and in any given design it is desirable to know the law connecting these quantities so that the performance of the heater under given conditions may be predicted. In addition to the above quantities the value of the heater is to some extent dependent upon the amount of power necessary to force the air through it, therefore the law connecting the friction loss with the volume of air flowing must also be known.

For each design and build of heater, therefore, an investigation must be made in such a way as to relate the above quantities. During 1922 and 1925 the writer investigated three different heaters, and some of the results are given herewith. In two makes cast iron was used exclusively, while in the third cast iron headers were connected by wrought iron or steel pipes. In all of the investigations the experimental work was done with heat supplied by steam at fixed pressure and temperature, and the heaters were made up in different sizes so as to vary their heating surface, etc. In each size a series of experiments was made by varying the volume of air passing through, and measurements were made of the heat supplied by the steam, the temperature of the air entering and leaving the heater, and the volume of air passing through, together with the friction loss in each case.

The apparatus employed in all the tests was similar but varied in details and is described fully below. The equipment used for the first series of tests in 1922 is shown in Figs. 1, 2, and consisted of a galvanized iron box, 3 ft. 7½ in. wide by 5 ft. 1 in. high inside, for a length of 9 ft. 3¼ in., this box being air tight along the sides so that air could only enter it through the front. The heaters to be tested were built into the box and five stacks are shown at *A, B, C, D, E*, in the figure, these stacks being placed at 10 in. centres. A tapering galvanized iron piece

6 ft. 6 $\frac{1}{4}$ in. long connected the box with a circular pipe 2 ft. diameter inside, and 12 ft. long, the other end of this pipe being joined to the suction side of a fan. The fan was thus capable of drawing air through the front of the box past the heater and out.

In order to prevent radiation of heat to the room in which the equipment was placed, the pipe, tapering piece, and that part of the box containing the heater were all lagged with hair felt 1 $\frac{1}{2}$ in. thick.

Steam was supplied from a 50 H.P. Babcock and Wilcox boiler, carrying a pressure of approximately 25 pds. per sq. in., and from a header supplied by this boiler a 2 in. pipe ran directly to the heater. The pressure carried in the heater was about 5 pds. per sq. in. throughout, which was obtained from a 2 in. reducing valve placed on the line close to the heater, a drip being placed on the boiler side of this valve so as to drain out any accumulated moisture. On the heater side of the reducing valve the pipes down to the individual heaters were all 2 in. and fittings were placed for a manometer connection and a thermometer well, so that the exact condition of the steam entering the heater was known. As the steam reaching the reducing valve was nearly dry, that leaving it was superheated in all the tests, this superheat, however, was controllable and was kept below 12 deg. F.; some superheat was considered preferable to an uncertain amount of moisture. All the steam supply lines were lagged and the reducing valve was close to the heater.

The steam condensed in each heater section was drawn off separately in a receiver, each receiver being made of standard steel pipe, 6 in. or larger, capped at both ends, and there was a water glass on each receiver. The detail of each drip connection is shown on Fig. 2; the condensate left the heater through a 2 in. pipe with a tee at the outer end, and fittings arranged so that the temperature of the condensate could be taken, and in addition a petcock was placed so that the air could be drawn off. The petcocks were kept slightly open during all the tests, and no tests were used where the thermometer in the outlet showed a temperature differing by more than one degree from that corresponding to the pressure. From the 2 in. tee a $\frac{1}{2}$ in. pipe led the condensate to the receiver and the former was blown out of the receiver through a $\frac{1}{2}$ in. pipe and globe valve. All the piping and receivers were lagged with hair felt 1 $\frac{1}{2}$ in. thick.

The friction loss in the heater was determined by inserting one tube *F* in Fig. 3 (*a*) in front of the heater and another just behind it and connecting these tubes to a sensitive differential gauge. The pressure tubes were located at the centre of the cross-section in each case and consisted of a brass tube, 0.389 in. outside diameter, with its axis parallel to the air stream and with its front end closed, tapered and rounded as

shown in Fig. 3 (a). At a distance of $2\frac{9}{16}$ in. to $3\frac{1}{8}$ in. from the end of the tube, twelve holes $\frac{1}{32}$ in. diameter were bored through the wall of the tube, exactly normal to its surface, and the air entered these holes, and this pressure was transmitted to a differential gauge. In both cases the pressure openings were about 6 in. from the heater.

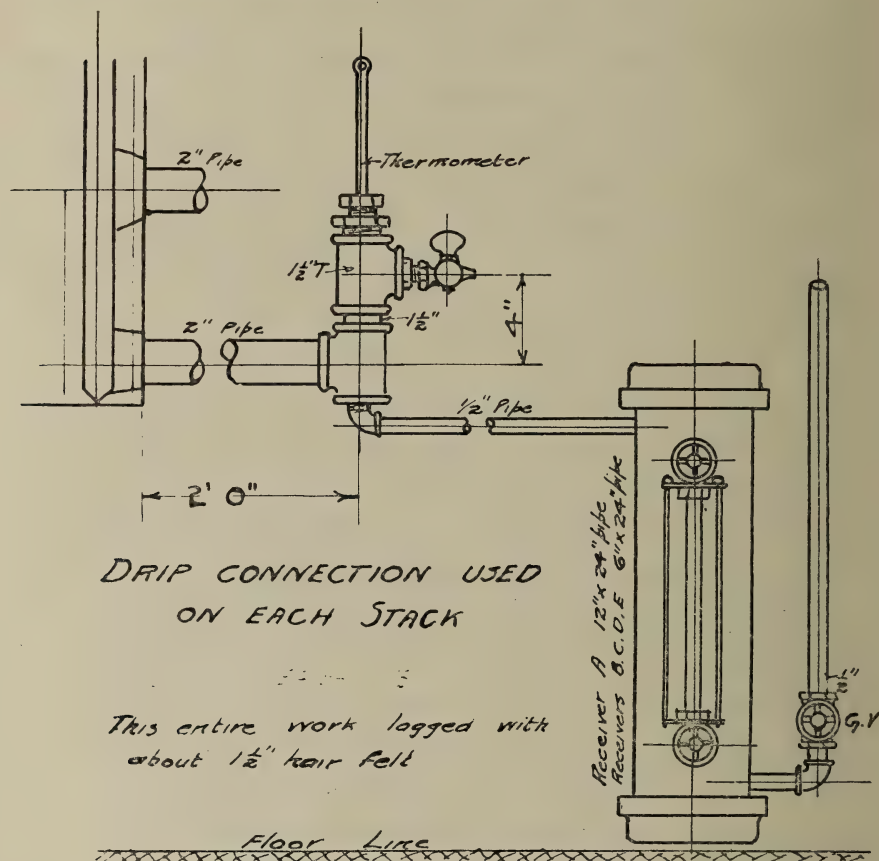


Fig. 2

The differential gauge used has been described in a former Engineering Research Bulletin, but a brief description is given here to make the reference convenient. It consists of a base *B*, Fig. 4, supported on adjustable screws *C* and carrying a frame *D* to which is fixed a steel scale *E* graduated to tenths of an inch. The frame also supports the screw *F* very accurately cut with ten threads per inch, and this screw passes through a tapped opening in the iron sliding block *G*, so that rotation of the screw causes *G* to rise or fall, and as the head *H* of the

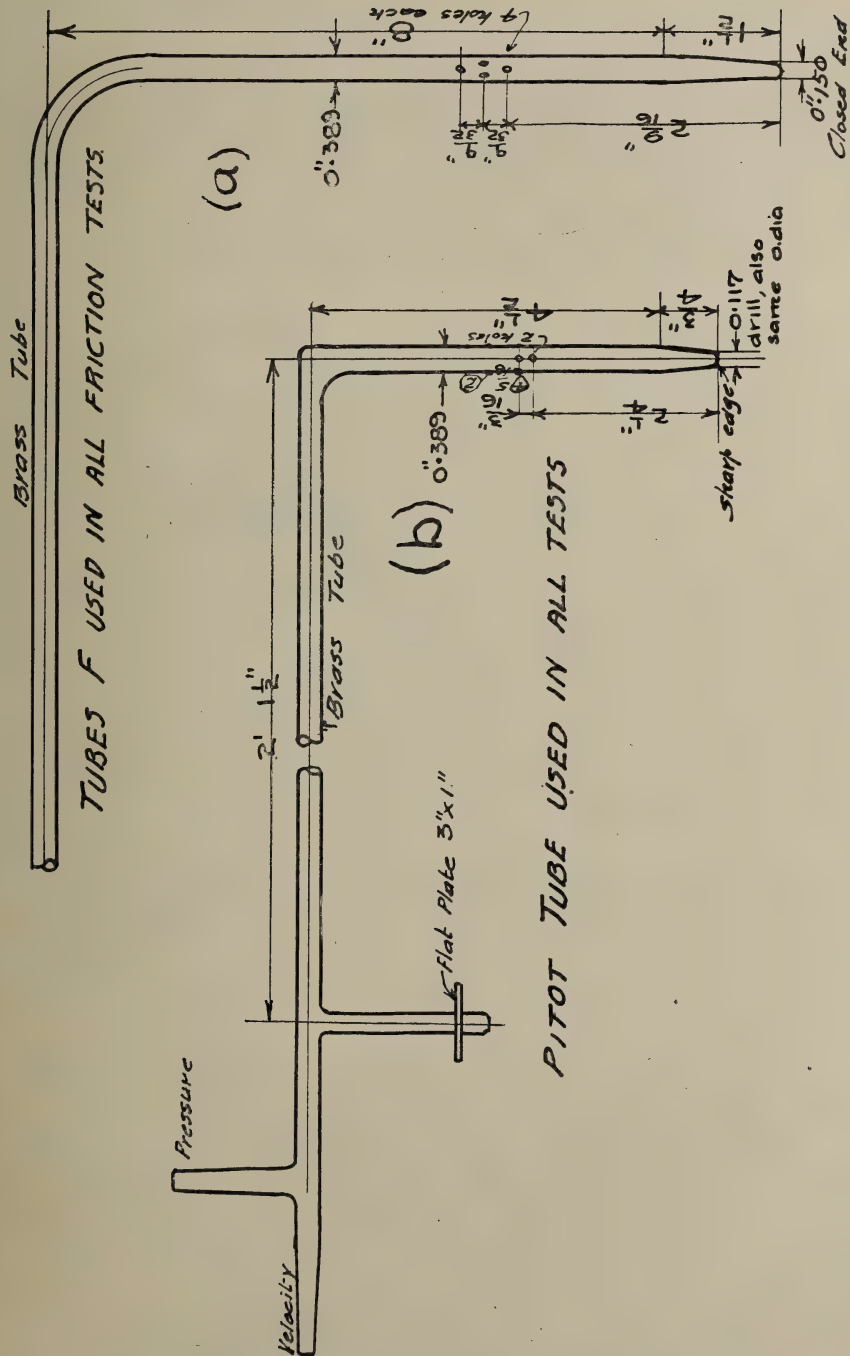


Fig-3

screw *F* is divided into 100 parts the change of level of *G* may easily be read to thousandths of an inch.

A glass tube *J* is attached to the block *G* by a swivel pin, not shown on the figure, so that the central straight part of the glass tube may be set at any desired angle to the horizontal, and the glass tube is connected by a rubber tube *K* to the large diameter reservoir *A*, carried on the adjusting screw *R* shown.

Coloured alcohol was used in the gauge and there was enough to

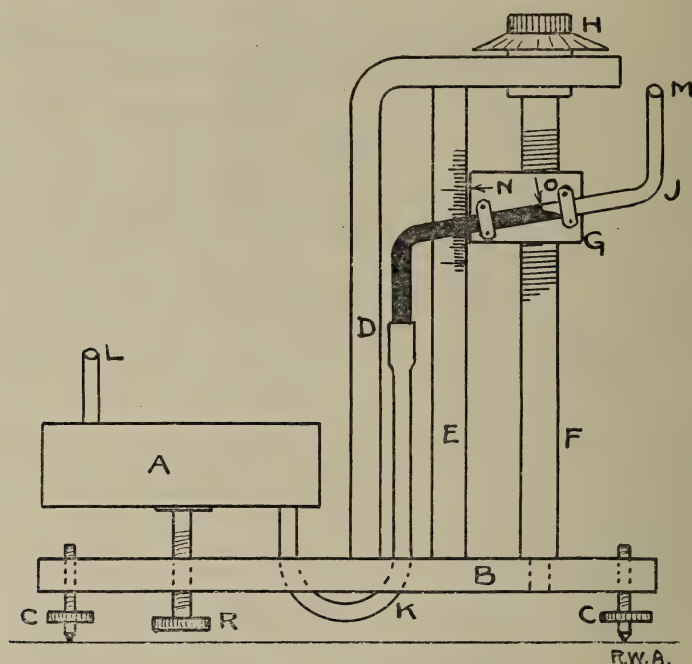


Fig. 4

keep a depth of an inch or so in *A*. In setting the gauge the block *G* was lowered till the line *N* on it coincided with the zero on the scale *E* at the same time as the head *H* read zero. The height of the reservoir *A* was then adjusted till the liquid in *J* came to an arbitrarily marked zero on the block as shown, the two connections *L* and *M* being open to the atmosphere. By setting the slope of the glass tube *J* to a very small angle it was quite easy to detect a movement of less than a thousandth of an inch in *G*.

When using the gauge the higher pressure connection was made at *L* and the lower one at *M*, and the height of the block *G* adjusted till the

liquid in J was again at zero, when the amount of the pressure was easily read off on the scales.

The pressure of the steam in the heater coils was measured by a mercury column attached close to the connection for the first heater, and a thermometer was also placed at this same point to determine the temperature of the steam, these two instruments being within 15 in. of the nearest stack. The thermometer at this point, as stated above always indicated a small amount of superheat. In addition to this thermometer, others were placed, one at the outlet from each of the stacks, in a way indicated on Fig. 2, and the readings of these latter thermometers were compared continually with the steam pressure corresponding to the mercury manometer reading, and in no case was there a difference exceeding a degree. At all times the petcock, close to this outlet thermometer, was kept very slightly open so as to keep the system properly relieved of air, and the result was that the thermometer read constant temperatures throughout each test.

The steam condensed in each stack was drained into the receiver already mentioned, the water glass indicating the depth of water, and at the beginning of each test condensation was allowed to collect to about the middle of the water glass, the exact reading being then taken. By hand adjustment the condensate was blown out of each receiver so as to keep the water level approximately constant, and at the end of each test great care was taken to see that the level was the same as at the beginning. The water blown from the receiver was passed through a cooling tank and delivered to a second tank on weigh scales where the weights were taken to half pounds, corresponding to less than one-half per cent. of the total weight.

The temperatures of the air entering and leaving the heater were taken with some care, that entering being taken at a point about five feet away from the front of the heater, so as not to be affected by radiation, and the same precaution was taken with regard to the outlet air, but it was found in the latter case that the thermometer had to be placed at greater distance away, and the temperature was, in fact, taken in the 24 in. pipe about 14 ft. 6 in. from the heater.

In the arrangement shown on the drawing, a number of thermometers were used to get the average temperature of the heated air, and as the air did not seem to mix very well, the temperatures in the different parts of the pipe varied, but in the two series of tests done subsequently in 1925, the fan was placed close to the heater, adjacent to the tapering box, and the air temperature was taken at the discharge side of the fan; this resulted in the complete mixing of the air in the pipe, and the air temperatures at all points in the cross section chosen were constant and easily read on a thermometer. Thermocouples were also used for

inlet and outlet temperature measurements, nine being used in front of the heater and nine behind it, and after the couples were properly set they agreed accurately with the mercury thermometers.

The volume of air flowing was determined by pitot tube, a sketch of which is shown on the accompanying drawing, Fig. 3 (b), this tube having an outside diameter of 0.389 in., and a part parallel to the stream $5\frac{1}{4}$ in. long, the pressure openings being between $2\frac{1}{4}$ in. and $2\frac{7}{16}$ in. from the point of the tube, and were $\frac{1}{32}$ in. diameter, while the velocity opening was 0.117 in. diameter. This tube was used in the 24 in. discharge pipe, the cross section of which was divided into five annular rings of equal area, these rings having outside diameters of 24 in., 21.47 in., 18.59 in., 15.17 in. and 10.73 in., and for each ring eight pitot tube readings were taken, these being along the horizontal and vertical diameters as well as along diagonals, the pitot tube measurements being made on circles having diameters of 22.73 in., 20.03 in., 16.88 in., 12.95 in. and 7.59 in. Each velocity measurement was thus the result of forty pitot tube observations, and in most of the experiments two complete sets of readings were taken for each particular test.

The velocity head in the pitot tube was measured on a sloping alcohol gauge of very accurate construction, and so arranged as to magnify the readings in the ratio of ten to one or twenty to one, depending on the slope of the gauge, this slope being varied with the magnitude of the velocity. The air temperature at the point where the pitot tube was used was measured, and also the air pressure inside the tube at the same point; hence, it was quite possible to calculate the weight of air passing the pitot tube, and this was taken for each of the tests from the following formulas. All of the following symbols and formulas refer to the section where the pitot tube is placed.

$$v = 60\sqrt{2gH} = 481.2\sqrt{H} = 481.2 \sqrt{\frac{62.3i}{12w}} = 1097 \sqrt{\frac{0.828a}{w}}$$

$$= 1000 \sqrt{\frac{a}{w}}$$

$$w = \frac{p}{RT} = \frac{p}{53.2(t+460)}$$

$$W = 3.142vw$$

where v = the mean air velocity in feet per minute

i = inches water column corresponding to the observed head on the sloping alcohol gauge

a = actual inches of alcohol read on gauge

w = weight of one cubic foot of air

p = absolute pressure, in pounds per square foot, of air in the pipe.

t = temperature of the moving air, degrees Fahrenheit.

3.142 = area, in square feet, of the pipe, its diameter being 2 ft.

W = weight of air discharged per minute in pounds.

In order to change the velocity of the air through the heater, a variable speed motor was employed and this gave a fairly great range of operation; but in addition to this, different sized orifices were placed in the 24 in. discharge pipe, and with the lower velocities these orifices were not very large, running down in size to 13-1/8 in. Unfortunately, the orifices had to be placed fairly close to the pitot tube, and for the smaller orifices they caused sufficient disturbance to affect the accuracy of the readings; but on all velocities of around 700 ft. per min., or more, the readings were proven to be accurate.

In order that the method of testing may be made clear, a sample observation sheet is shown at Fig. 5, and Table I herewith gives some of the results of the series of tests on a well known hot blast heater. This table contains the number of the test in the first column; $\sqrt{a}$, the averages of the square roots of the inches of alcohol obtained for each test in Column 2; Column 3 gives the barometer reading in pounds per square foot; Column 4 the absolute pressure p at the pitot tube in pounds per square foot; Column 5 the temperature t inside the pipe at the tube, degrees Fahrenheit; Column 6 the computed weight of air per cubic foot w , from the pressure and temperature; Column 7 the mean velocity r of the air passing the pitot tube section; Column 8 the volume of air passing the pitot tube per minute, this being calculated from the figures in Column 7 by multiplying by the area of the pipe; Column 9 gives W the weight of air per minute passing through the heater and is obtained by multiplying the figures in Column 8 by those in Column 6; Column 10 gives the velocity V through the heater reduced to standard conditions of air at 70° F. and barometer 29.92 inches, under which condition the air weighs 0.075 pounds per cubic foot. Dividing the results in Column 9 by 0.075 and by the clear area through the heater, the results in Column 10 are obtained, and are stated in feet per minute.

Columns 11 to 15 contain information about the steam; Column 11 gives the weight of steam condensed per hour, as measured on the test; Column 12 gives the entering steam temperature and shows superheat in most cases, as already stated; Column 13 is the temperature T of the drips leaving the heater, which was always the same as that of saturated steam corresponding to heater pressure; Column 14 gives the heat contained in each pound of steam between the entering temperature and water at the leaving temperature; while Column 15 gives the heat delivered by the steam to the air heated per hour.

A comparison of Column 9 and 15 enables the air temperature rise in the heater to be computed, and this has been done in Column 16,

TEST 50

DATE Sat. Sept. 12th, 1925

STACKS 4

TIME 10.54-11.34

BAROMETER = 29.57 in.

BOILER PRESSURE 25 pds.

HEATER PRESSURE, in. mercury, 11.40, 11.65, 11.15. Av. 11.40

ENTERING STEAM TEMPERATURE, deg. F., 231, 231.5, 235. Av. 232.5

CONDENSATE TEMPERATURE EACH STACK, deg. F., 227, 227, 227, 227

WET BULB TEMPERATURE, deg. F., 70.5

DRY BULB TEMPERATURE, deg. F., 74.2, 74.3, 74.0. Av. 74.2

AIR TEMPERATURE LEAVING HEATER, deg. F., 155.5, 155, 155, 154.5
Av. 155

FRICTION LOSS, inches alcohol, .751, .751, .736. Av. .746

PRESSURE OF AIR PASSING PITOT TUBE, 0.15 in. water

PITOT TUBE READINGS FOR DISCHARGE, INCHES ALCOHOL

MULTIPLICATION 10/1

5.50	4.25	5.00	5.00
7.10	4.95	6.65	5.50
7.70	5.05	7.10	5.30
7.65	5.05	7.00	5.15
6.95	5.35	6.15	5.40
6.00	5.70	6.15	6.10
6.15	5.20	6.80	5.90
6.20	4.80	7.10	5.90
6.45	4.15	6.25	5.55
4.70	2.35	3.15	3.15

Average of square roots of one-tenth of these readings = 0.746

CONDENSATION

		Pounds per hour
Stack No. 1 (entry side	= 154.0 lbs.	231.0
" No. 2	= 128.5 lbs.	192.7
" No. 3	= 105.5 lbs.	158.3
" No. 4 (leaving side)	= 84.0 lbs.	126.0
		708.0

Fig. 5

TABLE I

TESTS ON HOT BLAST HEATER—SEPTEMBER 1925

(1) Test No.	Pitot Tube						Steam						(16) Computed air temp. rise, F.	(17) Temp. supply air, F.	(18) Temp. leaving air, F.	(19) Actual temp. rise, F. $t_2 - t_1$
	(2) Av. sq. ft. of reading, V_a	(3) Barometer pds. sq. ft.	(4) Absolute pressure in pipe, pds. sq. ft.	(5) Temp. in pipe, deg. F.	(6) Weight of air per cu. ft., lbs.	(7) Mean vel. of air, ft. min.	(8) Cu. ft. air per min. discharged	(9) Weight of air dis- charged, lbs. per min. W	(10) Velocity thro. heater, ft. per min. V	(11) Steam cond. per hr., lbs.	(12) Temp. of entering steam, F.	(13) Temp. condensate, F. T	(14) Heat Contents per lb. B.T.U.	(15) Heat Supplied, B.T.U. per hr.		
48	.955	2101	2102	154.2	.06436	3764	11824	761.1	1408	912	227	226	961.4	87800	154.2	80.9
49	.878	2101	2102	153.5	.06436	3461	10871	699.7	1294	820	226	226	961.4	78800	153.5	79.3
50	.746	2101	2102	155.0	.06425	2943	9246	594.0	1098	708	232	227	964.4	682800	155.0	80.8
51	.648	2109	2110	162.3	.06377	2566	8062	514.0	951	729	230	227.5	961.7	701000	162.3	93.1
41	.975	2096	2097	137.3	.06603	3794	11519	786.8	1455	802	226	225	962.5	771900	137.3	68.5
42	.884	2096	2097	135.5	.06627	3434	10788	714.8	1322	705	230	226.5	962.9	678800	135.5	66.0
43	.743	2096	2097	137.5	.06603	2891	9082	590.6	1109	606	234	226	964.4	584400	137.5	68.0
44	.638	2096	2097	141.3	.06591	2485	7807	514.6	952	538	233	225.5	965.7	519500	141.3	70.3
45	.536	2096	2099	148.0	.06490	2104	6610	429.0	793	469	235	226	965.9	453000	148.0	71.8
55	1.028	2105	2106	96.6	.07108	3856	12110	860.8	1592	326	235	227	964.7	314500	96.6	26.6
56	.916	2105	2106	97.4	.07108	3436	10793	767.3	1419	287	235	227	964.7	276850	97.4	26.3
57	.753	2105	2106	97.1	.07108	2824	8872	630.6	1166	244	235	227	964.7	235400	97.1	26.3
58	.677	2105	2106	97.3	.07108	2539	7977	567.0	1049	218	234	226	965.9	210550	97.3	26.8
59	.544	2100	2104	98.8	.07075	2045	6425	454.6	841	192	235	226.5	965.5	185350	98.8	28.3
60	.430	2100	2105	103.6	.07017	1623	5100	357.9	605	163	236	226.5	965.9	157420	103.6	33.5

4 stacks

3 stacks

1 stack

whereas Columns 17 and 18 give, respectively, the observed temperatures t_1 and t_2 of the air entering and leaving the heater, and Column 19 gives the observed temperature rise $t_2 - t_1$, all in degrees Fahrenheit. A comparison of Columns 16 and 19 indicates that the computed and actual temperature rises agreed very accurately, and this was always the case in the experiments, except at the low air velocities, in which part of the error was undoubtedly due to the disturbance caused by the pipe orifice. In case of some lower velocities, not shown in the table, the calculation from the steam was taken in preference to the observed conditions of the air.

There has always been considerable difference of opinion as to the proper method of measuring the free areas of heaters, and some care has been taken to get the results as nearly accurate as possible in this particular series of tests. The heating surface of each section was measured very accurately and allowance was made for pipes and all other sources of radiation, and the heating surface for one of the heaters referred to in this paper was measured and computed as 13.41 sq. ft. per section, which corresponds quite closely with the catalogue figure of 13.5 sq. ft. The clear area is not quite so definite, but the method employed was to take a vertical section through the centre of the supply pipe on a given section and calculate the plane area of the outside of this section. Multiplying this area by the number of sections in the front stack, and subtracting the product from the area of the box in which the heater was placed, gave the clear area used in the calculations; in the case of the heater referred to in this paragraph this amounted to 7.87 sq. ft., which differs from the 6.14 sq. ft. given in the heater catalogue.

The foregoing describes in detail the set up of the equipment and the methods of making the measurements of the various quantities taken during the tests, and the calculations resulting therefrom follow.

The general view of the plant used in the second season's work is shown on Fig. 6, where the discharge from the fan passes through 12 ft. of 24 in. pipe, not shown on the accompanying photographs. The apparatus was carefully lagged with magnesia, and as the installation was intended to be permanent, the joints throughout were tight; some refinements were also introduced, and the tables and data given from tests are all the results of the second season's work, on the equipment shown in Fig. 6.

The method of carrying out the work was the same on all the tests.

Two objects were kept in mind in making the tests, one being the construction of the tables used for commercial purposes, the other a study of the laws of heat flow in such cases, and the effect of the design

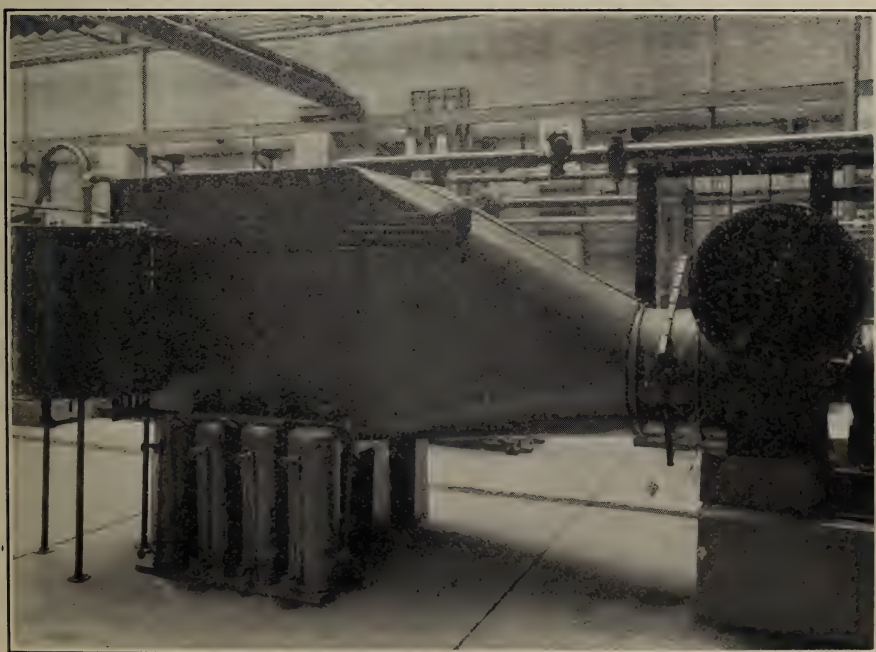
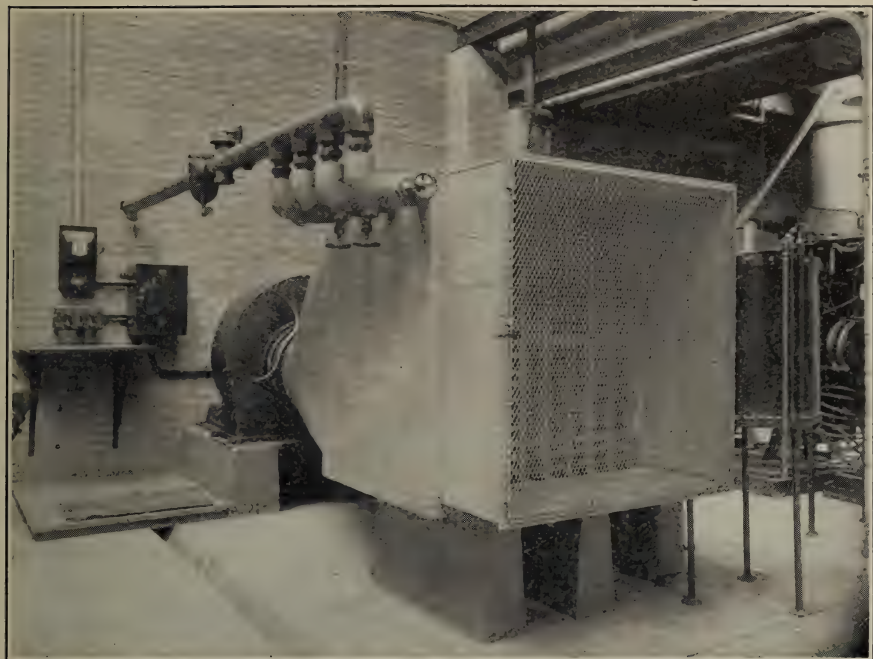


Fig. 6

on this rate of flow. The theory on which the investigation was based is given herewith.

LAW OF HEAT TRANSFER IN A HOT BLAST HEATER

In a hot blast heater the steam pressure remains constant throughout and the supply is saturated steam as nearly dry as possible. The heater is so trapped that the water of condensation is drawn off, and hence the steam in the heater is at constant temperature everywhere, heat transmitted coming from the latent heat of the steam. Thus, the air passing through the heater is raised in temperature, while the steam inside the heater is constant in temperature at that corresponding to the steam supply pressure.

Let p = the absolute pressure of the supply steam in pounds per square inch.

T = the temperature Fahrenheit of dry saturated steam at pressure p .

t_1 and t_2 be, respectively, the temperatures of the air entering and leaving the heater, in degrees Fahrenheit.

W = the weight of air in pounds passing through the heater per minute.

c = the specific heat of the air at constant pressure, taken as 0.2375.

K = the rate of heat transfer in B.T.U. per hour, per square foot of heating surface, per degree Fahrenheit temperature difference, between the steam and air. K is the coefficient of heat transfer.

t_m = mean temperature difference, in degrees Fahrenheit, between the steam and air.

Consider the flow of W pounds of air per minute over a heating surface of δS square feet area, which causes a rise in temperature δt degrees Fahrenheit in the air, the mean temperature of the air passing this surface being t , and the constant temperature of the steam being T . Then equating the heat passing through this surface to that received by the air gives

$$60cW\delta t = K(T-t)\delta S$$

$$\text{or } \frac{\delta t}{T-t} = \frac{-\delta(T-t)}{T-t} = \frac{K\delta S}{60cW}$$

and integrating this between t_1 and t_2 gives

$$\log_e \left(\frac{T-t_1}{T-t_2} \right) = \frac{KS}{60cW}.$$

It is usual to arrange this formula so as to give what might be called the mean temperature difference t_m between the steam and air. This may be found as follows:

The total heat per hour, received by the air, and hence that passing through the heating surface, is:

$$60cW(t_2 - t_1) = KS t_m$$

$$\text{or } t_m = \frac{60cW}{KS} (t_2 - t_1) = \frac{t_2 - t_1}{\log_e \left(\frac{T - t_1}{T - t_2} \right)} \text{ from above.}$$

This formula may be compared with the one used for rough approximations

$$t_m = T - \frac{t_1 + t_2}{2}.$$

If $T = 227^\circ \text{ F.}$, $t_1 = 57^\circ \text{ F.}$, $t_2 = 137^\circ \text{ F.}$, very common temperatures for hot blast heaters, the first formula gives $t_m = 125.6^\circ \text{ F.}$, while the second gives $t_m = 130^\circ \text{ F.}$, the two thus differing materially.

In order to adapt the formula for t_m to common logarithms to base 10, it may be written

$$t_m = \frac{60cW}{KS} (t_2 - t_1) = \frac{t_2 - t_1}{2.3026 \log_{10} \left(\frac{T - t_1}{T - t_2} \right)} \quad (1)$$

Referring now to the above formulas in the light of the experiments already referred to, it will be seen that the experiments give directly the values of W , K , T , t_1 , t_2 for each air velocity through the heater, and for each heating surface S used in the tests. The results of the tests have been plotted in the form of curves, shown on Figs. 7 to 12. Fig. 7 gives the B.T.U.'s transferred per hour, calculated from the steam condensed, for different velocities through heater A , the velocities being reduced to that of air at standard conditions of 70° F. and 29.92 in. These results plotted locate three separate straight lines, one for each heating surface, as controlled by the number of stacks used.

The next set of curves, Fig. 8, give the number of B.T.U.'s per hour, per degree Fahrenheit mean temperature difference, per square foot, for the different velocities through the same heater, and these locate two curved lines, the lower one corresponding to a single stack and the upper one to three and four stacks, the curves indicating that the single stack is relatively less effective than a larger number. The experiments showed this to be the case in all of the tests, because the condensation in the first stack was always close to or less than that in the second one.

The corresponding curves with heater B also tested in 1925, are shown in Figs. 10 and 11, and Fig. 10 shows that the B.T.U.'s per hour radiated, when plotted on a velocity base, gave straight lines, as in the

former tests, these lines all intersecting the axis of velocities close to the same point. The values of the coefficients of heat transfer, in B.T.U.'s per square foot, per hour, per degree Fahrenheit mean temperature difference, as plotted in Fig. 11, are somewhat different to those plotted in Fig. 7, and in this case the curves for the three stacks and four stacks do not quite coincide. And, further, the curves are concave towards the axis of velocities, whereas in the tests represented in Fig. 7 the curves are convex to that axis.

A few points were also obtained for a two stack heater, and are plotted on the same sheet. It would appear that the curve drawn dotted between the two outside curves, represents approximately the results

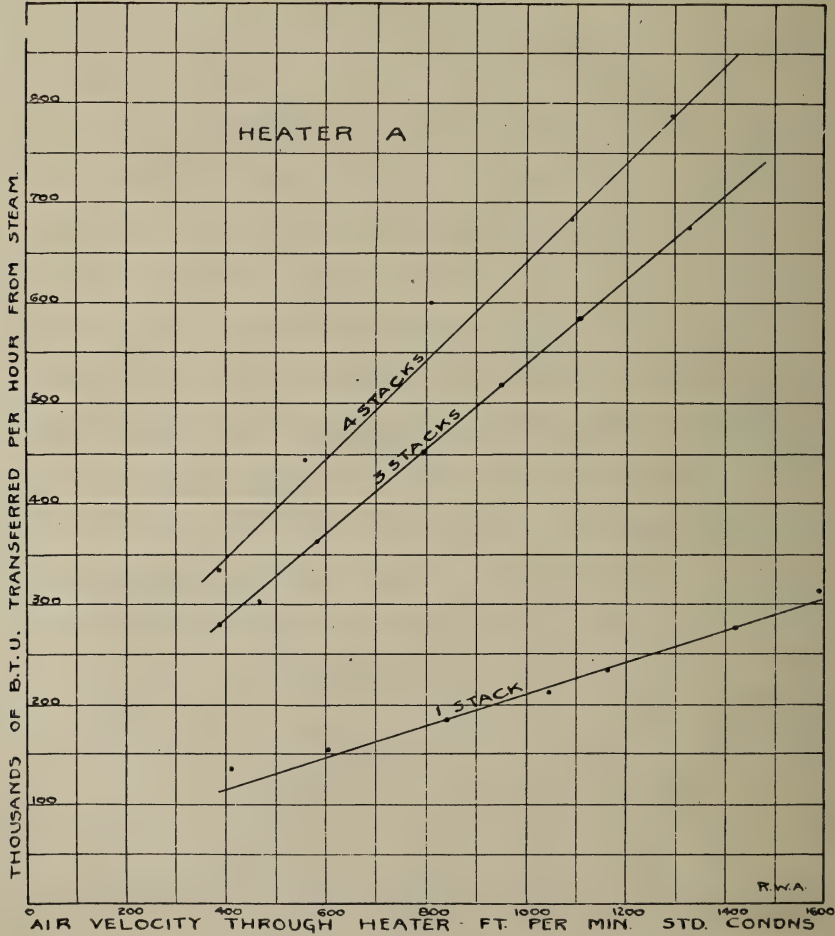


Fig. 7

for three and four stacks, and the values of K for the dotted curve do not differ, at the worst, more than two per cent. from the outside curves, and at the higher velocities are much closer.

In curves shown on Figs. 9 and 12 the friction loss for the two heaters has been plotted on a velocity base, the curves on the left hand side in each case being on logarithmic axes, those on the right hand side being

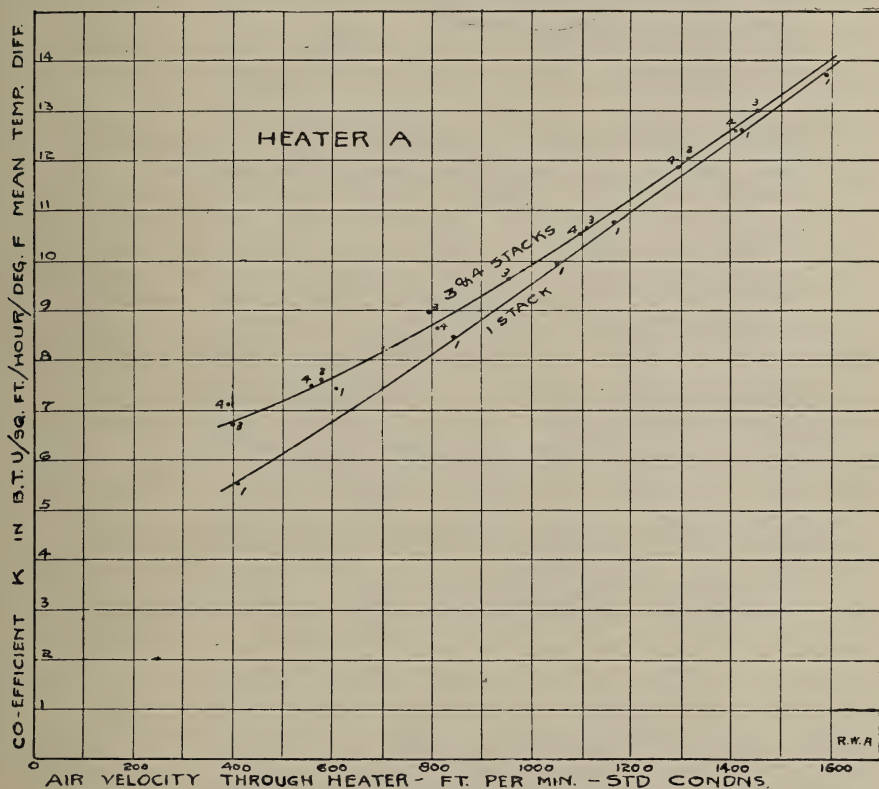


Fig. 8

plotted on the ordinary axes, and it is evident that the curves are of exponential form, since the plotting on the logarithmic axes gives straight lines. The lines are parallel in the case of the three and four stacks, but for the one stack the line is a little steeper.

CONSTRUCTION OF COMMERCIAL TABLES, HEATER A

Using the curves just described, and also the figures given in the experiments, a complete table for each heater may be readily worked out. Taking the formula (1) and re-arranging it, the following formula

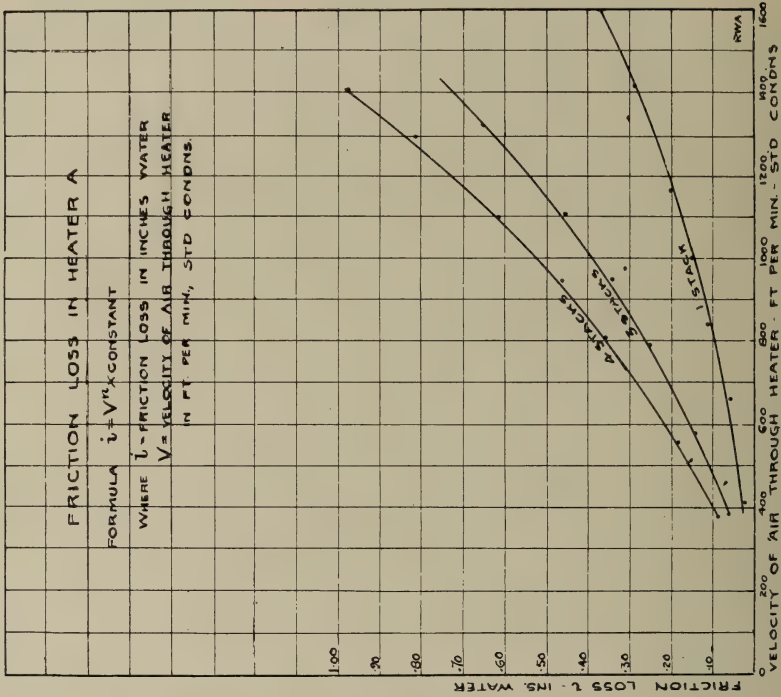
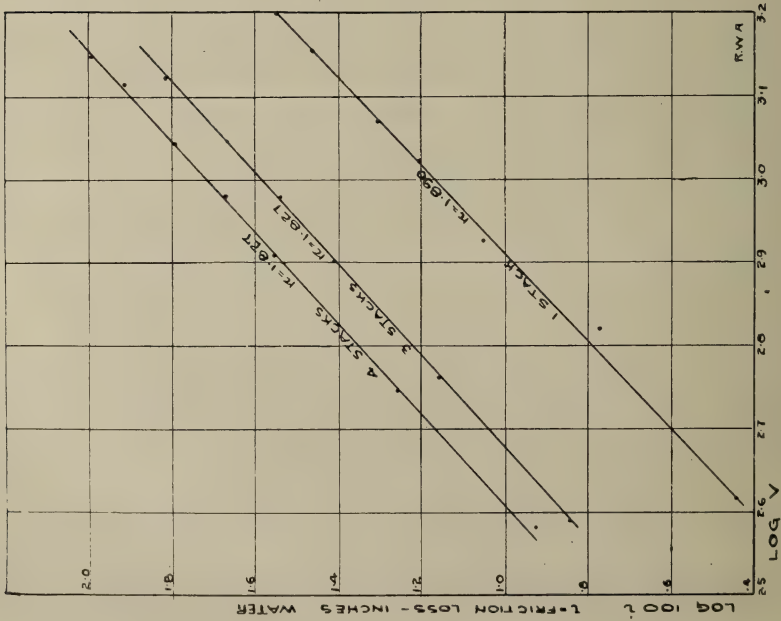
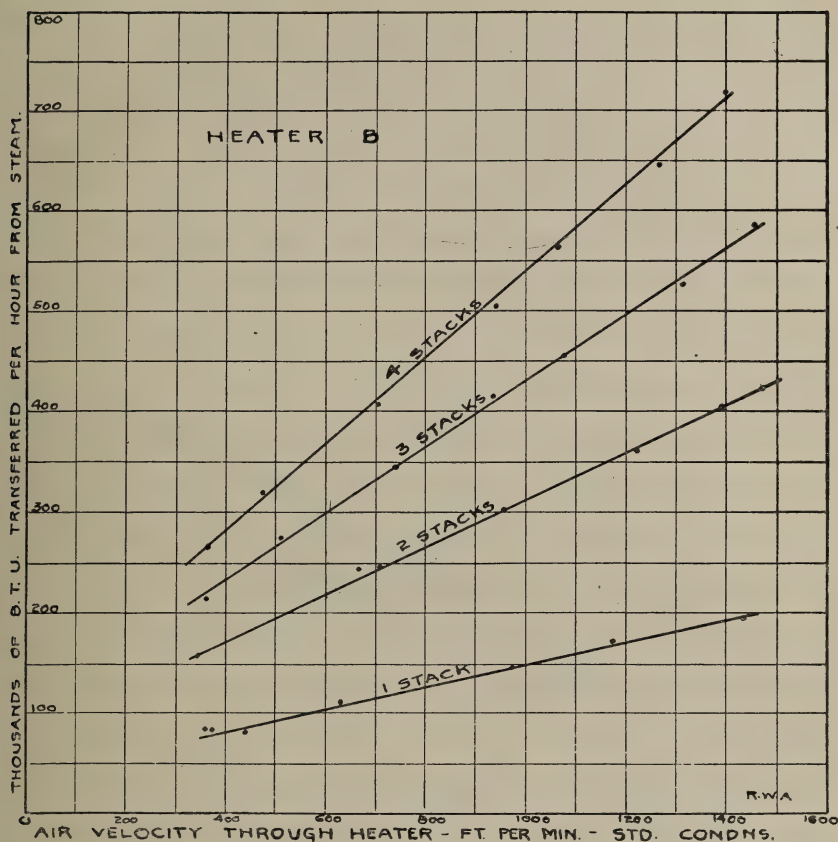


Fig. 9





Fig; 10

(2) is obtained, which connects up directly the temperatures of the steam and of the air, and the rate of heat transfer at given velocities for corresponding areas of heating surface and free areas.

$$\begin{aligned}
 \log_{10} \left(\frac{T - t_1}{T - t_2} \right) &= \frac{KS}{2.3026 \times 60cW} \\
 &= \frac{KS}{2.3026 \times 0.2375 \times 60 \times 0.075A V} \\
 &= \frac{KS}{2.461A V} \quad (2)
 \end{aligned}$$

where A is the free area in square feet, and V is the velocity through the free area in feet per minute.

In calculating results from this formula it is found easiest to construct a table for different velocities through the heater, and such a table (Table II) is given herewith for velocities of 400, 600, 800, 1000, 1200, 1400, 1600 and 1800 ft. per minute, the coefficient of heat transfer at each of these speeds being read directly from the curves already plotted. Two sets of coefficients must be used, the one for one stack only and the other set for more than one stack, see Fig. 8.

For any desired number of stacks, and for any given size stack, that is, for a given number of sections of selected height in a stack, the heating

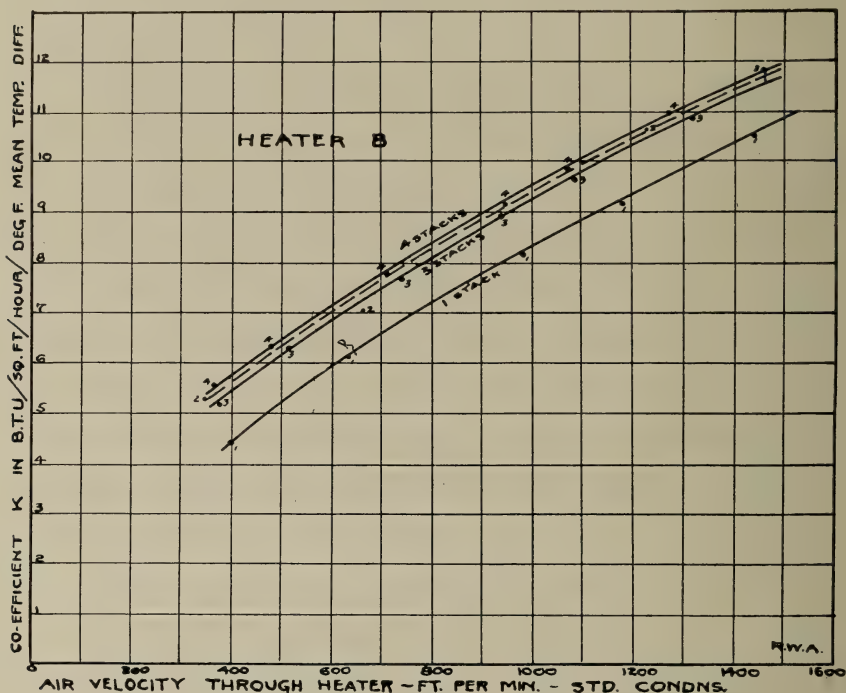


Fig. 11

surface and free area are known, and hence the formula (2) gives at once the relation between the temperatures T , t_1 and t_2 of the steam and air for each velocity. To assist in the calculations it is well to continue Table II, already mentioned, and this has been worked up so as to show the numerical value of the quantity on the left hand side of formula (2) for each velocity. If, now, the steam temperature is assumed—and this must always be done in practice, because one must know the pressure at which the heater is to be operated—then for any entering air temperature t_1 , the final air temperature t_2 may be computed. A sample

FRICTION LOSS IN HEATER B

FORMULA $f = V^{1.75} \times \text{CONSTANT}$

WHERE f = FRICTION LOSS IN INCHES WATER

V = VELOCITY OF AIR THROUGH HEATER
IN FEET PER MINUTE, STD CONDNS.

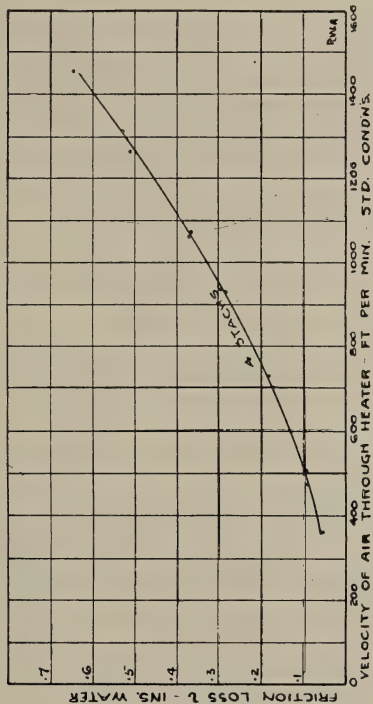
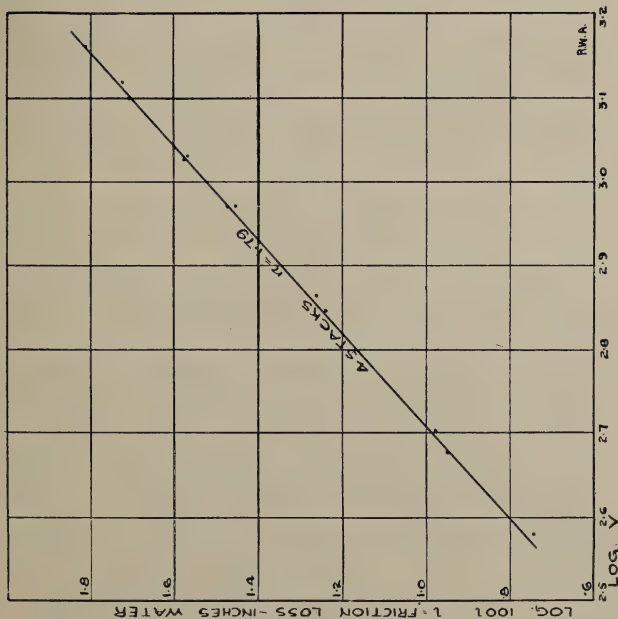


Fig. 12

TABLE II
COEFFICIENTS OF HEAT TRANSFER AT VARIOUS AIR VELOCITIES THROUGH THE
HEATER A

Velocity ft. per min.	400	600	800	1000	1200	1400	1600	1800
Values of K								
(a) One Stack	5.471	6.784	8.204	9.594	11.007	12.416	13.810	15.232
(b) Over one Stack	6.738	7.701	8.766	9.910	11.207	12.596	14.018	15.426
$\log_{10} \left[\frac{T-t_1}{T-t_2} \right]$								
(a) One Stack	.1195	.0987	.0893	.0839	.0800	.0774	.0753	.0738
(b) Over one Stack, multiply number of Stacks by	.1470	.1120	.0948	.0685	.0815	.0786	.0765	.0748

The values of K given in this table are higher than those usually accepted, but this is due to the fact that part of the radiating surface was in the form of fins, only the edges of which came in contact with the steam. In figuring the heating surface only part of this heating surface was taken into account, and any variations in the assumptions on this score will naturally affect the value of K although the final working tables are not affected.

of the table used in making such a calculation is shown on Table III, in which case steam has been taken at 227° F., corresponding to 5 pds. gauge approximately, and calculations have been made for air entering at 40° F. below zero; and for air entering at 70° F. above zero, for each of which the final temperature t_2 has been computed by the method already described.

TABLE III
FINAL TEMPERATURES—STEAM AT 227° F.
Two Stacks

Velocity V ft. per min.	$\log_{10} \left[\frac{T-t_1}{T-t_3} \right]$	$\frac{T-t_1}{T-t_2}$	$t_1 = -40^\circ \text{ F.}$		$t_1 = +70^\circ \text{ F.}$	
			$T-t_2$	t_2	$T-t_2$	t_2
400	.294	1.968	135.7	91.3	79.8	147.2
600	.224	1.675	159.4	67.6	93.7	133.3
800	.1895	1.547	172.6	54.4	101.5	125.5
1000	.137	1.489	179.3	47.7	105.4	121.6
1200	.163	1.455	183.5	43.5	107.9	119.1
1400	.1572	1.436	185.9	41.1	109.3	117.7
1600	.1530	1.422	187.5	39.5	110.4	116.6
1800	.1496	1.411	189.2	37.8	111.3	115.7

Much time can be saved in making out the table for practical use by plotting curves with constant air velocity V , showing, for the given steam temperature, the final air temperatures corresponding to different assumed initial air temperatures. Fig. 13 shows a sample of such curves, the vertical axis indicating the entering air temperatures, while the horizontal axis gives the final air temperatures. Consideration of the formula will show that all of these curves are straight lines, as shown in the figure.

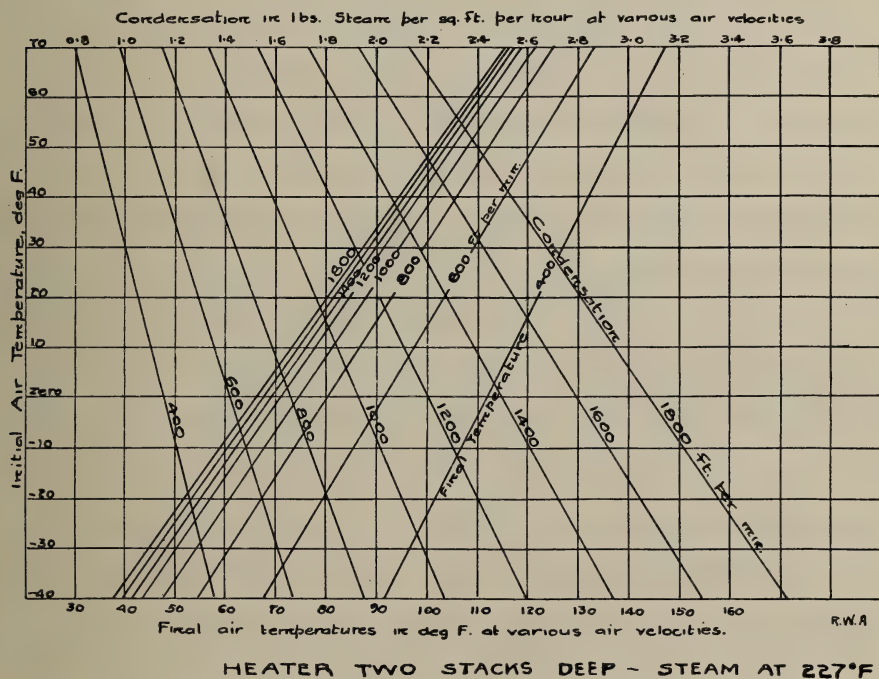


Fig. 13

In order to complete the table the amount of steam condensed per hour, corresponding to each size heater, and each entering air temperature is required. This condensation may be readily calculated from the initial and final temperatures of the air and of the steam, and the velocity through the heater. Thus, if C represents the pounds of condensation per square foot of heating surface per hour, then C may be calculated from the formula below, where A is the free area in square feet, S the heating surface in square feet, and 960.7 is the latent heat of steam at the temperature 227° F., which has been assumed as the temperature in the heater in this particular case.

$$C = \frac{60A V \times 0.075 \times 0.2375 (t_2 - t_1)}{960.7 S} \quad (3)$$

For each stack there is a known and fixed relation between A and S , thus, in one stack of eight sections of the heater A , referred to above, the free area A was 7.21 sq. ft. and the heating surface S was 154.8 sq. ft., and for convenience in making up practical tables it is usually assumed that for a particular make of heater the ratio S/A is constant per stack, independent of the number of sections or their heights. This statement is approximately correct if the number of sections is not less than six or seven.

The above assumption enables formula (3) to be further simplified, and in the particular case illustrated above the formula reduces to

$$C = \frac{.0000518 V(t_2 - t_1)}{\text{number of stacks deep in the heater}} = M(t_2 - t_1) \quad (4)$$

TABLE IV

VALUES OF THE CONSTANT M IN THE FORMULA $C = M(t_2 - t_1)$ FOR COMPUTING THE CONDENSATION

	Air Velocity through the heater, ft. per min.							
	400	600	800	1000	1200	1400	1600	1800
1 Stack	.0207	.0311	.0414	.0518	.0622	.0725	.0829	.0932
2 Stacks	.0104	.0155	.0207	.0259	.0311	.0362	.0414	.0466
3 Stacks	.0069	.0104	.0138	.0173	.0207	.0242	.0276	.0311

and so on for any number of stacks deep

Using this formula Table IV is readily made, giving the multipliers for $t_2 - t_1$ for each velocity and number of stacks deep, and by combining Tables III and IV, the results in Table V are readily found. The calculations may be very much shortened by the use of curves, and it has already been stated in connection with Fig. 13 that for a given steam temperature and velocity of air V , the curves connecting t_1 and t_2 are straight lines. On the same figure it will, therefore, be convenient to plot a series of curves giving the condensation C , and it will also be evident that for any given number of stacks, such curves will be straight lines, if the vertical axis gives values of t_1 and the horizontal axis the values of C . These condensation curves are also plotted on Fig. 13.

For the complete construction of commercial tables a diagram like Fig. 13 would be made for each number of stacks likely to be used, and then, by reference to the proper curve sheet, one could readily see the final temperature and the condensation when air at, say, zero entered a heater having any given number of stacks, and passed through the heater at any known velocity. Referring to the illustration of Fig. 13, which is for a two stack heater, it is evident that if air of initial temperature zero passed through the heater at 1000 ft. per min. it would acquire

TABLE V

CONDENSATION IN POUNDS PER HOUR PER SQUARE FOOT, STEAM AT 227° F.

TWO STACKS

Velocity through heater ft. per min.	Constant <i>M</i>	$t_1 = -40^\circ \text{ F.}$		$t_1 = +70^\circ \text{ F.}$	
		$t_2 - t_1$	<i>C</i>	$t_2 - t_1$	<i>C</i>
400	.0104	131.3	1.360	77.2	.800
600	.0155	107.6	1.672	63.3	.984
800	.0207	94.4	1.956	55.5	1.150
1000	.0259	87.7	2.271	51.6	1.336
1200	.0311	83.5	2.595	49.1	1.526
1400	.0362	81.1	2.940	47.7	1.729
1600	.0414	79.5	3.294	46.6	1.931
1800	.0466	77.8	3.625	45.7	2.130

a final temperature of 74.5° F., and the condensation per square foot of heater would be 1.93 lbs. If an engineer wishes to deliver a given quantity of heat he will have considerable choice of heater arrangements, since he may vary either the air velocity, the number of stacks or the size of the latter, and his choice will be influenced by an examination of such friction curves as are shown on Fig. 9 or Fig. 12, since the power required to force the air through the heater is an important factor.

A partial commercial table for a heater is given herewith, Table VI, although this has not been extended as fully as would be necessary for practical use, the curtailment being effected to reduce the amount of printing. The table may readily be extended to suit other air velocities and steam pressures, but obviously the one series of tests only permit the construction of a table for a given spacing of sections in a stack. In other words, the table and data are only applicable to a given make of heater with constant ratio of heating surface to clear area per stack.

In a second paper, to be prepared as soon as possible, the writer intends to discuss the results given on Figs. 7 to 12, along with some other data obtained.

TABLE VI
HOT BLAST HEATER
FINAL TEMPERATURES AND CONDENSATION. STEAM 5 PS. = 227° F.
Condensation C in lbs. per sq. ft. per hour

Number of Stacks	Temperature of Entering Air Deg. F.	Velocity Through Heater in Feet per Minute Measured at 70° F.																	
		400		600		800		1000		1200		1400		1600		1800			
		Final Air Temp. ° F.	Conden- sation per Sq. Ft. per Hr. lbs.	F.T.	C.	° F.	lbs.	F.T.	C.	° F.	lbs.	F.T.	C.	° F.	lbs.	F.T.	C.	° F.	lbs.
1	+60	100	.83	94	1.06	91	1.28	90	1.52	89	1.75								
	40	85	.78	78	1.18	75	1.44	73	1.71	72	1.96								
	30	77	.98	70	1.24	66	1.52	63	1.80	63	2.06								
	20	70	1.04	62	1.31	58	1.59	56	1.89	55	2.17								
	10	62	1.08	54	1.37	50	1.67	48	1.98	47	2.27								
	0	54	1.13	46	1.44	42	1.75	40	2.08	38	2.38								
2	-10	47	1.18	38	1.50	34	1.83	32	2.17	30	2.48								
	-20	39	1.23	30	1.56	26	1.91	23	2.25	21	2.59								
	-40	25	1.33	14	1.69	10	2.06	7	2.44	5	2.79								
	60	142	.86	127	1.05	119	1.23	115	1.42	112	1.63								
	40	132	.95	116	1.17	106	1.37	101	1.59	99	1.82								
	30	127	1.01	110	1.23	100	1.44	95	1.68	92	1.92								
3	20	122	1.06	103	1.30	93	1.52	88	1.77	85	2.02								
	10	117	1.11	98	1.36	87	1.59	81	1.85	78	2.11								
	0	112	1.16	91	1.42	80	1.66	74	1.93	71	2.21								
	-10	107	1.21	86	1.49	74	1.74	68	2.02	64	2.31								
	-20	102	1.26	80	1.55	67	1.81	61	2.11	57	2.40								
	-40	92	1.36	68	1.67	55	1.95	48	2.27	44	2.60								
4	60	167	.74	150	.93	140	1.10	135	1.30	132	1.49								
	40	160	.82	141	1.03	130	1.23	124	1.45	121	1.67								
	30	156	.87	136	1.10	125	1.31	119	1.54	115	1.76								
	20	152	.91	131	1.15	120	1.37	113	1.61	109	1.85								
	10	149	.95	127	1.21	114	1.44	108	1.69	104	1.94								
	0	145	1.00	122	1.26	109	1.51	102	1.77	98	2.03								
5	-10	141	1.04	118	1.32	104	1.58	97	1.84	92	2.11								
	-20	138	1.08	113	1.38	99	1.64	91	1.92	86	2.20								
	-40	131	1.17	104	1.49	89	1.77	80	2.08	75	2.38								
	60	184	.64	167	.83	157	1.00	152	1.19	148	1.37								
	40	179	.72	160	.93	149	1.13	143	1.33	138	1.54								
	30	177	.76	157	.99	145	1.19	138	1.40	134	1.62								
6	20	174	.80	153	1.03	141	1.25	134	1.47	129	1.70								
	10	171	.84	150	1.08	137	1.31	129	1.54	125	1.78								
	0	169	.87	146	1.13	132	1.37	125	1.61	120	1.87								
	-10	166	.91	142	1.19	128	1.43	120	1.69	115	1.95								
	-20	163	.95	139	1.23	124	1.50	116	1.90	110	2.19								
	-40	159	1.02	132	1.34	116	1.61	107	1.95	101	2.23								

FURTHER TESTS ON A TWO STROKE CYCLE OIL ENGINE

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In a previous paper¹ the author described a series of tests made for the purpose of determining the general characteristics of a heavy oil engine operating on the two stroke cycle. During this investigation certain questions arose, and the experiments described in the following pages were designed to throw some further light on the functioning of the stroke cycle in general, and on the working of this engine in particular.

The same engine was used as in the previous tests,² but in this case special attention was paid to the exhaust and charging period. In a two stroke engine this part of the cycle replaces the exhaust and charging strokes of the four stroke cycle, and therefore the whole functioning of the engine is controlled by it. In this particular engine the exhaust ports open when the crank reaches an angle of 120° from top dead centre, and the inlet ports (which are on the opposite side of the cylinder) open 17.5° later.

Changes of pressure during the whole cycle were recorded, as before, by Crosby gas engine indicators, connected to the cylinder and crank case respectively. The speed of the engine made the use of a small diagram imperative to avoid oscillations due to inertia, and therefore in these tests two other indicators of the balanced diaphragm type were connected to the cylinder and exhaust pipe respectively. These instruments, being practically free from inertia, enabled a more careful study of the pressure changes to be made, and the results obtained with them are recorded in the following pages.

A further point brought out by the previous trials was the fact that the chemical composition of samples of exhaust gas obtained by the usual method differed materially from those calculated from the quantities of air and oil put into the cylinder. The advisability of taking samples of gas from the cylinder was first considered, but this project was abandoned as being unlikely to produce useful results.³ Uniform distribution of gas in the cylinder is improbable, and samples taken at

*Proc. Inst. Mech. Eng., May, 1927.

¹"Some Tests on a Two Stroke Cycle Oil Engine", Proc. I. Mech. E., May, 1925.

²Viz., $9\frac{1}{2}'' \times 11''$ Allen engine 375 r.p.m. crank case compression.

³C.f. results obtained at Cambridge University.

Discussion of previous paper, p. 890, Proc. I. Mech. E., May, 1925.

the same time from different places would probably vary considerably. Any such investigation, therefore, would necessitate sampling at several different points in the cylinder at each period in the cycle. This would give valuable information, but the mechanical difficulties in the way were very great. It was decided, therefore, to insert a sampling valve in the exhaust pipe in order to find out what changes in composition of exhaust gas were taking place during the cycle.

DESCRIPTION OF APPARATUS

The sampling valve was placed in the exhaust pipe at a distance of about 6 inches from the ports, the seating being about $2\frac{1}{2}$ inches from the side of the pipe and midway between the top and bottom surfaces. The valve was of the mushroom type, as it was considered that this form was most likely to remain free from leakage at the temperature of the exhaust gases, and it was purposely made small in diameter ($5/16''$) to reduce inertia and to avoid undue disturbance of the exhaust gas stream. The period of opening was 15 degrees or .0066 second. The path of the sample from valve to collecting bottle is clearly indicated in Fig. 1. A stuffing box and screwed gland were used to prevent leakage of air along the spindle, and the valve was held tightly on its seat by a spring. The valve was opened at the desired point in the cycle by

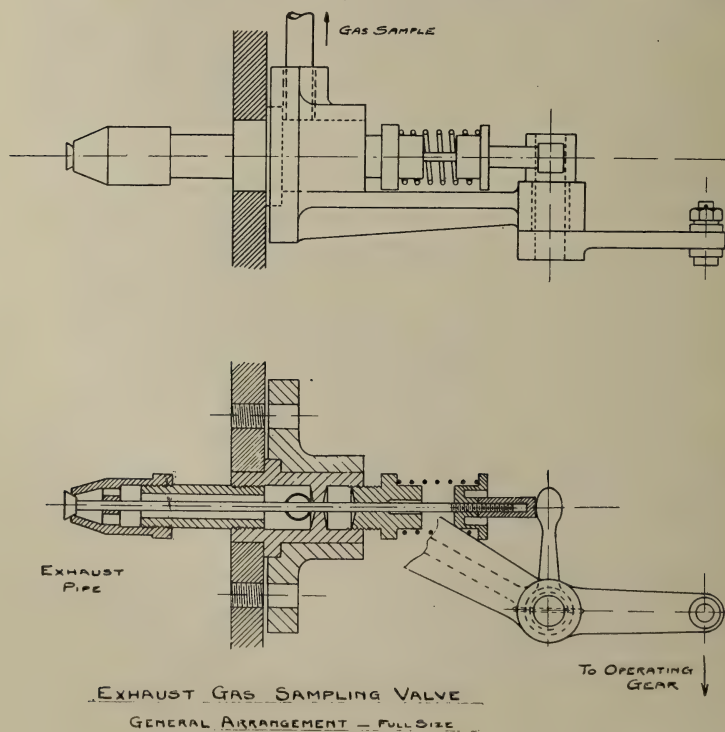


FIG. 1.—Exhaust gas sampling valve.

levers operated by a cam and push rod (Figs. 1 and 2). The clearances in each case were adjustable by means of screw threads, and as the cam was necessarily rather steep, a rocking lever was interposed between it and the end of the push rod, for the purpose of eliminating side thrust on the rod. The lift of the valve was $1/16$ inch.

The most difficult problem in connection with the operating gear was that of opening the valve at predetermined points in the cycle without stopping the engine to make the necessary adjustments. This difficulty was accentuated by the fact that there was less than $\frac{1}{4}$ inch of the crankshaft available for mounting gears or other transmission devices. It was therefore decided to bolt a gear wheel on the side of the flywheel. This gear (Fig. 3 (A)) was made in halves, bolted to the flywheel, and centred by a projection $1/8$ inch thick, extending to the shaft. A similar wheel (B) was mounted on the countershaft and drove the first bevel wheel (C) at engine speed. The second bevel wheel (E), carrying the cam (F), was driven through the intermediate agency of

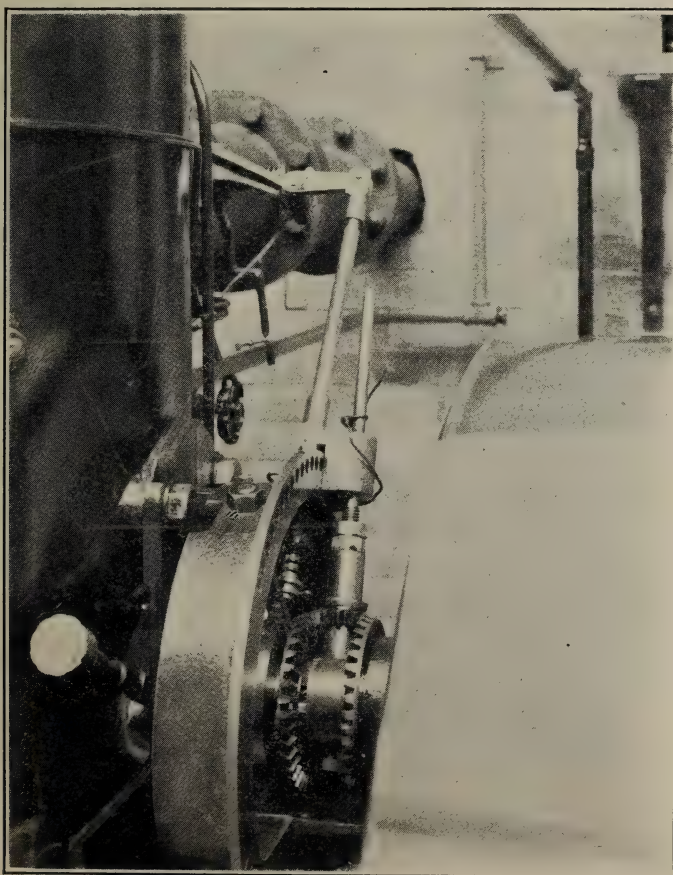


FIG. 2.—Operating gear for exhaust gas sampling valve.

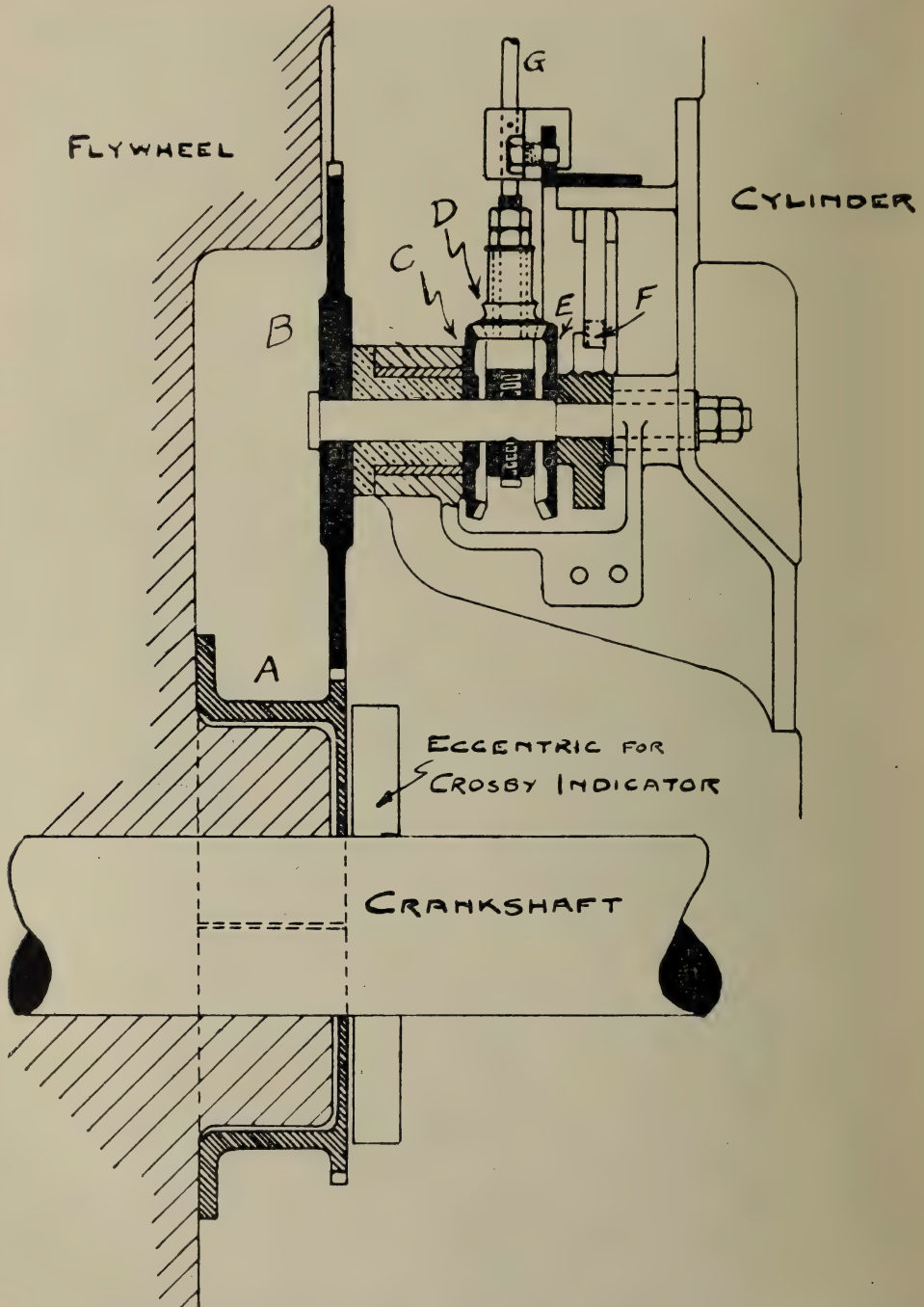


FIG. 3.—Diagram of operating gear.

the small bevel wheel (D). This was carried by an arm (G), which could be rotated about the countershaft and fixed in various positions by inserting a pin in the drilled holes (Fig. 2). Thus, as the arm and bevel wheel were rotated about the countershaft, the position of the cam, relative to the crank, was altered and a sample of gas could be taken at any desired point in the cycle without stopping the engine or taking off the load. The positions of the crank, when the sampling valve opened and closed, were obtained by actual measurement for each individual position of the gear, and the mean of these two readings, or the crank angle corresponding to the maximum lift of the valve, is that used in the curves (Figs. 6, 8, 9).

The special indicators used for recording pressure variations during the cycle (Fig. 4) were of the type in which a thin metal diaphragm is

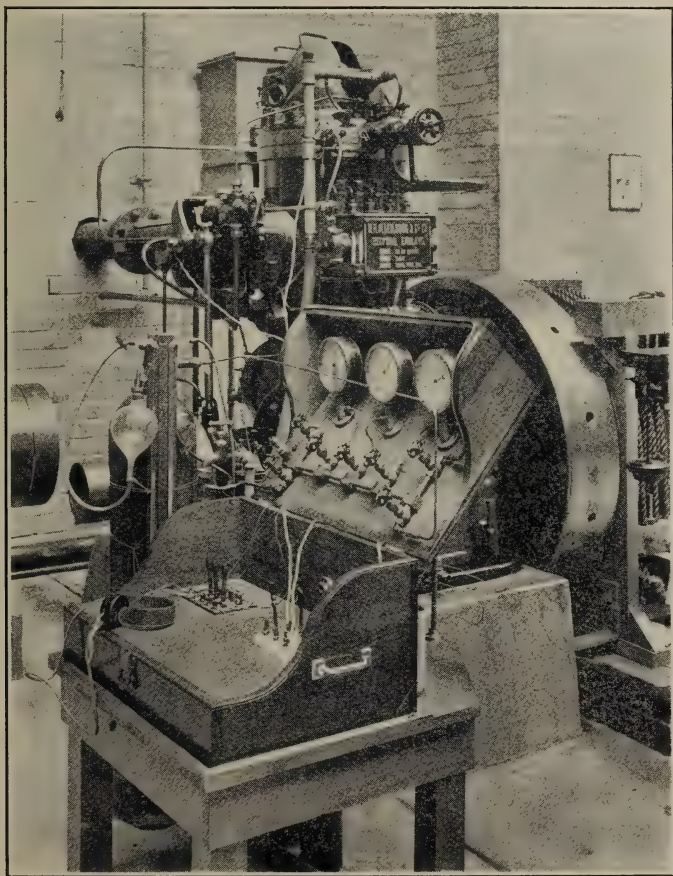


FIG. 4.—Arrangement of diaphragm indicators and gauges.

⁴For complete description see Bureau of Standards Report, No. 107, Washington U.S.A.

subjected to the varying pressure in the cylinder on one side, and to an adjustable measured pressure on the other side.⁴ A timing gear, mounted on the crankshaft, forms part of a battery circuit, and this gear is set to the particular crank angle for which the pressure reading is required. A contact on the indicator and a telephone receiver complete the circuit, and the pressure in the cylinder at any moment may be counterbalanced by raising the pressure on the outer side until the diaphragm ceases to touch the contact pin, when the clicks in the telephone cease. The counterbalancing pressure is then read from a pressure gauge, and this indicates the pressure in the cylinder at that point in the cycle. These pressures are then plotted to form the indicator diagram.

One of the principal difficulties with this instrument was the short life of the diaphragm, which usually failed by puncturing or corrosion, or both. The steel diaphragms (.005" thick) first used, failed very rapidly and therefore during these tests nickel-plated diaphragms (.004" to .003" thick) were used, and to avoid trouble arising during the test, new diaphragms were inserted before each test. Thicker diaphragms could not be used conveniently, as they were not sufficiently flexible. Zero readings were taken before and after each test and all the gauges were calibrated. Pressure was obtained from an oxygen bottle through a reducing valve and vacuum by means of a water aspirator.

A number of diagrams were taken with the Crosby indicator during each test, the spring strengths being $1/320$ and $1/10$ for cylinder and crank case diagrams, respectively.

PRESSURE READINGS AND INDICATOR DIAGRAMS

The pressure readings from which indicator diagrams were drawn naturally resolved themselves into two divisions:

- (A) The high pressure region between crank angles 240° (beginning of compression) and 120° (release)).
- (B) The low pressure region between 120° and 240° .

The pressure changes during period (A) were plotted against crank angles and piston displacements. The latter are shown for tests 1 to 5 in Fig. 5, and were compared in detail with the cards obtained from the Crosby indicator. It will be seen from Table 1 that the differences between them are small.

About 25 diagrams were taken with the Crosby indicator during each test, and the mean ordinates were measured at various points in the stroke to construct the average diagram referred to in Table 1. One difficulty in handling the diaphragm indicator was that of fixing the mean values of the maximum and other high pressures on each diagram. Starting with a balancing pressure well above the highest explosion pressure and gradually lowering it, an occasional click was heard in the telephone, corresponding to the highest pressures reached in occasional

cycles.⁵ As the balancing pressure was decreased the clicks increased in number until contact was made at every revolution. This was the lowest pressure, and the mean lay somewhere between the two readings, but not necessarily midway between them. The determination of the true mean pressure in this part of the cycle is therefore a matter of

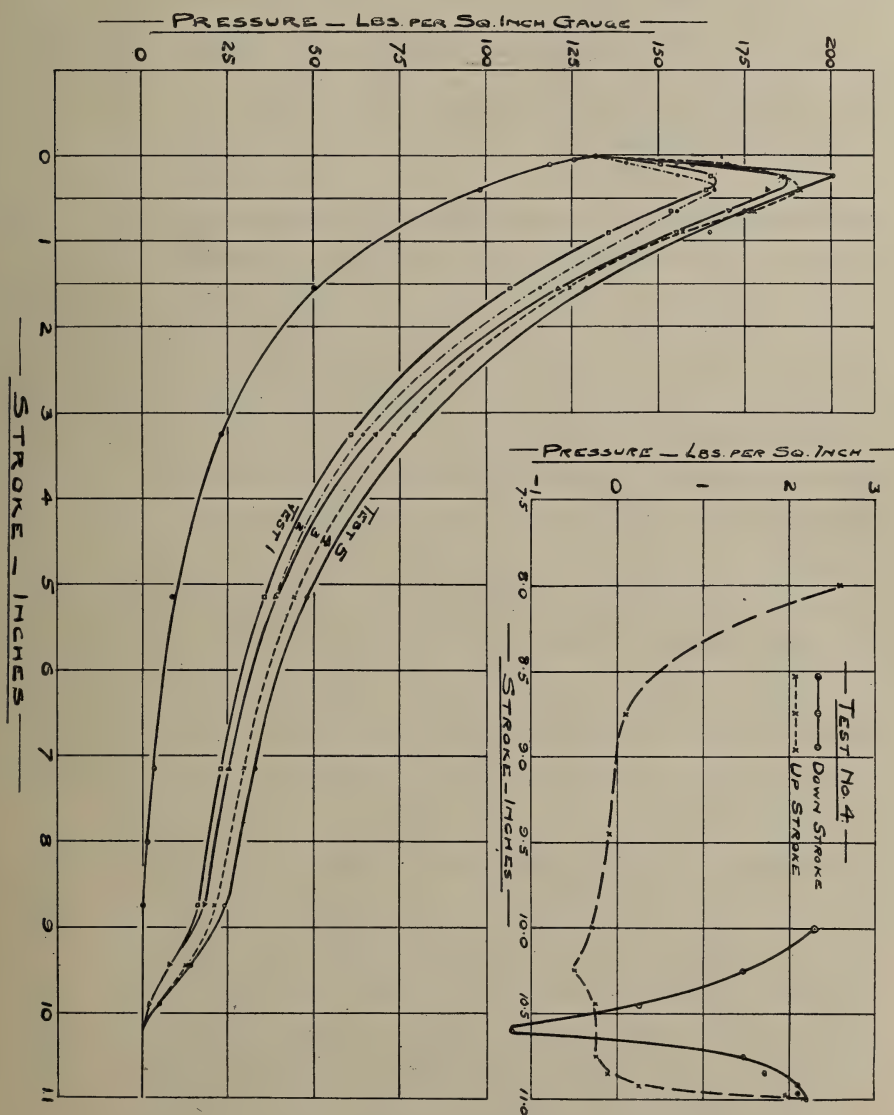


FIG. 5.

⁵See "Some Tests on a Two Stroke Cycle Oil Engine", Fig. 3.

TABLE 1

Test No.	1	2	3	4	5
I.H.P.—Diaphragm Indicator.....	21.5	23.2	25.7	28.7	31.8
I.H.P.—Crosby Indicator.....	21.1	23.4	26.9	28.8	31.7
Max. Pressure—Diaphragm Indicator					
lbs/sq. in.	166	166	187	191	201
Max. Pressure—Crosby Indicator					
lbs/sq. in.	175	178	196	203	203
Value of “ <i>n</i> ”—Diaphragm Indicator.....	1.19	1.19	1.25	1.19	1.16
Value of “ <i>n</i> ”—Crosby Indicator.....	1.26	1.26	1.25	1.23	1.23
Point in stroke at which	Diaph. Indicator..	.24	.40	.24	.40
max. pressure occurs	Crosby ,,	About .38" in all tests.			
—inches					

judgment and experience, and the difficulty of doing so correctly increases as the load decreases, when greater differences of pressure exist in successive cycles. As the pressure falls along the expansion curve this difficulty diminishes until at about half stroke it disappears entirely.

The compression curve was well marked and coincided almost exactly with that obtained from the Crosby indicator. It was the same for all loads, and therefore the values of “*n*” ($PV^n = \text{constant}$) for the expansion curve only are given in Table 1.⁶ The complete pressure readings, both for the cylinder and exhaust pipe, are shown in Table 2, so that individual values, which are difficult to obtain from the curves, may readily be obtained.

The light spring diagrams (period B) for all the tests are given in Fig. 6, and it will be observed that they are similar in form for all loads. In all cases, the lowest pressure (about 1 lb. per sq. in. below atmosphere) is obtained at about 150°, and this is followed by an abrupt rise to about 1½ to 2 lbs. above atmosphere, between 170° and 180°. The pressure then falls slightly below atmosphere and remains constant until the closing of the exhaust port at 240°, compression then starting from atmospheric pressure.

Calculations based on scavenging figures obtained from the exhaust gas analyses (see p. 9) and on temperatures taken from an entropy diagram fix the temperature at this point at a figure varying from 220° F. at the light loads (Tests 1 to 3) to 250° F. at full load (Test 5). The air compression diagram taken from the crank case is shown in Fig. 7, superimposed on the light spring diagram for Test No. 5.

This indicates that although the inlet ports open at 137½° the amount of air entering the cylinder is not sufficient to counteract the rapid fall

⁶These values were taken from a piston position of 1 inch after dead centre, as the subsequent points lay in a straight line on the log *P*-log *V* diagram. When the values from maximum pressure were taken and the mean line drawn lower values of “*n*” were obtained, as in the previous tests.

TABLE 2
BALANCED DIAPHRAGM INDICATOR DIAGRAMS

Test No.	Cylinder					Exhaust Pipe				
	1	2	3	4	5	1	2	3	4	5
Crank Angle										
0	131	131	132	130	132	.4	.5	.7	.5	.25
10	151	141	156	170	160	.45	.6	.5	.15	.2
15	166	156	187	186	201	.4	.3	.2	..	.2
20	164	166	182	191	191	.2	0	.1	.15	.2
25	154	156	171	178	175	0	-.1	.2	..	.25
30	136	144	156	157	155	0	0	.2	.2	.25
40	107	116	122	124	129	-.05	.2	.3	.25	.25
60	61	64	68	73	79	0	0	.2	.2	.3
80	36	39	39	44	48	..	.4	.5	.4	.45
100	23	25	25.5	29.6	33	.6	.6	.4	.5	.4
120*	16.5	18	18	21	24	1.5	1.3	.9	1.6	1.55
130	8.0	8.4	8.0	13	14	3.5	4.0	4.0	4.2	4.6
137.5†	3.3	3.0	2.5	..	5.1	2.3	2.3	2.7	2.7	3.3
140	2.5	2.0	2.0	2.3	2.3	1.7	2.0	1.9	2.1	2.3
145	.9	1.3	1.2	1.47	2.0	.6	.5	.4	-.1	.1
150	-.3	.3	-.2	.25	-1.1	-.4	-.7	-1.1	-1.4	-1.4
155	-.8	-.9	-1.4	-1.23	-1.1	-.8	-.6	-.9	-1.23	-1.0
160	.3	-.5	+.4	1.47	1.1	0	0	.2	.3	.5
165	1.7	1.1	1.5	1.72	1.2	1.2	1.2	..	1.3	1.3
170	1.4	1.1	1.4	2.1	1.9	1.4	1.5	1.5	1.7	2.0
175	..	1.2	1.4	2.1	2.1	1.5	1.8	1.7	2.0	2.3
180	1.5	1.6	1.7	2.21	2.0	1.4	1.8	1.8	1.8	2.1
185	..	1.4	..	1.96	.7	1.2	1.4	1.0	.8	.8
190	..	1.1	.55	.25	-.1	.5	.3	.2	.0	-.2
195	..	0	..	-.12	-.3	0	-.1	-.1	-.4	-.3
200	-.1	-.2	-.4	-.12	-.4	-.1	-.2	-.2	-.5	-.5
205	-.1	-.15	..	-.25	-.5	-.2	-.3	-.3	-.6	-.6
210	-.1	-.2	-.5	-.25	-.5	-.35	-.4	-.3	-.3	-.2
215	-.1	-.25	..	-.5	-.5	-.35	-.3	-.2	-.2	.1
220	-.1	-.2	-.4	-.3	-.1	-.2	-.15	-.1	-.15	.15
222.5§	-.1	-.2	..	..	..	-.2	-.15	-.1	..	.1
230	-.1	-.2	..	-.1	-.05	-.2	-.2	-.1	-.1	.05
240"	0.0	-.2	0	+.1	0	-.15	-.2	0	-.2	0
245	.5									
250	1.5	.9	..	2.6	1.2	-.25	-.3	-.2	-.25	-.1
255	2.2									
260	3.5	2.8	3.5	3.9	3.5	-.35	-.3	-.2	-.1	-.1
280	9.5	9.5	9.5	8.2	8.5	.3	.3	.4	.35	.3
300	23.5	24	24	23	23	.1	.3	.3	.2	.5
320	50	50	50	49.5	50	.75	.8	.9	.7	.4
340	96.5	96	98	94	98	.6	.4	.5	.4	.4
350	120	119	119	118	118	.4	.5	.5	.4	.5
355	128	127	131	..	126	.4	.5	.55	..	.4

*Exhaust port opens. †Inlet port opens. §Inlet port closes. "Exhaust port closes.

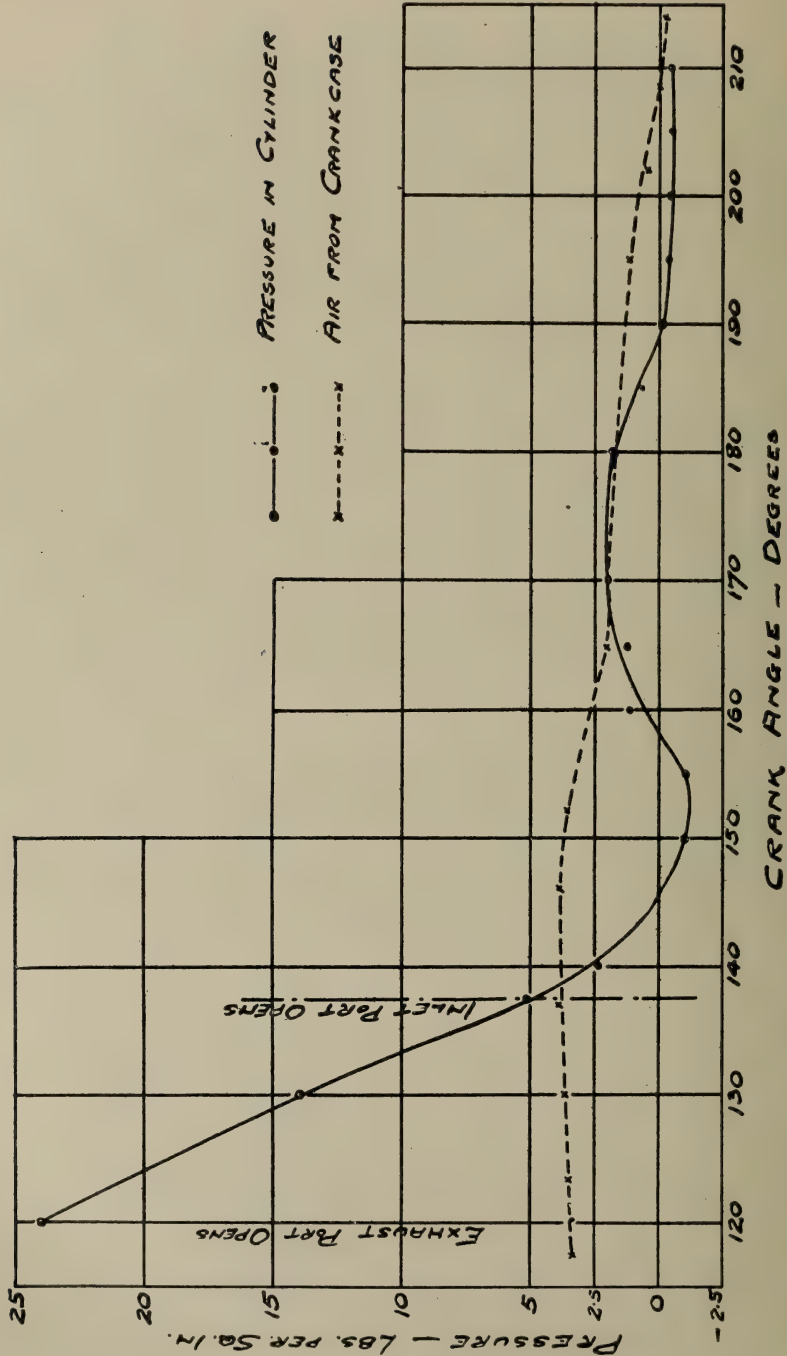
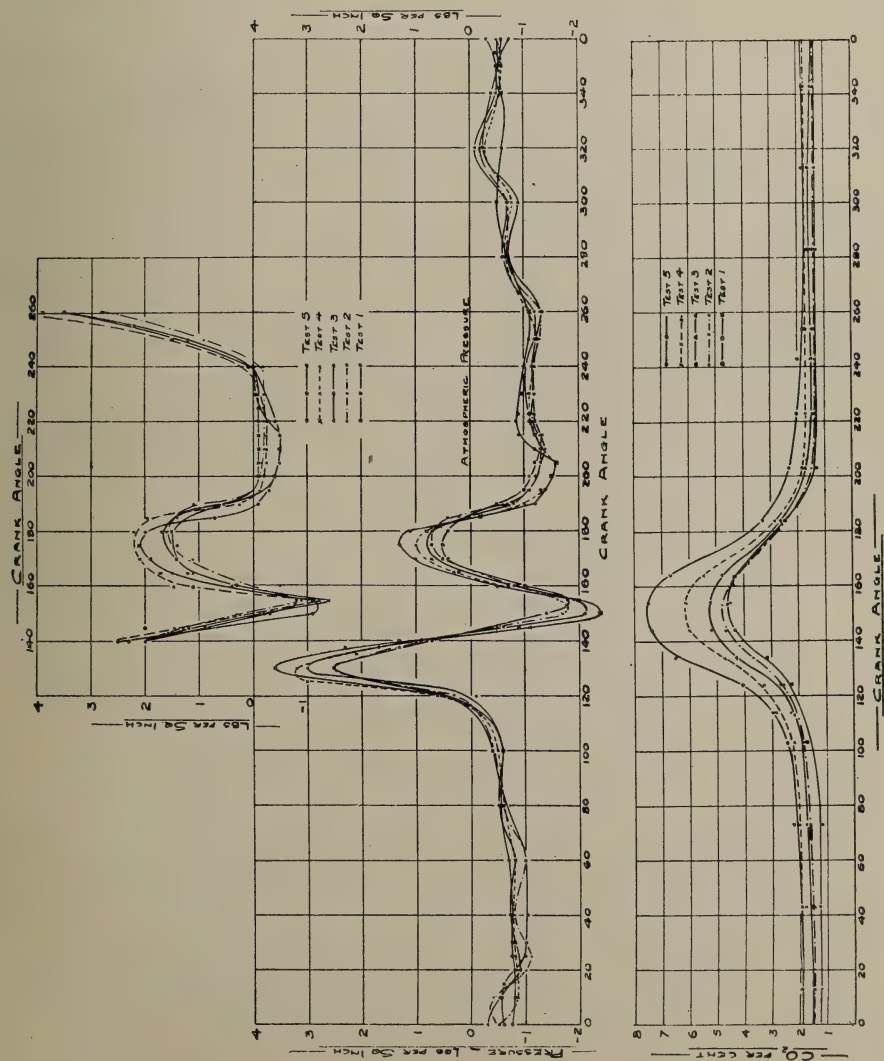


Fig. 7—Gas and Air Pressure, Test No. 5

of pressure until the crank reaches 150° , when the piston begins to slow down, and the air pressure in the crank case then commences to fall. At 200° the air has expanded nearly to atmospheric pressure, and the low pressure period begins. The diagrams for all tests show similar characteristics and indicate that during the high pressure period, around 180° , there is no "blow-back" of exhaust products into the crank case. These diagrams were also used to estimate the weight of exhaust products leaving the cylinder at successive periods during the cycle (see p. 284). The light spring diagram for Test No. 4 is shown plotted on a stroke basis in Fig. 5.



FIGS. 6, 8, 9.

A similar diagram (Fig. 8) giving fluctuations of pressure in the exhaust pipe shows that products of combustion begin to leave the cylinder at a crank angle of about 100° , although the nominal opening of the exhaust ports occurs at 120° . This is also indicated by the exhaust analyses (Fig. 9). The pressure variations are similar to those in the cylinder, and after the exhaust port closes, at 240° , oscillations of pressure take place, evidently due to surging in the exhaust pipe. It will be noted that the periods of maximum pressure during this part of the cycle synchronize in most of the tests, and also occur at the same points as those in the cylinder.

EXHAUST GAS ANALYSIS

The sampling apparatus has already been described, and Table 3 gives the analyses of exhaust gas samples taken at different points in the cycle. These are plotted on a crank angle basis in Fig. 9. The curves for all tests are similar in form, the CO_2 (volumetric) percentage reaching its maximum at about 150° , which is also the point of minimum pressure in the cycle. These curves indicate that exhaust gas begins to leave the cylinder when the crank is at an angle of 100° .

An average sample of exhaust gas was also taken in each test by holding the sampling valve open for several minutes, and the incorrectness of this method of sampling may be deduced from the curves. From 0° to 100° and 240° to 360° no gas is passing from the cylinder, and yet with the usual method of taking exhaust samples this period contributes 61 per cent. of the volume of sample collected.⁷ The percentage of CO_2 obtained by analysis, from average samples collected in this way, agrees very closely with the average height of the CO_2 curve (see Table 4 and Fig. 10), but a closer approximation to the true value is obtained by measuring the mean height of the curve during the actual exhaust period, viz., 100° to 240° . This gives figures substantially higher than the mean values obtained in the usual way (Table 4), but corrections must still be made for variations in velocity of exhaust gas during this period. Fig. 7 was used in conjunction with a step-by-step method to calculate the weight of exhaust gas leaving the cylinder for every 10 degrees between 100° and 240° . This investigation showed that maxima existed at about 160° and 210° and that the delivery of exhaust gas from the cylinder practically ceased at about 170° . In some instances a small quantity of gas returned to the cylinder from the exhaust pipe at this point. When this velocity or quantity component was combined with the actual exhaust composition for each section, the average figures given in the last line of Table 4, and plotted in Fig. 10, were obtained.

This discrepancy evidently affects calculations of air consumption,

⁷In a single cylinder engine (see p. 286).

TABLE 3
SUMMARY OF ANALYSES OF EXHAUST GAS SAMPLES FOR ALLEN OIL ENGINE

Test No.	1			2			3			4			5		
	CO ₂	O ₂	N ₂	CO ₂	O ₂	N ₂	CO ₂	O ₂	N ₂	CO ₂	O ₂	N ₂	CO ₂	O ₂	N ₂
Analysis															
Crank Angle															
13	1.22			1.48			1.48			2.02			1.91		
43	1.26			1.48			1.58			1.80			1.91		
73	1.16			1.59			1.69			2.01			2.21		
103	1.79			1.79			1.68			2.12			2.45		
114	2.02	17.62	80.36	2.32	17.81	79.87	2.11			2.87	16.30	80.83	2.97	16.29	80.74
124	2.21	17.95	79.84	2.56	17.37	80.07	2.64	16.82	80.54	3.27	16.45	80.28	4.04	14.87	81.09
134	3.16	15.91	80.93	3.67	15.62	80.71	3.29	14.91	81.80	4.70	14.12	81.18	6.50	11.75	81.75
144	4.34	14.92	80.74	4.68	14.35	80.97	5.20	13.36	81.44	5.99	12.62	81.39	7.34	10.39	82.27
154	4.54	14.48	80.98	4.46	14.88	80.66	5.18	13.58	81.24	6.10	12.61	81.29	7.47	10.10	82.43
164	4.33	14.69	80.98	4.28	14.55	81.17	4.82	13.98	81.20	5.45	12.99	81.56	6.65	11.29	82.06
184	2.49	17.32	80.19	2.42	16.95	80.63	2.65	16.94	80.41	2.85	16.56	80.59	3.27	16.13	80.60
203	1.27	19.02	79.71	1.46	18.83	79.71	1.66	18.42	79.92	1.80	18.45	79.75	2.32	17.27	80.41
223	1.42	18.78	79.80	1.47	18.44	80.09	1.35	19.19	79.46	1.69	18.58	79.73	2.02	17.98	80.00
243	1.32	18.99	79.69	1.36	18.68	79.96	1.48	18.19	80.33	1.47	18.57	79.96	1.99	17.83	80.18
254	1.37			1.49			1.49			1.80			1.70		
283	1.27			1.47			1.68			1.69			1.57		
313	1.37			1.37			1.58			1.90			1.80		
243	1.36			1.48			1.47			1.80			1.72		

TABLE 4
EXHAUST GAS COMPOSITION

Test No.	CO ₂ per cent. by volume				
	1	2	3	4	5
Analysis of mean samples.....	1.55	1.85	2.11	2.35	2.68
Mean Analysis from total area of curve (0°-360°).....	1.75	1.93	2.09	2.42	2.72
Mean Analysis from area of curve between 100° and 240°.....	2.50	2.66	2.87	3.32	4.04
Mean Analysis corrected for velocity changes	2.58	2.77	2.98	3.55	4.37

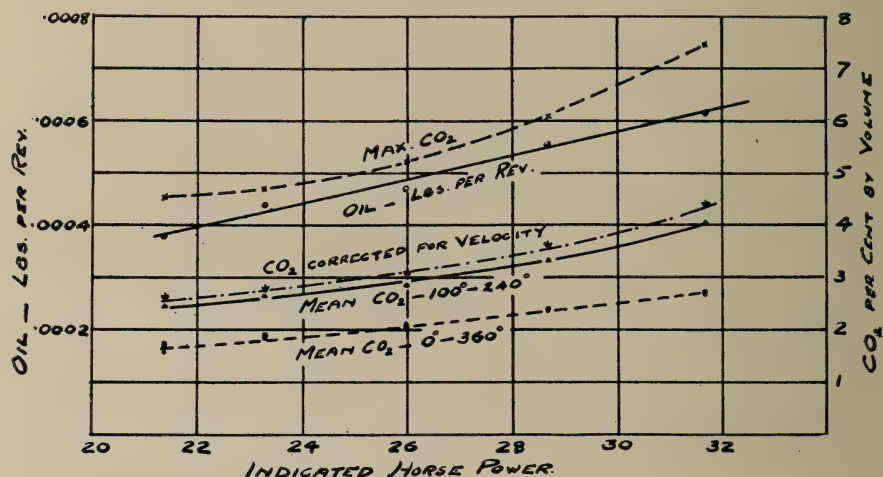


FIG. 10.—CO₂ in exhaust gases.

etc., based on the mean analysis. The difference between the true and apparent mean analyses becomes less as the number of cylinders increases. If the active exhaust period equals or exceeds 120° the idle period disappears when the engine has three or more cylinders, but the velocity correction will continue to affect the result until the number of cylinders is increased to such an extent that the exhaust periods overlap very considerably.

The figures in Table 5 (Fig. 11) were obtained by displacing the

TABLE 5
Number of Cylinders CO₂ per cent. by volume in
 average sample

1	2.72
2	3.6
3	4.48
4	5.03
6	6.22
8	6.74

CO₂ curve in Fig. 9 through angles of 180, 120, 90, 60 and 45 deg. respectively, and show the percentages of CO₂ that may be expected with 2, 3, 4, 6 and 8 cylinders respectively, if the cranks are at equal angles to one another.⁸

The first two of these must be corrected for the idle period, as stated above, and all of them require velocity correction.

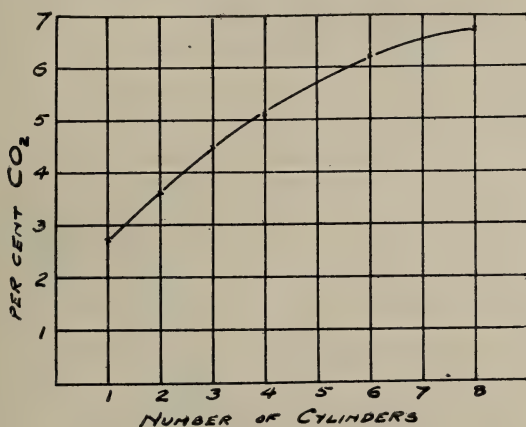


Fig. 11

SCAVENGING AND VOLUMETRIC EFFICIENCY

One of the principal difficulties in the two stroke cycle is to determine the quantity of scavenging air that remains in the cylinder when the exhaust ports are closed by the upward movement of the piston.

The best results are obtained with that design which replaces the greatest amount of products of combustion by scavenging air, and the exhaust analyses given above provide a means of calculating the amount of such replacement. It was assumed in each test that the exhaust analysis which gave the maximum percentage of CO₂ represented very closely the composition of the expanding charge, as at this period little, if any, of the scavenging air had reached the exhaust pipe.

From the volumetric composition of this gas and the weight of carbon in the fuel burned the weight of the expanding charge was deduced, and by this means the weight of air trapped in the cylinder was calculated, allowance being made for moisture in air and products of combustion. In this way the following figures were obtained:

From Table 6 it is evident that the average volume of air trapped in

⁸The exhaust characteristics of 6 or 8 cylinder engines, when the cranks are 120° and 90° apart, respectively, will obviously coincide with those of the 3 and 4 cylinder engines in the curve.

the cylinder at the beginning of the compression stroke amounted to 60 per cent. of the stroke volume, and this figure was accordingly used in the subsequent calculations to obtain the temperatures at various points in the cycle.

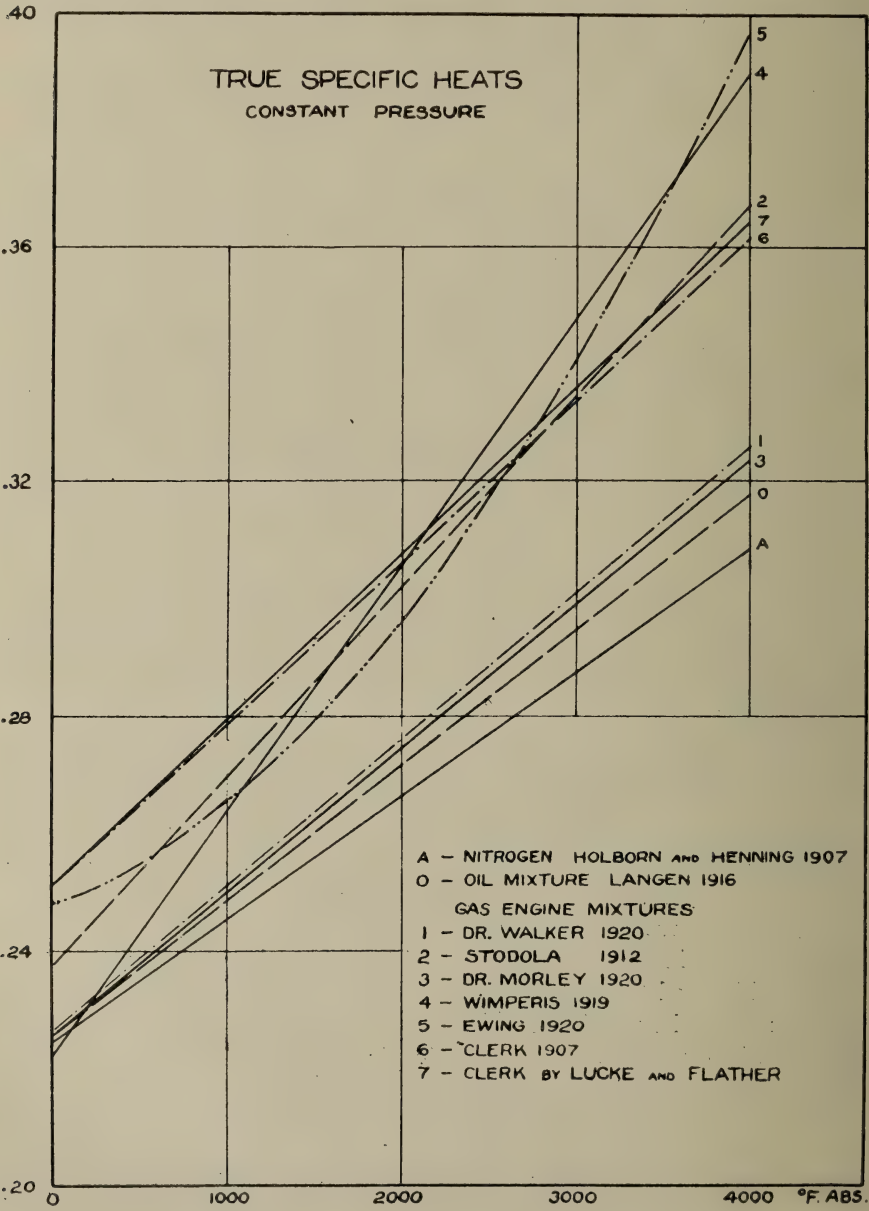


Fig. 12

TABLE 6

Test No.		1	2	3	4	5
Weight of air in cylinder during combustion period.....	lbs.	.0176	.0196	.0186	.0192	.0176
Weight of air delivered from crank case per cycle.....	lbs.	.0275	.0272	.0273	.0278	.0273
Percentage of stroke volume occupied by scavenging air at crank case temperature	%	57.4	63.3	60.3	60.9	57.8
Weight of air supplied for combustion per lb. of oil fuel	lbs.	46.3	44.7	39.8	34.6	28.6
Excess air in cylinder beyond that required for complete combustion.....	%	224	212	178	142	100

TEMPERATURES AND ENERGY DIAGRAM

For the purpose of investigating the temperature changes which take place during the cycle, an energy-entropy diagram was prepared, and after some consideration the specific heats of Holborn & Henning⁹ were adopted as a basis. This diagram will not be described in detail as it is similar to those used by Alexander¹⁰ and Stodola.¹¹

The temperatures at beginning of compression were calculated from the scavenging conditions described in the previous paragraph, assuming adiabatic expansion of the exhaust products from the release pressure and a further fall of temperature amounting to about 50-85° F. at different loads. As this temperature only refers to the weight of exhaust products left in the cylinder, a considerable error in this assumption does not affect the final result to any great extent. The temperatures obtained were as follows:

Test No.	1	2	3	4	5
Compression Temperature °F.....	710	710	720	720	760
Maximum Temperature °F.....	1350	1456	1560	1610	1725
Temperature at release °F.....	960	1030	1035	1165	1375

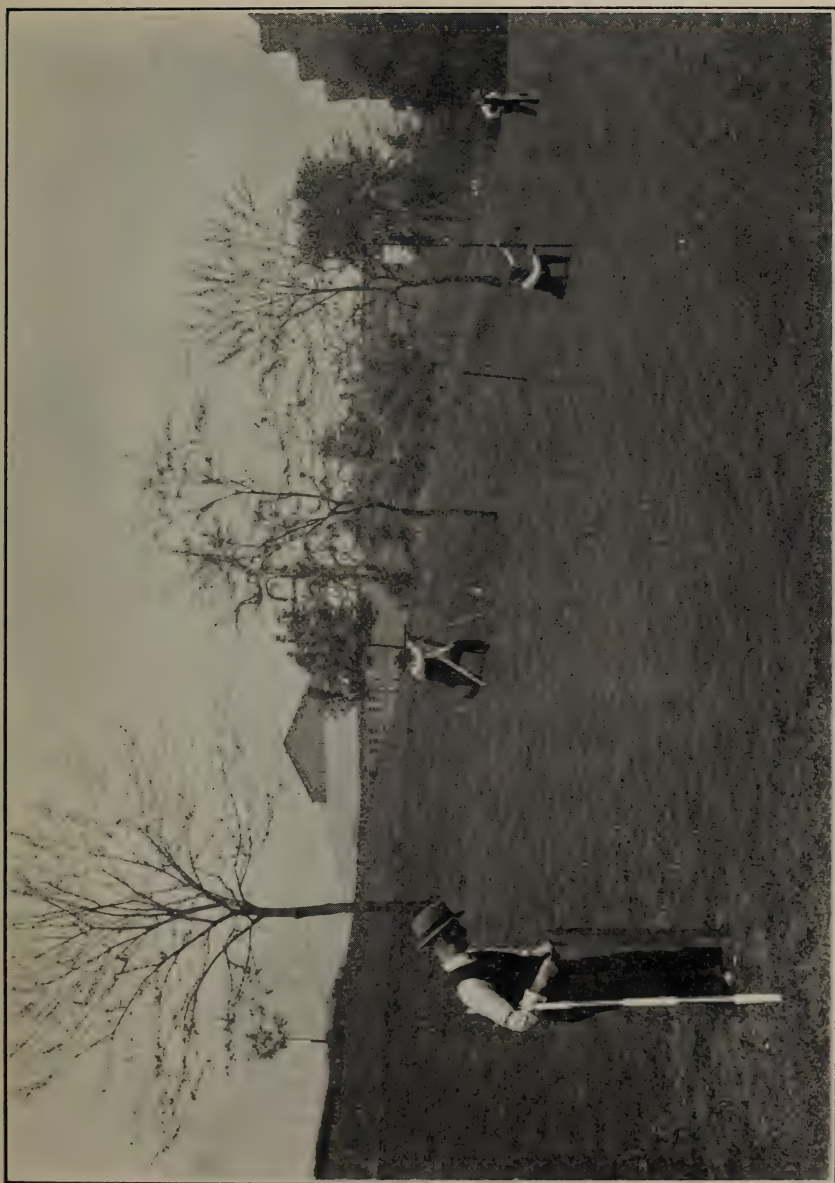
All the diagrams indicate that a considerable amount of after burning exists during the expansion stroke and that the maximum temperature occurs when the piston has moved about 0.9 inches or 8% of the stroke from top dead centre. This fact, together with the considerable amount of after-burning that exists, and the weakness of the charge in the cylinder accounts for the comparatively low combustion temperatures obtained.

⁹Fig. 12' shows the relationship between the values of specific heat for various gas and oil engine mixtures, and Holborn & Henning's figures for nitrogen.

¹⁰Proc. I. Mech. E., May, 1917.

¹¹See "Automotive Industries", June 10th, 1926.

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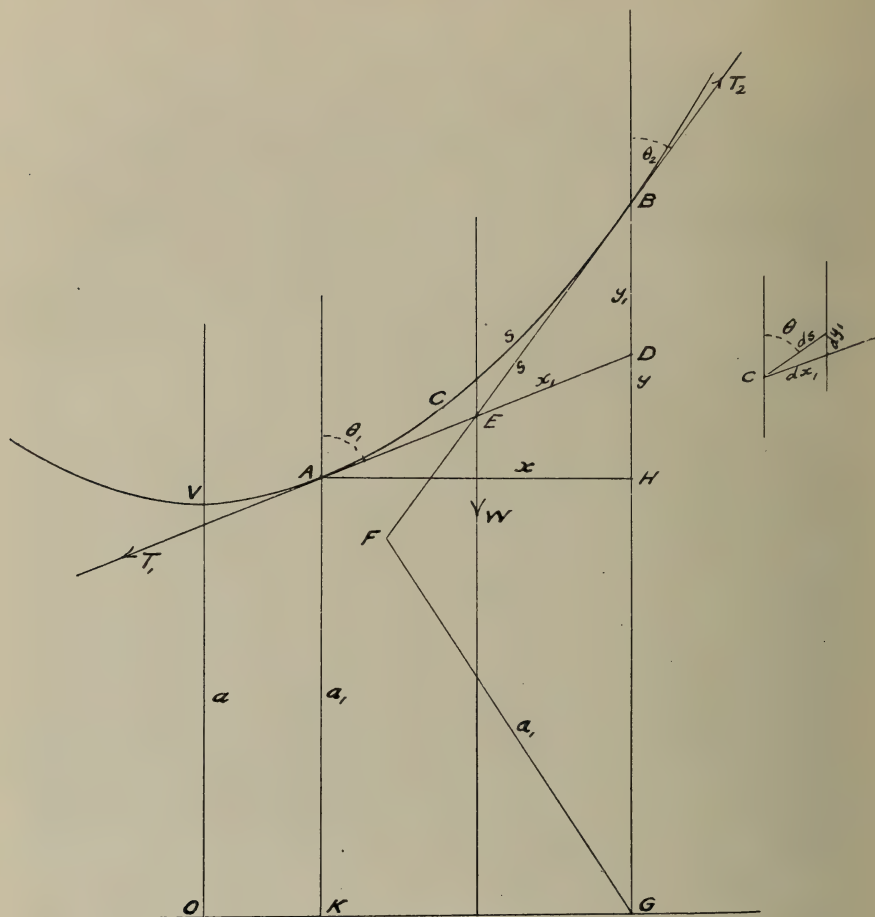


A METHOD OF BASE LINE MEASUREMENT

By L. B. STEWART

1. The object of this investigation is to develop a method by which precise linear measurements may be made with the ordinary instrumental equipment of the engineer. At the same time it is thought that the precision of the method to be described is such that it will not only meet all professional requirements, but also prove sufficient for the measurement of the base lines of a first order geodetic triangulation.

The method of measurement is as follows: The route of the base line having been cleared of obstructions, it is then marked out in the usual way by a series of carefully aligned posts spaced a trifle less than a tape-length apart and firmly driven into the ground, their tops standing 30 or 36 inches above the ground. To the top of each post is nailed a small piece of sheet zinc or copper, on which a fine line is ruled with a steel point at right angles to the direction of the line. In measuring the base the tape is stretched between each consecutive pair of posts in turn and the necessary tension applied to bring the zero lines of the tape precisely over the lines on the metal plates. While held thus, the height of the middle point of the tape with reference to the ends is found by holding a leveling rod vertically and against the tape at its middle point and reading on the rod the height of the tape, at the same time a reading of the rod being taken with an engineer's level; the rod is then held on the two posts in turn and readings taken with the level. These four readings of the rod, together with the temperature of the tape, constitute the field record. From these readings the heights of the higher end of the tape and its middle point above the lower end are found; and, in terms of these two heights, and the length of the tape, the extension of the tape due to the tension to which it has been subjected may be determined, and also the length of its projection on a horizontal plane. The constants of the tape: the weight of a unit of length, the stretch of a unit of length due to a unit tension, and the coefficient of expansion, must of course have been carefully determined. The derivation of the formulae by means of which the reductions above outlined may be made now follows; and in deriving them two assumptions are made that are not completely realized in practice, *viz.*, that the tape is perfectly flexible, and that the weight per unit of length is constant throughout notwithstanding the varying tension from point to point. The error due to the non-fulfilment of these conditions, however, must be extremely small.



2. In the figure AB represents a tape, stretched as in making a measurement. It is in equilibrium under the action of the tensions T_1 and T_2 , applied at its extremities, and its weight W acting vertically through its centre of gravity. The directions of these three forces pass through some point E . Then, taking A as the origin of coordinates, let x_1 and y_1 denote the oblique coordinates AD and DB of the point B , x and y the rectangular coordinates AH and HB , θ_1 and θ_2 the angles which the tangents to the curve at A and B make with the vertical, and s the length of the tape.

The sides of the triangle DEB are then parallel to, and, taken clockwise, in the same sense as, the directions of the three forces T_1 , T_2 and W , so that their lengths are proportional to those forces, and we have at once the equations:

$$T_1 = W \frac{\sin \theta_2}{\sin (\theta_1 - \theta_2)} \quad , \quad (1)$$

$$T_2 = W \frac{\sin \theta_1}{\sin (\theta_1 - \theta_2)} \quad , \quad (2)$$

whence $T_1 \sin \theta_1 = T_2 \sin \theta_2$. (3)

From (3) it follows that the horizontal component of the tension at any point of the tape is constant.

From (1) and (2) we have:

$$\begin{aligned} T_2 - T_1 &= W \frac{\sin \theta_1 - \sin \theta_2}{\sin (\theta_1 - \theta_2)} \quad , \\ &= ws \frac{\sin \theta_1 - \sin \theta_2}{\sin (\theta_1 - \theta_2)} \quad , \end{aligned} \quad (4)$$

in which w denotes the weight of a unit of length of the tape. Applying (4) to an element of the curve at any point C we have:

$$\begin{aligned} dT &= ws \frac{\sin \theta - \sin (\theta - d\theta)}{\sin d\theta} \quad , \\ &= ws \frac{d \sin \theta}{d\theta} = ws \cos \theta, \\ &= w dy; \end{aligned}$$

therefore, integrating between the limits y and 0 , we have

$$T_2 - T_1 = wy.$$

Comparing this result with (4) we have:

$$y = s \frac{\sin \theta_1 - \sin \theta_2}{\sin (\theta_1 - \theta_2)} \quad . \quad (5)$$

Also, denoting by a_1 the length of tape whose weight is equal to the tension T_1 we have

$$T_2 - wa_1 = wy,$$

or $T_2 = w(a_1 + y)$; (6)

from which we infer that the tension at any point of the curve is proportional to the vertical distance of the point from a horizontal line KG , where $AK = a_1$.

Again, for the element of the curve at C , we have

$$\frac{dx_1}{ds} = \frac{\sin \theta}{\sin \theta_1} \quad , \quad \frac{dy_1}{ds} = \frac{\sin (\theta_1 - \theta)}{\sin \theta_1}.$$

Also, from (1), s' denoting the length AC ,

$$\begin{aligned} s' &= a_1 \frac{\sin (\theta_1 - \theta)}{\sin \theta} \\ &= a_1 (\sin \theta_1 \cot \theta - \cos \theta_1); \end{aligned}$$

whence, by differentiation,

$$\begin{aligned}\frac{ds}{d\theta} &= -a_1 \frac{\sin \theta_1}{\sin^2 \theta} \cdot \\ \therefore \frac{dx_1}{d\theta} &= \frac{dx_1}{ds} \cdot \frac{ds}{d\theta} = -\frac{\sin \theta}{\sin \theta_1} \cdot \frac{a_1 \sin \theta_1}{\sin^2 \theta} = -\frac{a_1}{\sin \theta} ; \\ \frac{dy_1}{d\theta} &= \frac{dy_1}{ds} \cdot \frac{ds}{d\theta} = -\frac{\sin (\theta_1 - \theta)}{\sin \theta_1} \cdot \frac{a_1 \sin \theta_1}{\sin^2 \theta} = -a_1 \frac{\sin (\theta_1 - \theta)}{\sin^2 \theta} \\ &= -a_1 \sin \theta_1 \frac{\cos \theta}{\sin^2 \theta} + a_1 \frac{\cos \theta_1}{\sin \theta} .\end{aligned}$$

We therefore have,

$$\begin{aligned}x_1 &= a_1 \int_{\theta_2}^{\theta_1} \frac{d\theta}{\sin \theta} = a_1 (\log \tan \frac{1}{2}\theta_1 - \log \tan \frac{1}{2}\theta_2) \\ &= a_1 \log \frac{\tan \frac{1}{2}\theta_1}{\tan \frac{1}{2}\theta_2}\end{aligned}\quad (7)$$

$$\begin{aligned}\text{Also, } y_1 &= a_1 \int_{\theta_2}^{\theta_1} \left(\sin \theta_1 \frac{\cos \theta}{\sin^2 \theta} - \frac{\cos \theta_1}{\sin \theta} \right) d\theta \\ &= a_1 \sin \theta_1 \left(\frac{1}{\sin \theta_2} - \frac{1}{\sin \theta_1} \right) - a_1 \cos \theta_1 \log \frac{\tan \frac{1}{2}\theta_1}{\tan \frac{1}{2}\theta_2} \\ &= a_1 \frac{\sin \theta_1 - \sin \theta_2}{\sin \theta_2} - a_1 \cos \theta_1 \log \frac{\tan \frac{1}{2}\theta_1}{\tan \frac{1}{2}\theta_2}\end{aligned}\quad (8)$$

$$= a_1 \frac{\sin \theta_1 - \sin \theta_2}{\sin \theta_2} - x_1 \cos \theta_1 \quad (9)$$

Then, as $DH = x_1 \cos \theta_1$, we have

$$y = a_1 \frac{\sin \theta_1 - \sin \theta_2}{\sin \theta_2} \quad (10)$$

This last equation may also be obtained by writing (1) in the form,

$$a_1 = s \frac{\sin \theta_2}{\sin (\theta_1 - \theta_2)} , \quad (11)$$

and substituting the value of s from this in (5).

If the point A be at the vertex V of the curve, so that $\theta_1 = 90^\circ$, then the equations (11), (7), (10), (5), and (6), become, respectively (a denoting the length $V0$):

$$s = a \cot \theta_2 \quad (12)$$

$$x = a \log \cot \frac{1}{2}\theta_2 \quad (13)$$

$$y = a \frac{1 - \sin \theta_2}{\sin \theta_2} = a (\operatorname{cosec} \theta_2 - 1) \quad (14)$$

$$y = s \frac{1 - \sin \theta_2}{\cos \theta_2} = s (\sec \theta_2 - \tan \theta_2) \quad (15)$$

$$T_2 = w(a + y). \quad (16)$$

If a line GF now be drawn making an angle $\theta_1 - \theta_2$ with GH and intersecting the tangent at B in the point F , we have, in the triangle thus formed,

$$BF = (y + a_1) \frac{\sin (\theta_1 - \theta_2)}{\sin \theta_1},$$

$$GF = (y + a_1) \frac{\sin \theta_2}{\sin \theta_1}.$$

But (2) and (3) may be written:

$$s = (y + a_1) \frac{\sin (\theta_1 - \theta_2)}{\sin \theta_1},$$

$$a_1 = (y + a_1) \frac{\sin \theta_2}{\sin \theta_1};$$

hence it follows that $BF = s$, and $GF = a_1$. We therefore have the equation:

$$(y + a_1)^2 = s^2 + a_1^2 + 2sa_1 \cos \theta_1;$$

whence,

$$a_1 = \frac{s^2 + y^2}{2(y - s \cos \theta_1)} \quad (17)$$

3. Again, from (5),

$$\begin{aligned} \frac{s}{y} &= \frac{\sin (\theta_1 - \theta_2)}{\sin \theta_1 - \sin \theta_2}, \\ &= \frac{2 \sin \frac{1}{2}(\theta_1 - \theta_2) \cos \frac{1}{2}(\theta_1 + \theta_2)}{2 \sin \frac{1}{2}(\theta_1 - \theta_2) \cos \frac{1}{2}(\theta_1 + \theta_2)} \\ &= \frac{1 + \tan \frac{1}{2}\theta_1 \tan \frac{1}{2}\theta_2}{1 - \tan \frac{1}{2}\theta_1 \tan \frac{1}{2}\theta_2}; \end{aligned}$$

whence
$$\frac{s+y}{s-y} = \frac{1}{\tan \frac{1}{2}\theta_1 \tan \frac{1}{2}\theta_2}$$

or
$$\tan \frac{1}{2}\theta_1 \tan \frac{1}{2}\theta_2 = \frac{s-y}{s+y}. \quad (18)$$

Again, let y' denote the ordinate to the curve at its middle point; then by (17),

$$2a_1 = \frac{s^2 - y^2}{y - s \cos \theta_1} = \frac{s^2 - 4y'^2}{4y' - 2s \cos \theta_1},$$

so that, solving for θ_1 , we find

$$\cos \theta_1 = \frac{(s^2 - y^2) 4y' - (s^2 - 4y'^2)y}{(s^2 - y^2)2s - (s^2 - 4y'^2)s} \quad (19)$$

Again, writing

$$m = \frac{\tan \frac{1}{2}\theta_1}{\tan \frac{1}{2}\theta_2}, \quad (20)$$

and expanding $\log m$, (7) becomes

$$\frac{x_1}{a_1} = 2 \left\{ \frac{m-1}{m+1} + \frac{1}{3} \left(\frac{m-1}{m+1} \right)^3 + \frac{1}{5} \left(\frac{m-1}{m+1} \right)^5 + \right\} \quad (21)$$

Also

$$\frac{m-1}{m+1} = \frac{\tan \frac{1}{2}\theta_1 - \tan \frac{1}{2}\theta_2}{\tan \frac{1}{2}\theta_1 + \tan \frac{1}{2}\theta_2};$$

so that, eliminating θ_2 by (18), and making use of the equation:

$$\tan^2 \frac{1}{2}\theta_1 = \frac{1 - \cos \theta_1}{1 + \cos \theta_1},$$

we find, after a little reduction,

$$\frac{m-1}{m+1} = \frac{y-s \cos \theta_1}{s-y \cos \theta_1}. \quad (22)$$

Also, introducing the value of $\cos \theta_1$ in (19) and reducing, we have

$$\frac{m-1}{m+1} = \frac{2s(y-2y')}{s^2-4y'(y-y')}. \quad (23)$$

Substituting now in (21),

$$x_1 = \frac{x}{\sin \theta_1},$$

and the value of a_1 in (17), we obtain:

$$x = \frac{(s^2-y^2) \sin \theta_1}{s-y \cos \theta_1} \left\{ 1 + \frac{1}{3} \left(\frac{m-1}{m+1} \right)^2 + \frac{1}{5} \left(\frac{m-1}{m+1} \right)^4 + \right\} \quad (24)$$

Then, for brevity, writing

$$M = \frac{m-1}{m+1},$$

equation (22) gives

$$\cos \theta_1 = \frac{y-Ms}{s-My},$$

which, substituted in (24), give on reduction,

$$x = \sqrt{s^2-y^2} \cdot (1-M^2)^{\frac{1}{2}} \cdot (1 + \frac{1}{3}M^2 + \frac{1}{5}M^4 +).$$

Then, expanding, multiplying out and rearranging, we have finally:

$$x = \sqrt{s^2-y^2} \left(1 - \frac{1}{6}M^2 - \frac{11}{120}M^4 - \frac{103}{1680}M^6 - \frac{1823}{40320}M^8 - \right). \quad (25)$$

We shall now apply this expression to a numerical example in order to determine the relative magnitudes of its terms, taking as data:

$$s = 100, y = 50, y' = 24;$$

we then find—

$$\begin{aligned}
 & \text{1st term} = 86.60254 \\
 & -2\text{nd} \quad " = -0.0410123 \dots \\
 & -3\text{rd} \quad " = -0.0000641 \dots \\
 & -4\text{th} \quad " = -0.0000001218 \dots \\
 & -5\text{th} \quad " = -0.00000000255 \dots
 \end{aligned}$$

$$x = 86.56146 \dots$$

∴

By means of equation (42) we find, by assuming $w = 0.012$ lb., that the tension in this case is

$$11.556 \text{ pounds.}$$

If the tension be 20 pounds, s and y remaining unchanged, equation (45) gives

$$y' = 24.446 \dots;$$

so that, taking as data:

$$s = 100, y = 50, y' = 24.45,$$

we find the following values for the terms of (25):

$$\begin{aligned}
 & \text{1st term} = 86.60254 \dots \\
 & -2\text{nd} \quad " = -0.01241544 \dots \\
 & -3\text{rd} \quad " = -0.0000058736 \dots \\
 & -4\text{th} \quad " = -0.00000003379 \dots \\
 & -5\text{th} \quad " = -0.00000000021435 \dots
 \end{aligned}$$

∴

$$x = 86.59012 \dots$$

In this case it appears that the third term amounts to less than 6 parts in 100000000, the tape length being the unit; so that for practical use we are safe in writing:

$$x = \sqrt{(s+y)(s-y)} \cdot (1 - \frac{1}{6}M^2), \quad (26)$$

where,

$$M = \frac{2s(y-2y')}{s^2 - 4y'(y-y')}.$$

As an additional example we shall take the following:

$$s = 100, y = 0, y' = -0.7,$$

so that the ends of the tape are at the same height. In this case the equations become:

$$x = s \left(1 - \frac{1}{6}M^2 - \frac{11}{120}M^4 - \dots \right) \quad (27)$$

$$M = -\frac{4y's}{s^2 + 4y'^2}$$

and the result is:

$$\begin{array}{rcl}
 \text{1st term} & = & 100 \\
 -2\text{nd} \quad " & = & -0.013061538 \dots \\
 -3\text{rd} \quad " & = & -0.00000562993 \dots \\
 \hline
 & & 99.986932832 \dots \\
 \hline
 \end{array}$$

We may check this result by computing the same example by means of the well-known expression:

$$s - x = \frac{8}{3} \frac{y'^2}{s} + \frac{32}{15} \frac{y'^4}{s^3},$$

by which we find:

$$x = 99.986932825 \dots,$$

the omitted terms in the two expansions only affecting the eighth decimal place.

4. It may be well at this point to compare the precision to be attained by the above method of determining the projection of the tape length on a horizontal plane, with that of the usual method in which the tension applied to the tape is measured by means of a spring balance or weights, and x is found in terms of the tension and the weight and length of the tape. This may be effected by finding the error in x due to an error in the observed quantity in each method. Thus, from (27) we have,

$$\begin{aligned}
 \frac{dx}{dy'} &= \frac{dx}{dM} \frac{dM}{dy'} = -\frac{1}{3} s M \frac{4s(s^2 - 4y'^2)}{(s^2 + 4y'^2)^2} \\
 &= -\frac{16}{3} y' s^3 \frac{s^2 - 4y'^2}{(s^2 + 4y'^2)^3}.
 \end{aligned}$$

Then, applying this to the last example: $s = 100$, $y = 0$, $y' = -0.7$, we find:

$$\frac{dx}{dy'} = 0.037304 \dots;$$

so that, if $dy' = 0.005$, $dx = 0.0001865 \dots$

In order to compare this result with that of an error in reading a spring balance we take the well-known equations:

$$\begin{aligned}
 s - x &= \frac{s}{24} \left(\frac{W}{T} \right)^2 \\
 s - x &= \frac{8}{3} \frac{y'^2}{s}
 \end{aligned} \tag{28}$$

and equating them, we find,

$$T = \frac{Ws}{8y'}.$$

Also,

$$\frac{dT}{dx} = \frac{sW^2}{48T(s-x)^2} = \frac{12T^2}{sW^2}.$$

Then, assuming $W=1.2$ pounds, and $dx=0.0001865$, we find:

$$T=21.42857 \text{ pounds,}$$

$$dT=0.15294 \text{ pounds.}$$

It appears then that, with the assumed data, an error of 0.005 ft. in measuring y' will cause the same error in x as an error of 0.153 pound in reading a spring balance.

5. We proceed next to determine the extension of the tape due to the tension to which it is subjected in making a measurement. Returning to equation (6) and denoting by

- s_0 the total length AB of the tape,
- y the ordinate of any point C on the tape,
- s the length AC , and
- T the tension at the point C ,

we have

$$T=w(y+a_1).$$

Then if the same construction be made about the point C as about B , viz., a tangent to the curve be drawn at C having the length s , and also a vertical line terminating in OG , and the point of intersection with OG be joined to the extremity of the tangent, the triangle thus formed will give the relation:

$$(a_1+y)^2=s^2+a_1^2+2sa_1 \cos \theta_1.$$

If also,

e denote the extension of a unit of length of the tape due to unit tension, and

E the total extension of the tape,

we have,

$$\begin{aligned} eTds &= ew(a_1+y)ds \\ &= ew\sqrt{(s^2+2sa_1 \cos \theta_1+a_1^2)}ds; \end{aligned}$$

also

$$E=ew \int_0^{s_0} (s^2+2sa_1 \cos \theta_1+a_1^2)ds.$$

To integrate this we substitute,

$$s+a_1 \cos \theta_1 = z,$$

so that,

$$ds=dz$$

and

$$\begin{aligned} s^2+2sa_1 \cos \theta_1+a_1^2 \cos^2 \theta_1 &= z^2 \\ s^2+2sa_1 \cos \theta_1+a_1^2 &= z^2-a_1^2 \cos^2 \theta_1+a_1^2 \\ &= z^2+a_1^2 \sin^2 \theta_1 \\ &= z^2+b^2 \text{ (assume).} \end{aligned}$$

Then as,

$$\int (z^2+b^2)dz = \frac{z\sqrt{(z^2+b^2)}}{2} + \frac{b^2}{2} \log \left\{ z + \sqrt{(z^2+b^2)} \right\},$$

we have, on introducing the limits and omitting the subscript of s_0 ,

$$E = \frac{ew}{2} \left\{ (s + a_1 \cos \theta_1) \sqrt{(s^2 + 2sa_1 \cos \theta_1 + a_1^2) - a_1^2 \cos \theta_1} \right. \\ \left. + a_1^2 \sin^2 \theta_1 \log \frac{s + a_1 \cos \theta_1 + \sqrt{(s^2 + 2sa_1 \cos \theta_1 + a_1^2)}}{a_1 \cos \theta_1 + a_1} \right\}. \quad (29)$$

This may be simplified by substituting,

$$\cos \theta_1 = \frac{y - Ms}{s - My},$$

and the value of a_1 given by (17), which becomes,

$$a_1 = \frac{s - My}{2M};$$

so that we have after reduction,

$$E = \frac{ew}{2} \left\{ \frac{s^2 + y^2}{2M} + \frac{(s^2 - y^2)}{4M^2} (1 - M^2) \log m \right\} \\ = \frac{ew}{2} \left\{ \frac{s^2 + y^2}{2M} + (s^2 - y^2) \frac{1 - M^2}{4M^2} \cdot 2(M + \frac{1}{3}M^3 + \frac{1}{5}M^5 + \dots) \right\} \\ = \frac{ew}{2} \left\{ \frac{s^2}{M} - (s + y)(s - y) \left(\frac{1}{3}M + \frac{1}{15}M^3 + \dots \right) \right\} \quad (30)$$

Applying now equation (30) to the following three examples:

$$s = 100, y = 50, y' = 24.3;$$

$$s = 100, y = 10, y' = 4.3;$$

$$s = 100, y = 0, y' = -0.7;$$

and assuming, $w = 0.012$, $e = 0.00000911$, we find the following values for E , respectively:

$$0.0146398 \dots,$$

$$0.0193249 \dots,$$

$$0.0195202 \dots;$$

the values of the second term of (30) being, respectively:

$$0.0000051003 \dots$$

$$0.0000051003 \dots$$

$$0.0000051003 \dots$$

As each of these values amounts to only about 5 parts in 100000000, that term and the following may safely be neglected, and equation (30) written in the simple form:

$$E = \frac{ew}{2} \frac{s^2}{M}. \quad (31)$$

6. The corrections for change of length of the tape due to tension and temperature may be applied directly to s before computing x ; a simpler method, however, is to compute x , using the standard length of the tape, and then apply a correction to the value thus found, due to

tension and temperature. The relation between small changes in x and s may be found as follows: Taking log's. in (26) and differentiating, we have,

$$\log x = \frac{1}{2} \log (s^2 - y^2) + \log (1 - \frac{1}{6} M^2),$$

$$= \frac{1}{2} \log (s^2 - y^2) - \frac{1}{6} M^2 - ,$$

and

$$\frac{1}{x} \frac{dx}{ds} = \frac{1}{2} \frac{2s}{s^2 - y^2} - \frac{1}{3} M \frac{dM}{ds} ,$$

or

$$\frac{dx}{ds} = \frac{sx}{s^2 - y^2} - \frac{x}{3} M \frac{dM}{ds} .$$

Also

$$M = 2(y - 2y') \frac{s}{s^2 - 4y'(y - y')}$$

and

$$\frac{dM}{ds} = 2(y - 2y') \frac{s^2 - 4y'(y - y') - 2s^2}{\{s^2 - 4y'(y - y')\}^2}$$

$$= -\frac{M}{3} \cdot \frac{s^2 + 4y'(y - y')}{s^2 - 4y'(y - y')} .$$

Therefore,

$$\frac{dx}{ds} = \frac{sx}{s^2 - y^2} + \frac{x}{3} \frac{M^2}{s} \cdot \frac{s^2 + 4y'(y - y')}{s^2 - 4y'(y - y')} .$$

Then, substituting the value of x and omitting powers of M above the second, this becomes:

$$\frac{dx}{ds} = \frac{s}{\sqrt{s^2 - y^2}} (1 - \frac{1}{6} M^2) + \frac{M^2}{3} \frac{\sqrt{s^2 - y^2}}{s} \cdot \frac{s^2 + 4y'(y - y')}{s^2 - 4y'(y - y')} \quad (32)$$

If we now apply this equation to the above three examples, assuming $ds = 0.03$, the terms containing M are found to be as follows, respectively:

0.00001203...

0.00000410...

0.00000392...;

which shows that these terms are not likely to amount to one part in 10000000, and may therefore be neglected. We may therefore, without material error, write (32) in the simple form:

$$\frac{dx}{ds} = \frac{s}{\sqrt{(s+y)(s-y)}} \quad (33)$$

The precision of this method of correcting for increase of length of the tape, in which the change in s and the resulting change in x are treated as infinitesimals, may be tested numerically by applying the correction dx for the first example just considered, viz.,

$$dx = 0.0346531 \dots ,$$

to the value of x found above for the same data:

$$x' = 86.5824180 \dots$$

giving the corrected value:

$$x = 86.6170711 \dots ;$$

and then computing x directly from the data:

$$s = 100.03, y = 50, y' = 24.3;$$

whence we find $x = 86.6170694 \dots$;

which agrees satisfactorily with the former value.

The correction for change of temperature is found by the expression:

$$as(t - t_0),$$

in which,

a denotes the coefficient of expansion of the tape,

t the temperature of the tape during a measurement, and

t_0 the temperature at which its length is standard when subjected

to no tension.

7. The complete equation for x may therefore be written:

$$x = \sqrt{(s+y)(s-y)} \cdot \left(1 - \frac{1}{6}M^2\right) + \left\{ \frac{ew}{2} \cdot \frac{s^2}{M} + as(t - t_0) \right\} \frac{s}{\sqrt{(s+y)(s-y)}}; \quad (34)$$

in which

$$M = \frac{2s(y - 2y')}{s^2 - 4y'(y - y')}.$$

A table is appended to this paper, giving the value of M for values of y up to 10 feet, the length of the tape being 100 feet, the arguments of the table being y and $\frac{1}{2}y - y'$, the latter quantity being the approximate value of the middle ordinate contained between the curve and its chord.

The following examples give a form for the field record and show the application of equation (34).

From stake to stake	Length of tape	Rod readings				Ther. F.
		Back stake	Forward stake	Middle of tape		
				Inst.	Tape	
27-28	100	6.134	1.271	7.427	3.141	72° .2
35-36	100	0.747	1.985	4.609	2.118	73° .1

To find y and $\frac{1}{2}y - y'$, let

A and B denote readings of rod on lower and higher stakes, respectively,

C the reading of rod when held at middle of tape, and

D the reading of the tape on the rod.

$$\text{Then} \quad y = A - B \quad (35)$$

$$y' = A - (C - D) = D + A - C$$

$$\frac{1}{2}y - y' = \frac{A - B}{2} + C - A - D \quad (36)$$

The constants of the tape may be assumed to be:

$$e = 0.00000911$$

$$w = 0.012$$

$$a = 0.0000065$$

$$t_0 = 62^\circ \text{ F.}$$

Example 1.

$y = 4.863$	$\frac{1}{2}y - y' = 0.583$
$s + y = 104.863$	
$s - y = 95.137$	
$\log (s + y) = 2.0206223$	$\log (\frac{1}{2}ews^2) = \bar{4}.73767$
" $(s - y) = 1.9783495$	" $M = \bar{2}.36870$
3.9989718	" $0.023387 = \bar{2}.36897$
" $1st\ t = 1.9994859$	" $(as) = \bar{4}.81291$
" $\frac{1}{6} = \bar{1}.22185$	" $(t - t_0) = \bar{1}.00860$
" $M^2 = \bar{4}.73739$	" $0.006629 = \bar{3}.82151$
" $2nd\ t = \bar{3}.95873$	" $0.030016 = \bar{2}.47735$
$1st\ t = 99.88170$	" $s = 2$
$2nd\ t = 0.00909$	" $\sqrt{s^2 - y^2} = 1.99949$
99.87261	" $0.03005 = \bar{2}.47786$
$Corr. = 0.03005$	
$x = 99.90266$	

Example 2.

$y = 0.238$	$\frac{1}{2}y - y' = 0.625$
$s + y = 100.238$	
$s - y = 99.762$	
$\log (s + y) = 2.0010324$	$\log (\frac{1}{2}ews^2) = \bar{4}.73767$
" $(s - y) = 1.9989651$	" $M = \bar{2}.39787$
3.9999975	" $0.021867 = 1.33980$
" $1st\ t = 1.9999987$	" $as = \bar{4}.81291$
" $\frac{1}{6} = \bar{1}.22185$	" $(t - t_0) = \bar{1}.04532$
" $M^2 = \bar{4}.79574$	" $0.007215 = \bar{3}.85823$

" 2nd t	$= \overline{2.01759}$	" 0.029082	$= \overline{2.46362}$
1st t	$= \overline{99.99970}$	" s	$= 2$
2nd t	$= \overline{0.01041}$	" $\sqrt{(s^2 - y^2)}$	$= 2$
	$\overline{99.98929}$	" 0.02908	$= \overline{2.46362}$
Corr.	$= \overline{0.02908}$		
x	$= \overline{100.01837}$		

8. The constants of the tape e and w may be obtained in the following manner: The tape is stretched as in making a measurement, with its ends resting on firm supports which are at the same elevation. While one end is held fast the other end is free to move as different tensions are applied to it in a horizontal direction. The tensions are best applied by means of weights attached by a cord to the end of the tape, the cord passing over a frictionless pulley. Two widely different tensions are then applied to the tape and the horizontal movement of the free end measured accurately, and also the ordinates y' and y'' of the middle point of the tape corresponding to the two tensions. We have then the following equations:

$$x_1 = s_1 \left(1 - \frac{1}{6} M_1^2\right), \text{ where } M_1 = \frac{4y's}{s^2 - 4y'^2};$$

$$x_2 = s_2 \left(1 - \frac{1}{6} M_2^2\right), \text{ where } M_2 = \frac{4y''s}{s^2 - 4y''^2}.$$

In computing M_1 and M_2 the standard length s of the tape may be used. These equations give:

$$s_1 = x_1 + \frac{1}{6}s M_1^2, \quad s_2 = x_2 + \frac{1}{6}s M_2^2;$$

$$\text{whence} \quad s_2 - s_1 = (x_2 - x_1) + \frac{1}{6}s(M_2^2 - M_1^2). \quad (37)$$

$$\text{Then,} \quad e = \frac{s_2 - s_1}{s(T_2 - T_1)}. \quad (38)$$

To find w , we have by transposing one of (28):

$$W^2 = \frac{24T^2(s-x)}{s}. \quad (39)$$

As an example, let us assume that tensions of 10 and 50 pounds are applied in turn to a tape, and it is found that,

$$\begin{aligned} x_2 - x_1 &= 0.0939, \\ y' &= 1.500, \\ y'' &= 0.300. \end{aligned}$$

We then find:

$$\begin{aligned} s_2 - s_1 &= 0.0939 \\ &+ 0.0023998 \\ &- 0.059892 \\ &\hline &= 0.036408 \end{aligned}$$

$$\therefore e = \frac{0.0364}{100 \times 40} = 0.0000091.$$

In finding w we have the two values of $s - x$:

$$s_1 - x_1 = \frac{1}{6}sM_1^2, \quad s_2 - x_2 = \frac{1}{6}sM_2^2.$$

These give,

$$W = 1.19892$$

$$W = 1.19974$$

$$\text{Mean} = 1.19933$$

$$\therefore w = 0.01199$$

9. The method of reducing base line measurements developed above, and embodied in equation (34), may readily be applied to measurements in which the tension is found by means of a spring balance or weights, and the correction to reduce to the chord length of the tape determined in terms of that tension and the length and weight of the tape. To effect this it is merely necessary to express the quantity M in terms of s , y , w , and T — T denoting the tension at the forward end of the tape. Thus from (17),

$$\cos \theta_1 = \frac{y}{s} - \frac{s^2 - y^2}{2a_1s},$$

which, substituted in (22), gives on reduction,

$$M = \frac{s}{2a_1 + y}. \quad (40)$$

Also, by (6),

$$a_1 = \frac{T}{w} - y;$$

whence,

$$M = \frac{ws}{2T - wy} = \frac{W}{2T - wy} \quad (41)$$

Also

$$T = \frac{w}{2} \left(y + \frac{s}{M} \right) \quad (42)$$

Thus M may be found by (41) and used in (34) to determine x , the remainder of the computation being unchanged.

If the above value of M be substituted in (31) it becomes

$$E = es \left(T - \frac{1}{2}wy \right); \quad (43)$$

then as

$$wy = T_2 - T_1,$$

we have
$$E = es \frac{T_1 + T_2}{2} \quad (44)$$

from which it appears that,

$$\frac{1}{2}(T_1 + T_2) \text{ or } T - \frac{1}{2}wy$$

expresses, very closely, the mean value of the tension throughout the length of the tape.

If T is given, and also s , y , and w , we may find y' , if required, as follows: we have,

$$\cos \theta_1 = \frac{y}{s} - \frac{s^2 - y^2}{2a_1 s} ;$$

also by (17),
$$(a_1 + y')^2 = a_1^2 + \left(\frac{s}{2}\right)^2 + 2a_1 \frac{s}{2} \cos \theta_1 ;$$

or, substituting $\cos \theta_1$,

$$(a_1 + y')^2 = \left(a_1 + \frac{y}{2}\right)^2 - \frac{s^2 - y^2}{4} \quad (45)$$

a_1 being given by the equation:

$$a_1 = \frac{T}{w} - y.$$

10. The following are some of the advantages of the method of base line measurement above described:

Only four men are required for the measurement of a base;

No special training is necessary on the part of the two tape men, only good eye-sight;

No divided scales need be read at the ends of the tape, thus diminishing the chance of error;

The formulæ which have been derived are applicable to practically any slope.

With reference to this last point, however, it must be noted that the steepness of the slope on which this, or any, method may be used depends upon the precision of the leveling operations. Thus, by a process similar to that of p. 300 we find—omitting terms containing M :

$$\frac{dx}{dy} = - \frac{y}{\sqrt{s^2 - y^2}} ;$$

so that, making

$$\frac{dx}{dy} = 0.1, \text{ and } s = 100,$$

we find

$$y = 9.95;$$

which is the value of y for which the error in x is equal to one-tenth of the error in y .

The following table gives the value of M in terms of y and $\frac{1}{2}y - y'$, from which any required value may be interpolated, for values of y up to 10 feet, assuming $s = 100$ feet.

TABLE
1y-y'

y	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	Diff.
0.0	0.012000	0.015999	0.019998	0.023997	0.027995	0.031992	0.035988	0.039984	0.043979	0.047972	3993-99
1.0	0.012000	0.015999	0.019998	0.023997	0.027995	0.031992	0.035988	0.039984	0.043979	0.047972	3993-99
2.0	0.012000	0.015999	0.019998	0.023997	0.027995	0.031992	0.035988	0.039984	0.043979	0.047972	3994-99
3.0	0.012000	0.015999	0.019998	0.023997	0.027995	0.031992	0.035989	0.039984	0.043979	0.047973	3994-99
4.0	0.012000	0.015999	0.019998	0.023997	0.027995	0.031992	0.035989	0.039985	0.043979	0.047973	3994-99
5.0	0.012000	0.015999	0.019999	0.023997	0.027995	0.031993	0.035989	0.039985	0.043980	0.047974	3994-99
6.0	0.012000	0.016000	0.019999	0.023997	0.027996	0.031993	0.035990	0.039985	0.043980	0.047974	3994-00
7.0	0.012000	0.016000	0.019999	0.023998	0.027996	0.031993	0.035990	0.039986	0.043981	0.047975	3994-00
8.0	0.012000	0.016000	0.019999	0.023998	0.027996	0.031994	0.035991	0.039987	0.043982	0.047975	3994-00
9.0	0.012000	0.016000	0.019999	0.023998	0.027997	0.031994	0.035991	0.039987	0.043982	0.047976	3994-00
1.0	0.012001	0.016001	0.020000	0.023999	0.027997	0.031995	0.035992	0.039988	0.043983	0.047977	3994-00
2.0	0.012001	0.016001	0.020000	0.023999	0.027998	0.031996	0.035993	0.039989	0.043984	0.047978	3994-00
3.0	0.012001	0.016001	0.020001	0.024000	0.027999	0.031996	0.035994	0.039990	0.043985	0.047979	3994-00
4.0	0.012002	0.016002	0.020001	0.024001	0.027999	0.031997	0.035994	0.039991	0.043986	0.047980	3994-00
5.0	0.012002	0.016002	0.020002	0.024001	0.028000	0.031998	0.035995	0.039992	0.043987	0.047982	3995-00
6.0	0.012002	0.016002	0.020003	0.024002	0.028001	0.031999	0.035996	0.039993	0.043989	0.047983	3994-00
7.0	0.012003	0.016003	0.020003	0.024003	0.028002	0.032000	0.035999	0.039994	0.043990	0.047985	3995-00
8.0	0.012003	0.016004	0.020004	0.024003	0.028003	0.032001	0.035999	0.039996	0.043991	0.047986	3995-01
9.0	0.012004	0.016005	0.020005	0.024004	0.028004	0.032002	0.036000	0.039997	0.043993	0.047988	3995-01
2.0	0.012004	0.016005	0.020005	0.024005	0.028005	0.032003	0.036001	0.039998	0.043995	0.047990	3995-01
3.0	0.012005	0.016006	0.020006	0.024006	0.028006	0.032005	0.036003	0.040000	0.043996	0.047992	3996-01
4.0	0.012005	0.016007	0.020007	0.024007	0.028007	0.032006	0.036004	0.040002	0.043998	0.047994	3996-01
5.0	0.012006	0.016007	0.020008	0.024008	0.028008	0.032007	0.036005	0.040003	0.044000	0.047996	3996-02
6.0	0.012006	0.016008	0.020009	0.024009	0.028009	0.032009	0.036007	0.040005	0.044002	0.047998	3996-01
7.0	0.012006	0.016008	0.020010	0.024010	0.028011	0.032010	0.036009	0.040007	0.044004	0.048000	3996-02
8.0	0.012007	0.016009	0.020011	0.024012	0.028012	0.032012	0.036011	0.040009	0.044006	0.048002	3996-02
9.0	0.012008	0.016010	0.020012	0.024013	0.028013	0.032013	0.036013	0.040011	0.044008	0.048005	3997-02
2.0	0.012008	0.016011	0.020013	0.024014	0.028015	0.032017	0.036015	0.040015	0.044013	0.048007	3997-03
3.0	0.012009	0.016012	0.020014	0.024015	0.028016	0.032017	0.036017	0.040015	0.044013	0.048010	3997-03
4.0	0.012010	0.016012	0.020015	0.024017	0.028018	0.032019	0.036019	0.040018	0.044016	0.048013	3997-02
5.0	0.012010	0.016013	0.020016	0.024018	0.028020	0.032021	0.036021	0.040020	0.044018	0.048016	3998-03
6.0	0.012011	0.016014	0.020017	0.024020	0.028021	0.032023	0.036023	0.040022	0.044021	0.048018	3997-03
7.0	0.012012	0.016015	0.020018	0.024021	0.028023	0.032025	0.036025	0.040025	0.044024	0.048022	3998-03
8.0	0.012013	0.016016	0.020020	0.024023	0.028025	0.032027	0.036028	0.040028	0.044027	0.048025	3998-03
9.0	0.012013	0.016017	0.020021	0.024024	0.028027	0.032029	0.036030	0.040030	0.044030	0.048028	3998-04

TABLE—Continued

y	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	Diff.
3.5	0.012014	0.016019	0.020023	0.024026	0.028029	0.032031	0.036032	0.040033	0.044033	0.048031	3998-05
6	0.012015	0.016020	0.020024	0.024028	0.028031	0.032033	0.036035	0.040036	0.044036	0.048035	3999-05
7	0.012016	0.016021	0.020025	0.024029	0.028033	0.032036	0.036038	0.040039	0.044039	0.048038	3999-05
8	0.012017	0.016022	0.020027	0.024031	0.028035	0.032038	0.036040	0.040042	0.044042	0.048042	4000-05
9	0.012018	0.016023	0.020028	0.024033	0.028037	0.032041	0.036043	0.040045	0.044046	0.048045	3999-05
4.0	0.012019	0.016025	0.020030	0.024035	0.028039	0.032043	0.036046	0.040048	0.044049	0.048049	4000-06
1	0.012020	0.016026	0.020032	0.024037	0.028042	0.032046	0.036049	0.040051	0.044053	0.048053	4000-06
2	0.012021	0.016027	0.020033	0.024039	0.028044	0.032048	0.036052	0.040055	0.044056	0.048057	4000-06
3	0.012022	0.016029	0.020035	0.024041	0.028046	0.032051	0.036055	0.040058	0.044060	0.048061	4001-07
4	0.012023	0.016030	0.020037	0.024043	0.028049	0.032054	0.036058	0.040062	0.044064	0.048065	4001-07
5	0.012024	0.016031	0.020039	0.024045	0.028051	0.032057	0.036061	0.040065	0.044068	0.048070	4002-07
6	0.012025	0.016033	0.020040	0.024047	0.028054	0.032060	0.036065	0.040069	0.044072	0.048074	4002-08
7	0.012026	0.016034	0.020042	0.024050	0.028056	0.032063	0.036068	0.040072	0.044076	0.048079	4003-08
8	0.012027	0.016036	0.020044	0.024052	0.028059	0.032066	0.036071	0.040076	0.044080	0.048083	4003-09
9	0.012028	0.016037	0.020046	0.024054	0.028062	0.032069	0.036075	0.040080	0.044085	0.048088	4003-09
5.0	0.012030	0.016039	0.020048	0.024051	0.028065	0.032072	0.036079	0.040084	0.044089	0.048093	4004-09
1	0.012031	0.016041	0.020050	0.024059	0.028068	0.032075	0.036082	0.040088	0.044093	0.048097	4004-10
2	0.012032	0.016042	0.020052	0.024062	0.028070	0.032079	0.036086	0.040092	0.044098	0.048102	4004-10
3	0.012033	0.016044	0.020054	0.024064	0.028073	0.032082	0.036090	0.040097	0.044103	0.048107	4004-11
4	0.012035	0.016046	0.020056	0.024067	0.028076	0.032085	0.036094	0.040101	0.044107	0.048113	4006-11
5	0.012036	0.016048	0.020059	0.024069	0.028079	0.032089	0.036097	0.040105	0.044112	0.048118	4006-12
6	0.012037	0.016049	0.020061	0.024072	0.028083	0.032092	0.036102	0.040110	0.044117	0.048123	4006-12
7	0.012039	0.016051	0.020063	0.024075	0.028086	0.032096	0.036106	0.040114	0.044122	0.048129	4007-12
8	0.012040	0.016053	0.020065	0.024078	0.028089	0.032100	0.036110	0.040119	0.044127	0.048134	4007-13
9	0.012041	0.016055	0.020068	0.024080	0.028092	0.032104	0.036114	0.040124	0.044132	0.048140	4008-14
6.0	0.012043	0.016057	0.020070	0.024083	0.028096	0.032107	0.036118	0.040128	0.044138	0.048146	4008-14
1	0.012044	0.016059	0.020073	0.024086	0.028099	0.032111	0.036123	0.040133	0.044143	0.048151	4008-15
2	0.012046	0.016061	0.020075	0.024089	0.028103	0.032115	0.036127	0.040138	0.044148	0.048157	4009-15
3	0.012047	0.016063	0.020078	0.024092	0.028106	0.032119	0.036132	0.040143	0.044154	0.048163	4009-16
4	0.012049	0.016065	0.020080	0.024095	0.028110	0.032123	0.036136	0.040148	0.044159	0.048170	4011-16
5	0.012050	0.016067	0.020083	0.024098	0.028113	0.032128	0.036141	0.040154	0.044165	0.048176	4011-17
6	0.012052	0.016069	0.020085	0.024102	0.028117	0.032132	0.036146	0.040159	0.044171	0.048182	4011-17
7	0.012054	0.016071	0.020088	0.024105	0.028121	0.032136	0.036151	0.040164	0.044177	0.048189	4012-17
8	0.012055	0.016073	0.020091	0.024108	0.028125	0.032140	0.036155	0.040170	0.044183	0.048195	4012-18
9	0.012057	0.016076	0.020094	0.024111	0.028128	0.032145	0.036160	0.040175	0.044189	0.048202	4013-19

TABLE—Continued
 $\frac{1}{2}y-y'$

y	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	Diff.
7.0	0.012059	0.016078	0.020096	0.024115	0.028132	0.032149	0.036165	0.040181	0.044195	0.048208	4013-19
1	0.012060	0.016080	0.020099	0.024118	0.028136	0.032154	0.036171	0.040186	0.044201	0.048215	4014-20
2	0.012062	0.016082	0.020102	0.024122	0.028140	0.032158	0.036176	0.040192	0.044208	0.048222	4014-20
3	0.012064	0.016085	0.020105	0.024125	0.028144	0.032163	0.036181	0.040198	0.044214	0.048229	4015-21
4	0.001266	0.016087	0.020108	0.024129	0.028149	0.032168	0.036186	0.040204	0.044221	0.048236	4015-21
5	0.012067	0.016089	0.020111	0.024132	0.028153	0.032173	0.036192	0.040210	0.044227	0.048244	4017-22
6	0.012069	0.016092	0.020114	0.024136	0.028157	0.032178	0.036197	0.040216	0.044234	0.048251	4017-23
7	0.012070	0.016094	0.020117	0.024140	0.028161	0.032183	0.036203	0.040222	0.044241	0.048258	4017-23
8	0.012071	0.016097	0.020120	0.024143	0.028166	0.032188	0.036209	0.040229	0.044248	0.048266	4018-24
9	0.012075	0.016099	0.020124	0.024147	0.028170	0.032193	0.036214	0.040235	0.044255	0.048273	4018-24
8.0	0.012077	0.016102	0.020127	0.024151	0.028175	0.032198	0.036220	0.040241	0.044262	0.048281	4019-25
1	0.012079	0.016105	0.020130	0.024155	0.028179	0.032203	0.036226	0.040248	0.044269	0.048289	4020-26
2	0.012081	0.016107	0.020133	0.024159	0.028184	0.032208	0.036232	0.040255	0.044276	0.048297	4021-26
3	0.012083	0.016110	0.020137	0.020163	0.028189	0.032214	0.036238	0.040261	0.044284	0.048305	4021-27
4	0.012085	0.016113	0.020140	0.024167	0.028193	0.032219	0.036244	0.040268	0.044291	0.048313	4022-28
5	0.012087	0.016115	0.020143	0.024171	0.028198	0.032225	0.036250	0.040275	0.044299	0.048321	4022-28
6	0.012089	0.016118	0.020147	0.024175	0.028203	0.032230	0.036256	0.040282	0.044306	0.048330	4024-29
7	0.012091	0.016121	0.020151	0.024180	0.028208	0.032236	0.036263	0.040289	0.044314	0.048338	4024-30
8	0.012093	0.016124	0.020154	0.024184	0.028213	0.032241	0.036269	0.040296	0.044322	0.048347	4025-31
9	0.012095	0.016127	0.020158	0.024188	0.028218	0.032247	0.036276	0.040303	0.044330	0.048355	4025-32
9.0	0.012097	0.016130	0.020161	0.024192	0.028223	0.032253	0.036282	0.040310	0.044338	0.048364	4026-33
1	0.012100	0.016133	0.020165	0.024197	0.028228	0.032259	0.036289	0.040318	0.044346	0.048372	4027-33
2	0.012102	0.016135	0.020169	0.024201	0.028233	0.032265	0.036295	0.040325	0.044354	0.048382	4028-33
3	0.012104	0.016138	0.020172	0.024206	0.028239	0.032271	0.036302	0.040333	0.044362	0.048391	4029-34
4	0.012106	0.016142	0.020176	0.024210	0.028244	0.032277	0.036309	0.040340	0.044371	0.048400	4029-36
5	0.012109	0.016145	0.020180	0.024215	0.028249	0.032283	0.036316	0.040348	0.044379	0.048490	4030-36
6	0.012111	0.016148	0.020184	0.024220	0.028255	0.032289	0.036323	0.040356	0.044388	0.048418	4030-37
7	0.012114	0.016151	0.020188	0.024224	0.028260	0.032296	0.036330	0.040364	0.044396	0.048428	4032-37
8	0.012116	0.016154	0.020192	0.024229	0.028266	0.032302	0.036337	0.040372	0.044405	0.048437	4032-38
9	0.012118	0.016157	0.020196	0.024234	0.028272	0.032308	0.036344	0.040380	0.044414	0.048447	4033-39
10.0	0.012121	0.016161	0.020200	0.024239	0.028277	0.032315	0.036352	0.040388	0.044423	0.048457	4034-40

CHARTS FOR APPROXIMATE STRESS DETERMINATION IN REINFORCED CONCRETE CHIMNEYS

By PETER GILLESPIE, *Professor of Civil Engineering*,
and W. K. IRWIN, *Research Assistant*

NOTATION

The following symbols among others have been used:

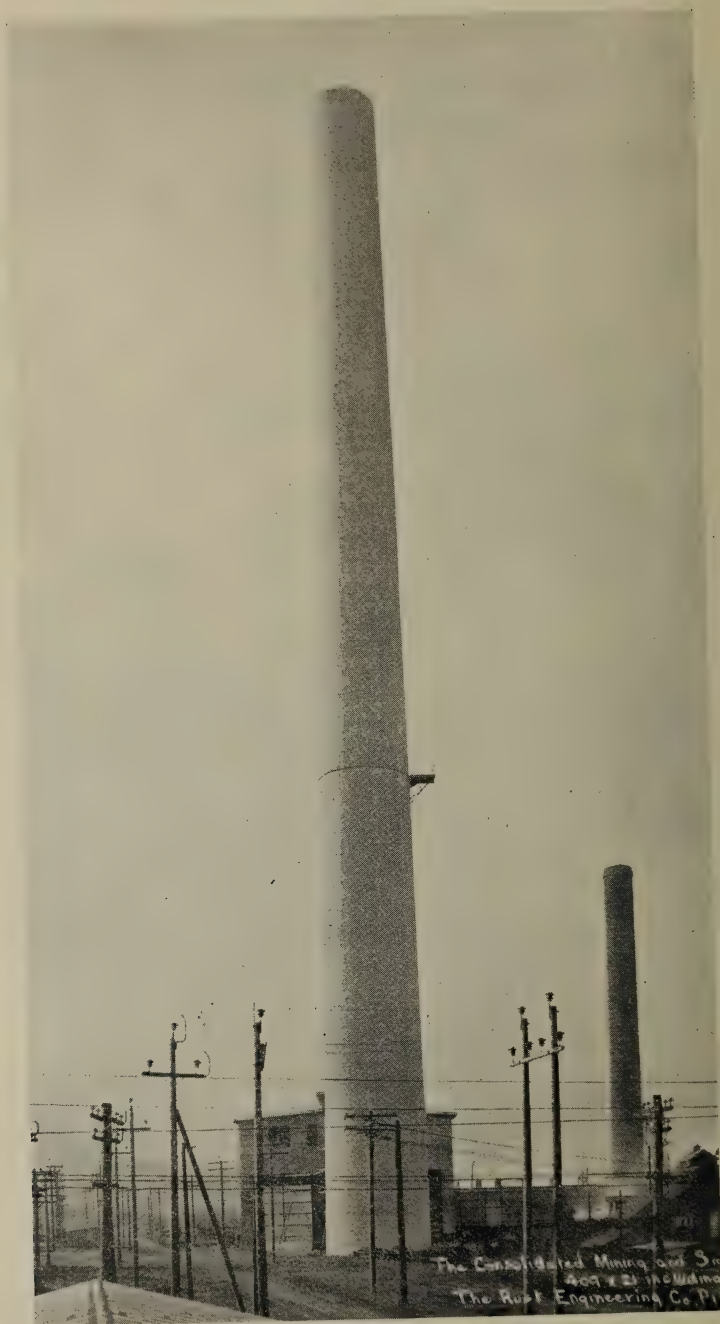
- E_c —Young's Modulus of Elasticity for Concrete.
- E_s —Young's Modulus of Elasticity for Steel.
- f_c —Stress in Concrete at centre of wall due to Weight and Wind only.
- f_s —Tensile Stress in Steel due to Weight and Wind only.
- f_{ct} —Stress in Concrete due to Temperature Rise only.
- f_{st} —Tensile Stress in Steel due to Temperature Rise only.
- f_{c-com} —Stress in Concrete Consequent on a Combination of Temperature Rise with Weight and Wind.
- f_{s-com} —Tensile Stress in Steel Consequent on a Combination of Temperature Rise with Weight and Wind.
- K —Coefficient of Linear Expansion, Steel and Concrete; Fahrenheit Scale.
- n — E_s/E_c here taken as 12.
- p —Ratio of Area of Metal to Gross Area of Section.
- q —A Coefficient in the Equation, $f_s = qf_c$.
- t —Wall Thickness.
- T —Difference in Temperature in Degrees F. between Faces of Shell.
- W —Weight of Chimney above Section under Consideration.
- X —A Coefficient in the Equation, $f_c = \frac{W}{r_m t} X$.

Charts 1 and 2 herewith are intended to be used in the determination of compressive stresses in concrete in chimney shells due to wind and weight, the eccentricity ratio c/r_m being .5 or less. The former is plotted from the equation

$$m = \frac{1 + 2c/r_m}{2\pi(1 + pn)}$$

and the latter from the equation

$$m = \frac{1 - 2c/r_m}{2\pi(1 + pn)}.$$



Frontispiece—Highest Reinforced Concrete Chimney in Canada, Erected at Trail, B.C., for The Consolidated Mining and Smelting Company of Canada by the Rust Engineering Company, Pittsburg, Pa.

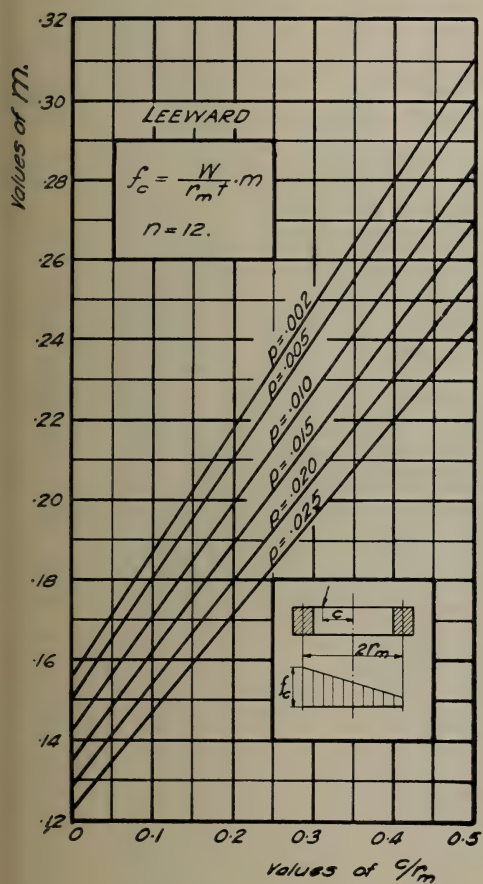


Fig. 1. Use for Determining f_c at Leeward, due to Weight and Wind only when the Eccentricity Ratio is .5 or less.

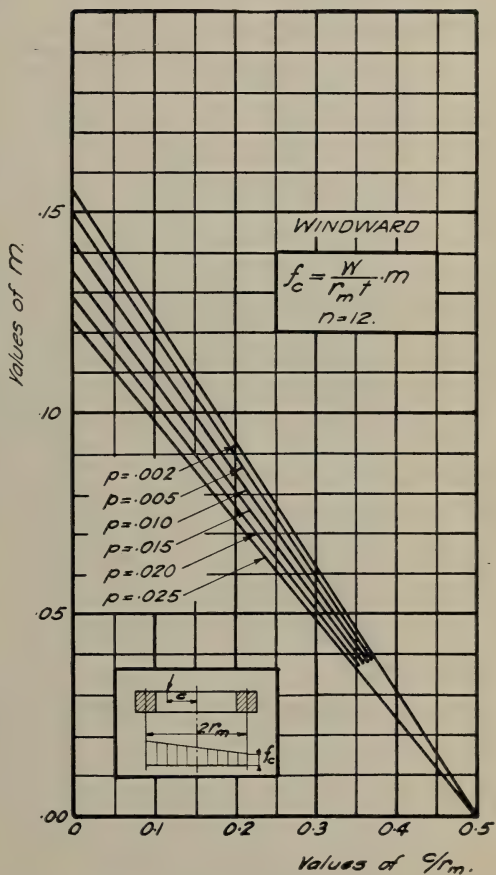


Fig. 2. Use for Determining f_c at Windward, due to Weight and Wind only when the Eccentricity Ratio is .5 or less.

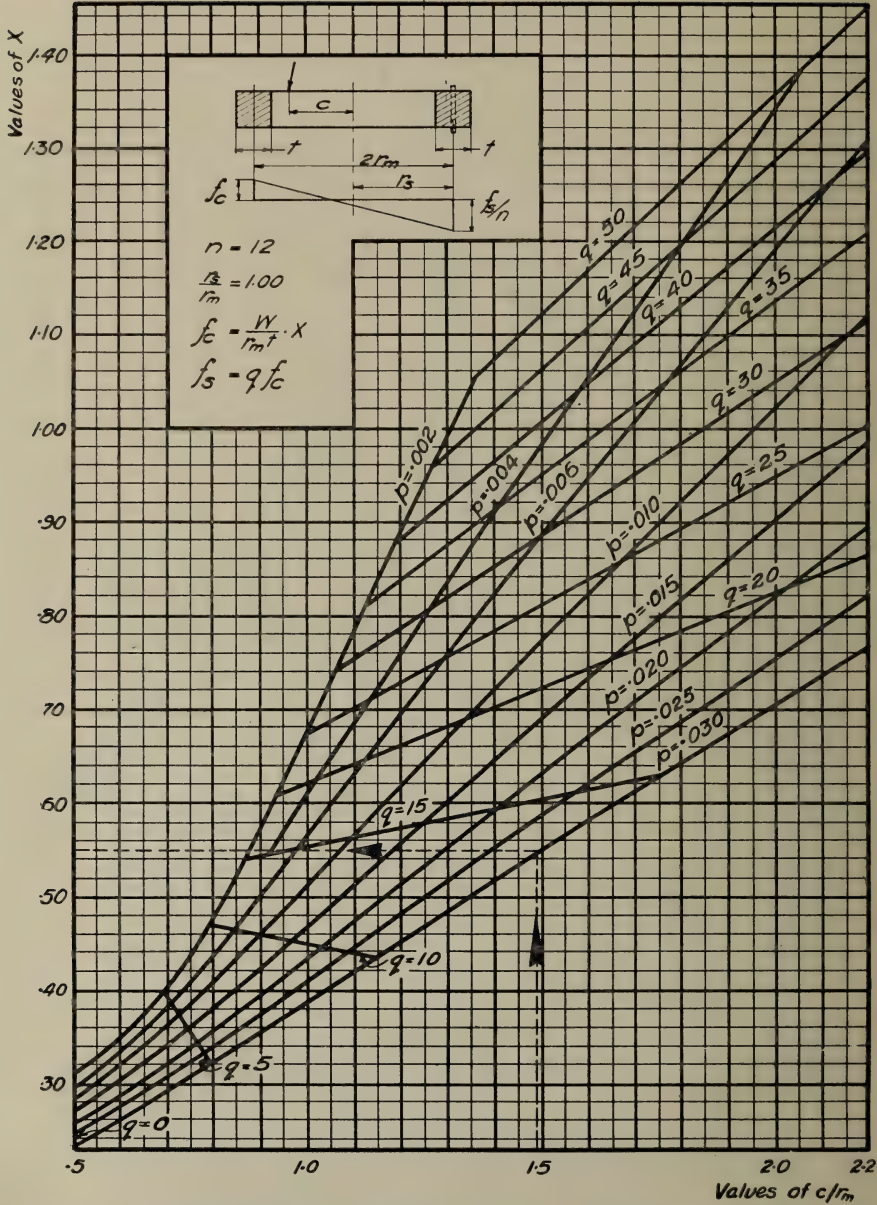


Fig. 3. Use For Determining Stresses Due to Weight and Wind only when the Eccentricity Ratio is .5 or more. Stress f_c occurs at Leeward; Stress f_s occurs at Windward. See Fig. A.

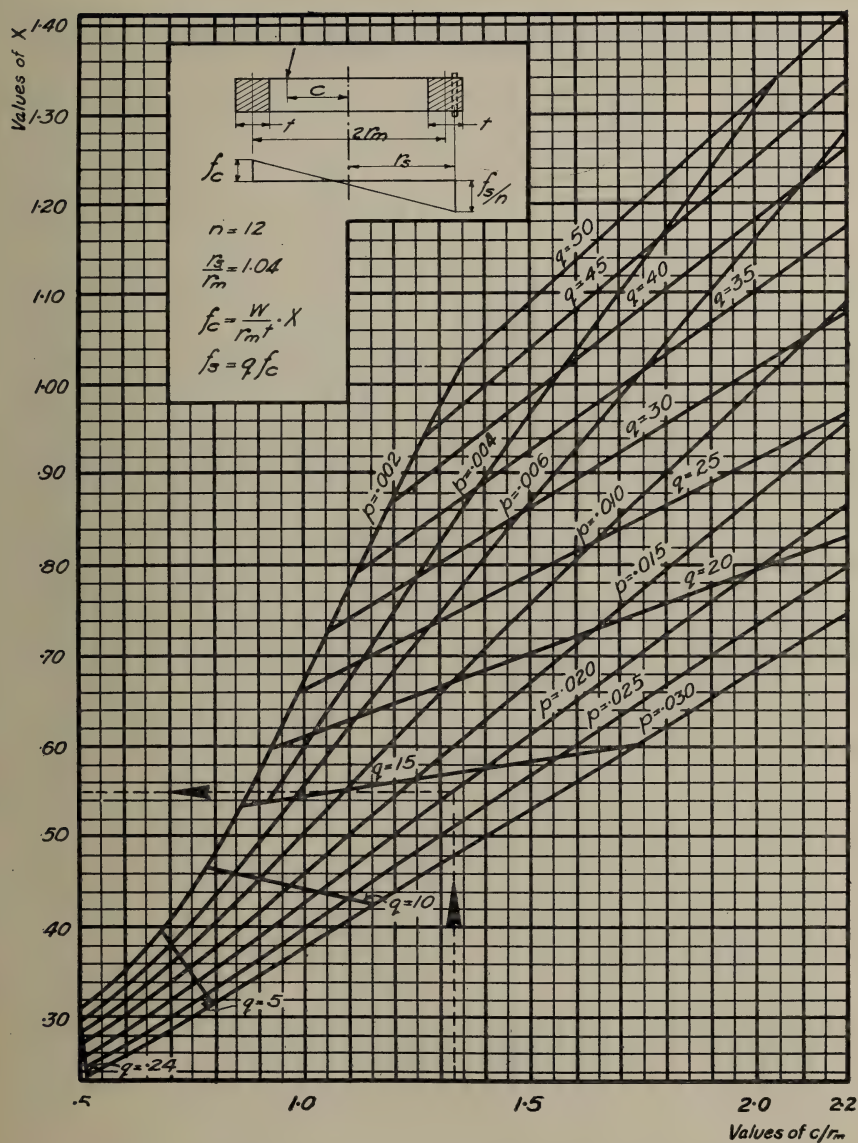


Fig. 4. Use For Determining Stresses Due to Weight and Wind only when the Eccentricity Ratio is .5 or more. Stress f_c occurs at Leeward; Stress f_s occurs at Windward. See Fig. A.

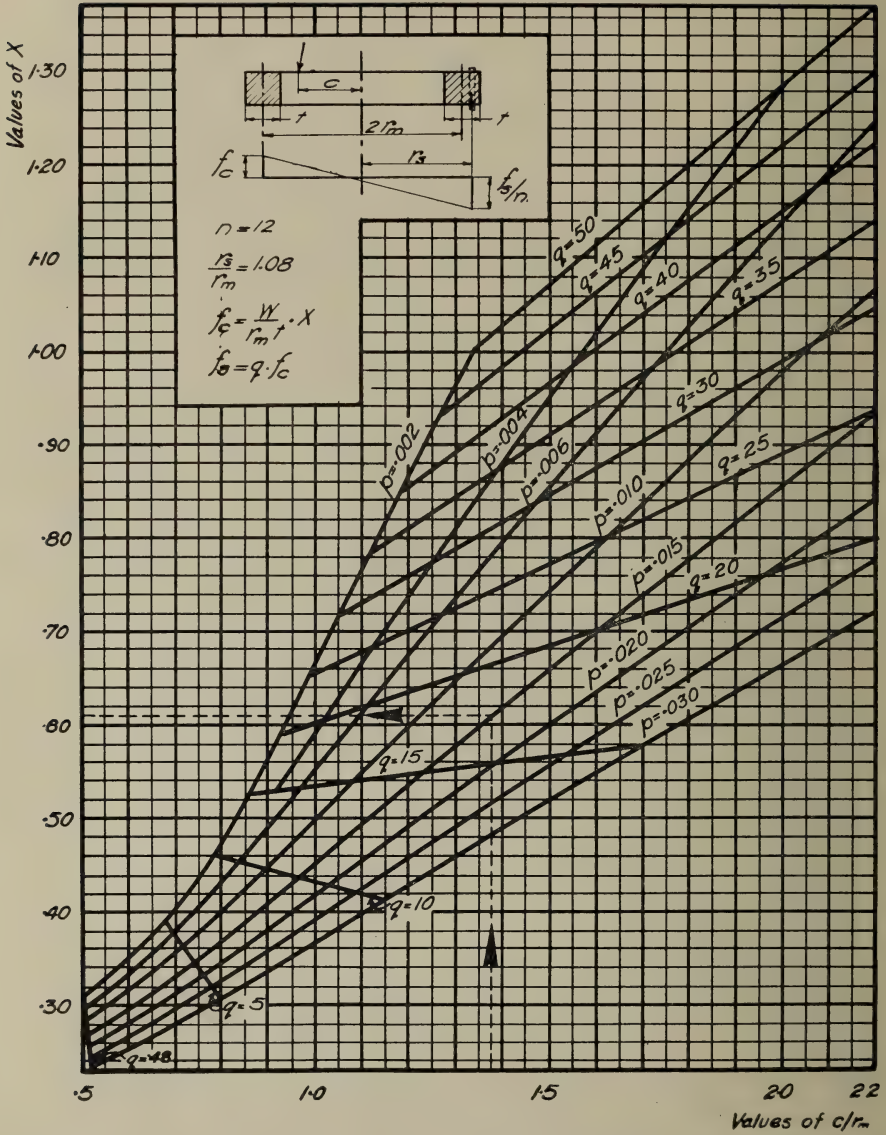


Fig. 5. Use For Determining Stresses Due to Weight and Wind only when the Eccentricity Ratio is .5 or more. Stress f_c occurs at Leeward; Stress f_s occurs at Windward. See Fig. A.

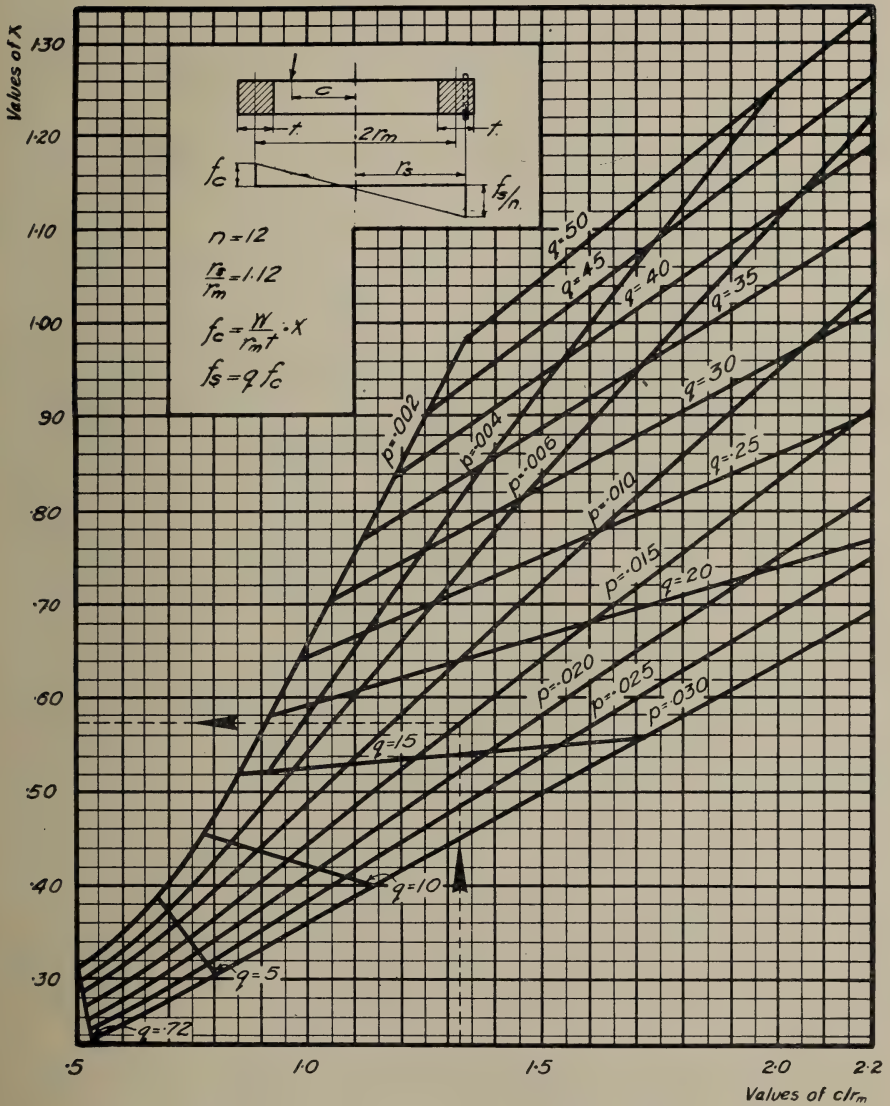


Fig. 6. Use For Determining Stresses Due to Weight and Wind only when the Eccentricity Ratio is .5 or more. Stress f_c occurs at Leeward; Stress f_s occurs at Windward. See Fig. A.

Charts 3 to 6 inclusive are intended for the determination of f_c , the mean compressive stress in concrete on the leeward side and f_s , the tensile stress in steel on the windward side, the eccentricity ratio being .5 or more. See Fig. A. All follow the usually accepted practice in respect to the analysis of stress in reinforced concrete chimneys except that n , the modular ratio, has been taken as 12 instead of 15 and for charts 4 to 6 inclusive, a position of the longitudinal reinforcement other than in the centre of the annulus is provided for.¹

¹Making the usual assumptions, the following, Fig. A, may be established:

$$\frac{c}{r_m} = \frac{2\phi - \sin 2\phi + 2np\pi \left(\frac{r_s}{r_m}\right)^2}{4(\sin\phi - \phi\cos\phi - np\pi\cos\phi)} \quad (a)$$

$$m = \frac{1 - \cos\phi}{2} \quad (b)$$

$$f_c = \frac{W}{r_m t} \frac{\sin^2 \frac{\phi}{2}}{\sin\phi - \phi\cos\phi - np\pi\cos\phi} = \frac{W}{r_m t} X \quad (c)$$

$$f_s = n \left[-1 + \frac{1}{2m} \left(\frac{r_s}{r_m} + 1 \right) \right] f_c = q f_c \quad (d)$$

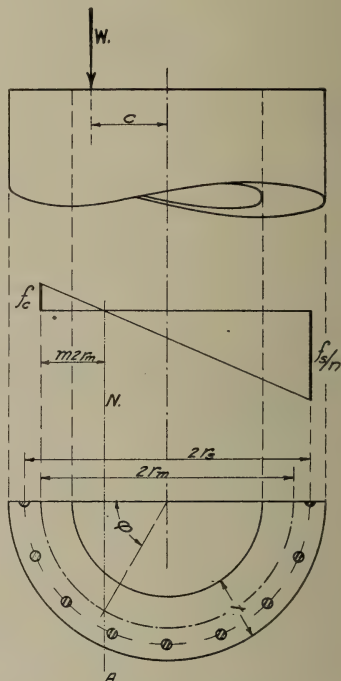


Fig. A

It will be apparent, $\frac{r_s}{r_m}$ and n being assumed constant, that any two of p , $\frac{c}{r_m}$, X and q being known, the other two follow. Figs. 3, 4, 5 and 6 are the graphical equivalent of this fact. See Turneure and Maurer, *Principles of Reinforced Concrete Construction*, third edition, p. 408, and R. Saliger in *Beton und Eisen*, Heft X, 1905, "Spannungen in Schornsteinen mit Kreisringquerschnitt" and Heft III, 1906, "Querschnittsabmessungen von Schornsteinen aus Eisenbeton."

Figs. 8 and 9 are to be used for the determination of temperature stresses primarily circumferential in direction. It can be shown (Fig. 7) that

$$f_{ct/T} = 15np \left[\sqrt{1 + \frac{2(1-k')}{np}} - 1 \right] \text{ and} \quad (1)$$

$$f_{st/T} = 15n \left\{ 1 - k' - np \left[\sqrt{1 + \frac{2(1-k')}{np}} - 1 \right] \right\} \quad (2)$$

where f_{ct} and f_{st} are stresses in concrete and steel respectively due to a rise in temperature only. These equations may be written

$$f_{ct} = yT \text{ and}$$

$$f_{st} = y'T.$$

The two charts result, n being taken as 12.

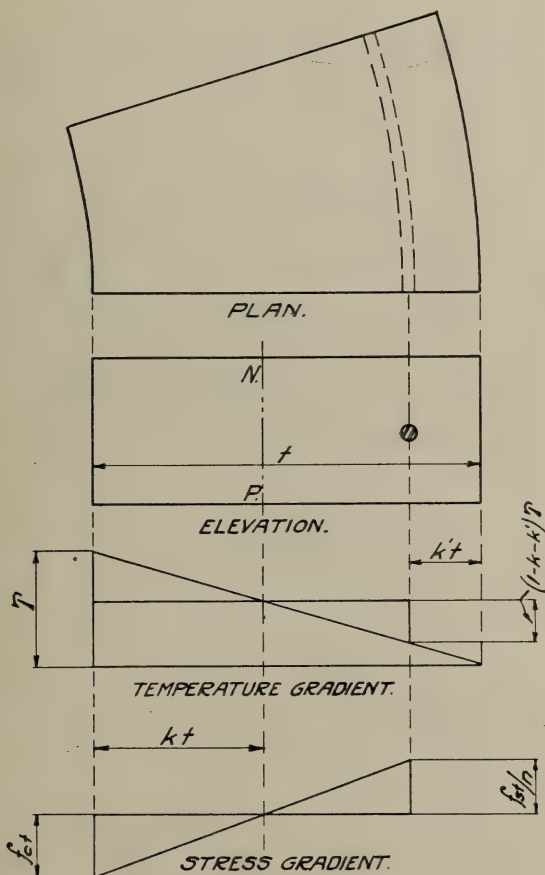


Fig. 7. Temperature Stresses in Relation to Circumferential Reinforcement primarily.

STRESSES DUE TO FLEXURE COMBINED WITH TEMPERATURE²

From Figs. 3, 4, 5 and 6 as stated above, stresses in steel and concrete due to wind and weight only may be estimated. Figs. 11 to 18 inclusive are intended for the determination of stresses resulting from a combination of wind and weight with a temperature rise. Such combined stresses are denoted by f_{c-com} and f_{s-com} in concrete and steel respectively. The

²See "Hohe Schornsteine", by Waldau and Hingerle, Vol. XIII, 3rd edition, *Handbuch für Eisen-Betonbau*.

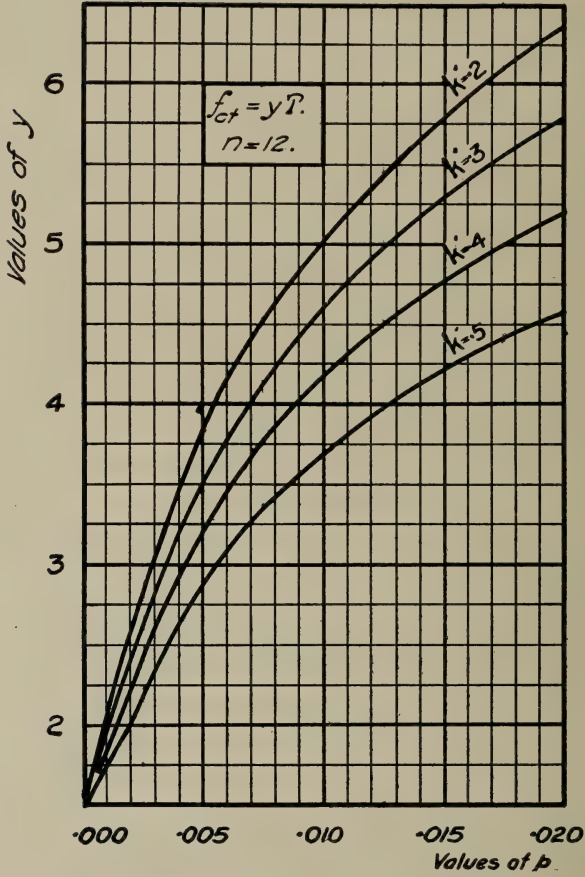


Fig. 8. Use For Determining primarily, Circumferential Stress f_{ct} in Concrete Due to Temperature Rise only.

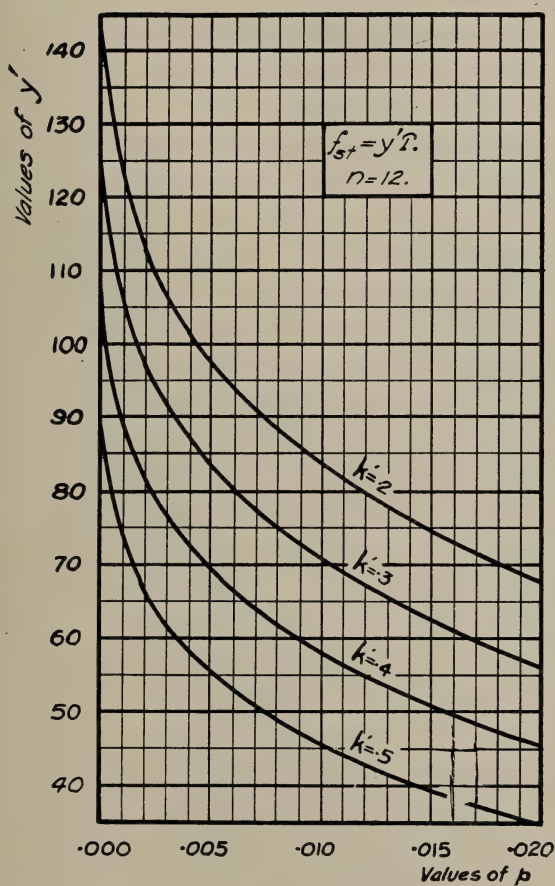


Fig. 9. Use For Determining primarily, Circumferential Stress f_{st} in Steel Due to Temperature Rise only.

stress denoted by f_{c-com} is compressive; that by f_{s-com} tensile. In Figs. 11, 12, 13 and 14, no values of z have been indicated below the zero line. This is not because combined compressive stresses cannot exist in the metal, but because in intensity such are always small and therefore not of vital interest.

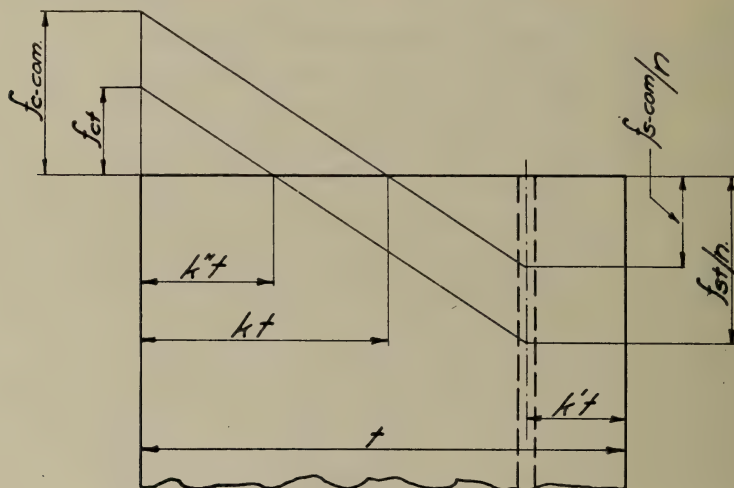


Fig. 10. Combining Stress Due to Temperature Rise with Stress Due to Weight and Wind.

Fig. 10 is a vertical section of a chimney shell of unit width perpendicular to the plane of the drawing. On it are shown stresses f_{ct} and f_{st} produced by a difference of temperature T , between the inner and outer surfaces of the shell. The position of the resulting neutral plane is indicated by the dimension $k''t$. Assume a flexural stress f_c superposed on the temperature stress diagram. Obviously this will bring an enlarged area into compression and will reduce somewhat the tensile stress in the metal. The position of the new neutral plane is indicated by the dimension kt . It has been assumed that the combined stress diagram is parallel to that for temperature only and that the excess of compression over tension is the constant quantity $tf_c(1+pn)$. It will thus be apparent that f_c is not in general represented by the difference between the ordinates for f_{c-com} and f_{ct} , f_c being the mean stress in the shell due to wind and weight only. This difference indeed will in general exceed f_c for the simple reason that only part of the area of the section is called upon to resist the load represented by the mean stress f_c . In other words, f_{c-com} is in general greater than $f_{ct}+f_c$. Once k becomes equal to unity, further additions to f_c are reflected by increments to f_{c-com} equivalent thereto. In Figs. 15, 16, 17 and 18 will be found limiting numerical values of $f_{s/T}$ beyond which " $f_{s-com}=f_s$ and $f_{c-com}=0$." These

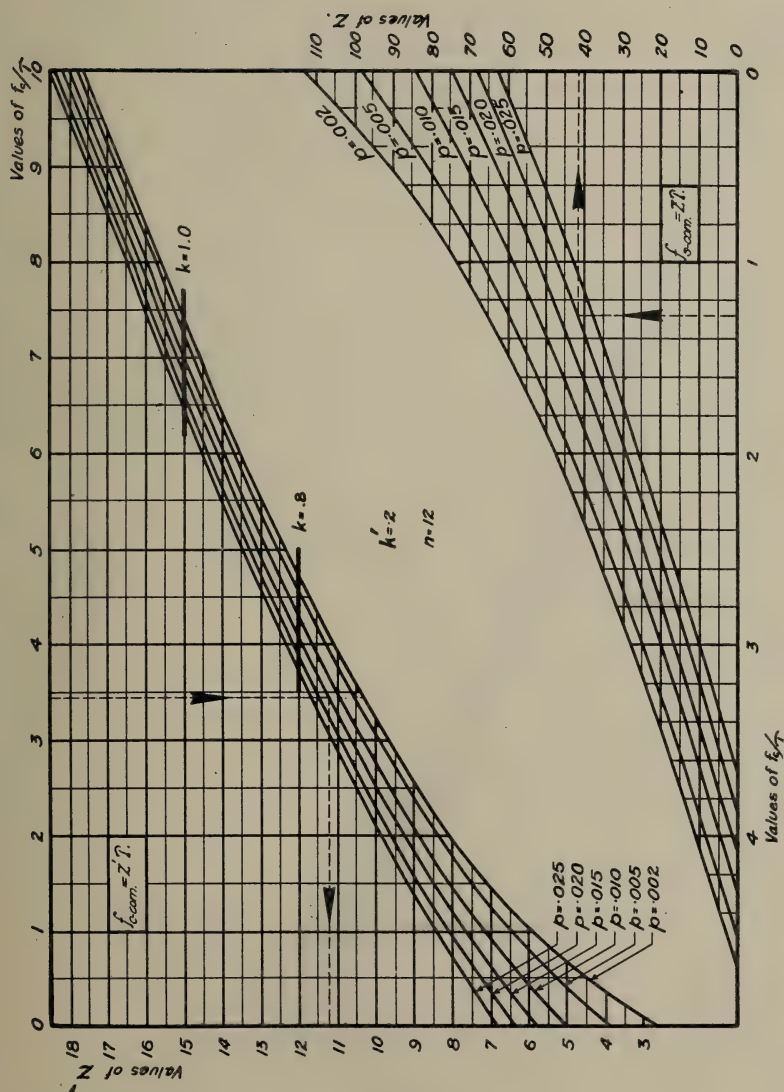


Fig. 11. Use For Combining Stress Due to Temperature Rise T , with Stress f_c Due to Weight and Wind. In General, f_c occurs at Leeward but for Values of c/r_m less than .5, f_c occurs at Leeward and at Windward.

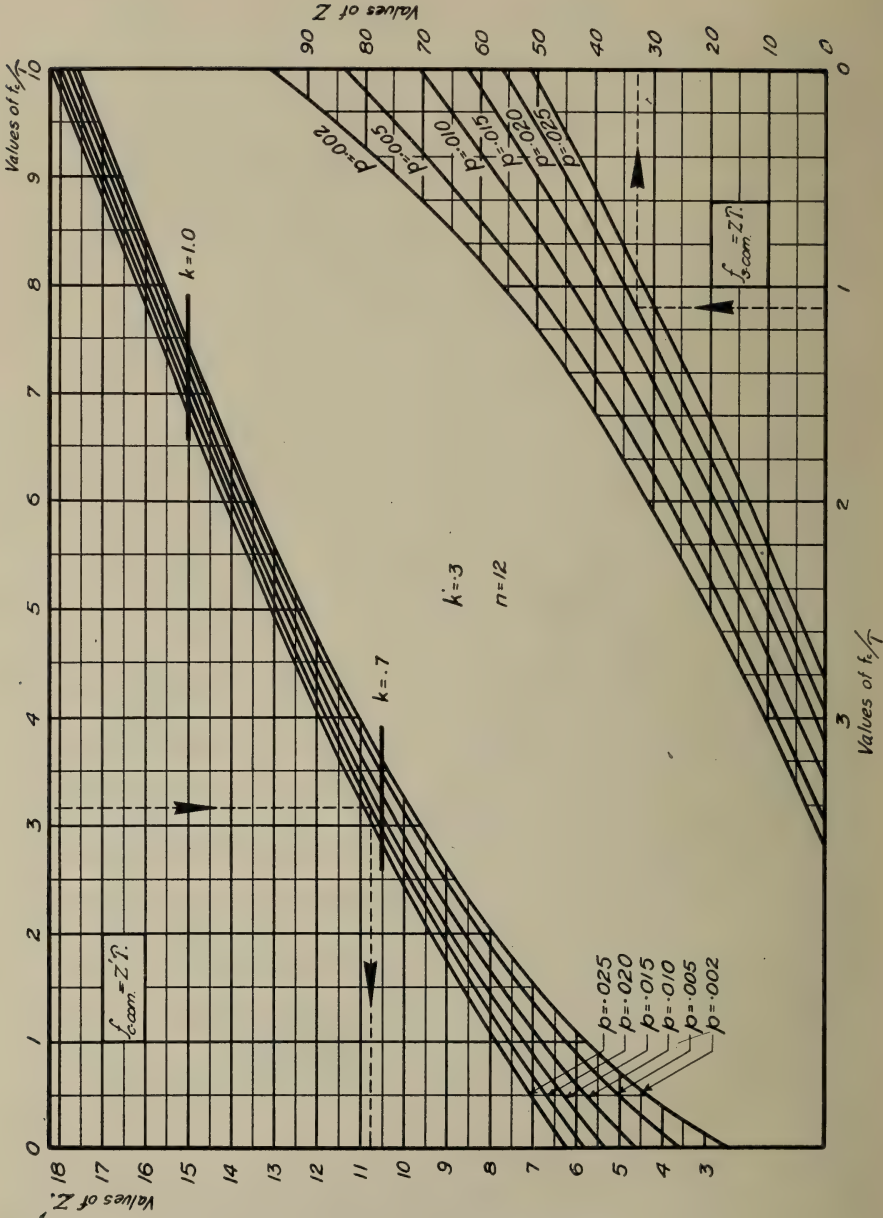


Fig. 12. Use For Combining Stress Due to Temperature Rise T , with Stress f_c Due to Weight and Wind. In General, f_c occurs at Leeward, but for Values of c/r_m less than .5, f_c occurs at Leeward and at Windward.

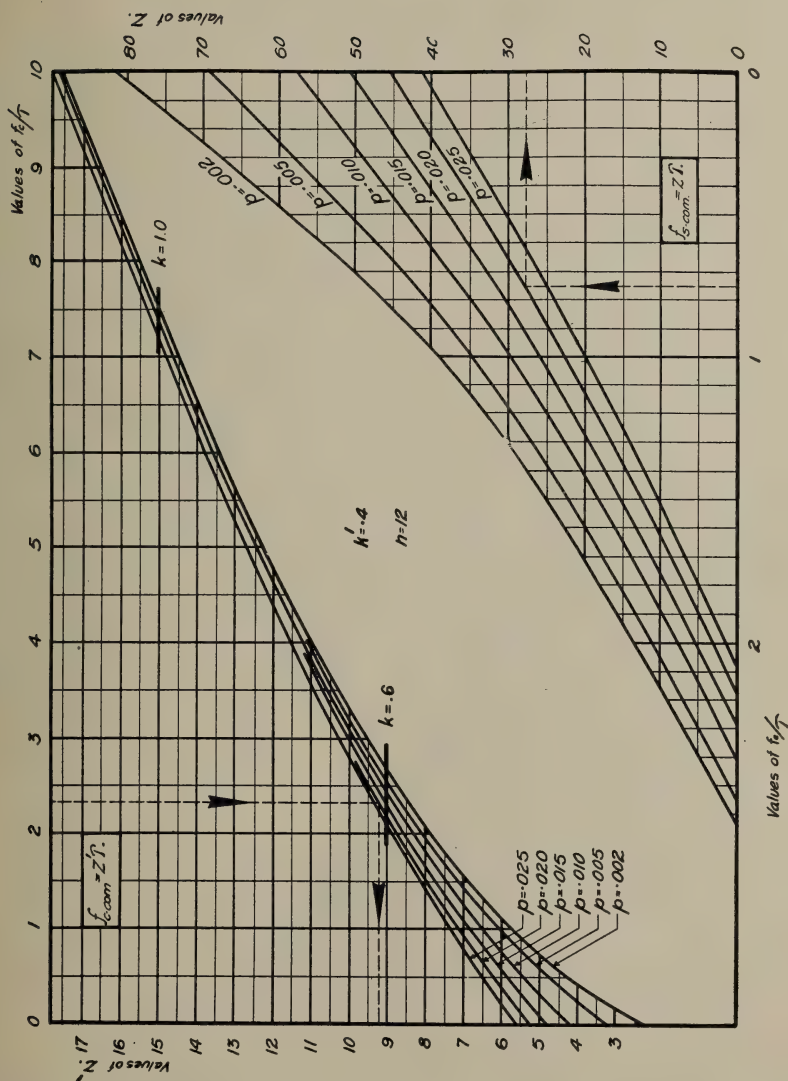


Fig. 13. Use For Combining Stress Due to Temperature Rise T , with Stress f_c Due to Weight and Wind. In General, f_c occurs at Leeward, but for Values of c/r_m less than .5, f_c occurs at Leeward and at Windward.

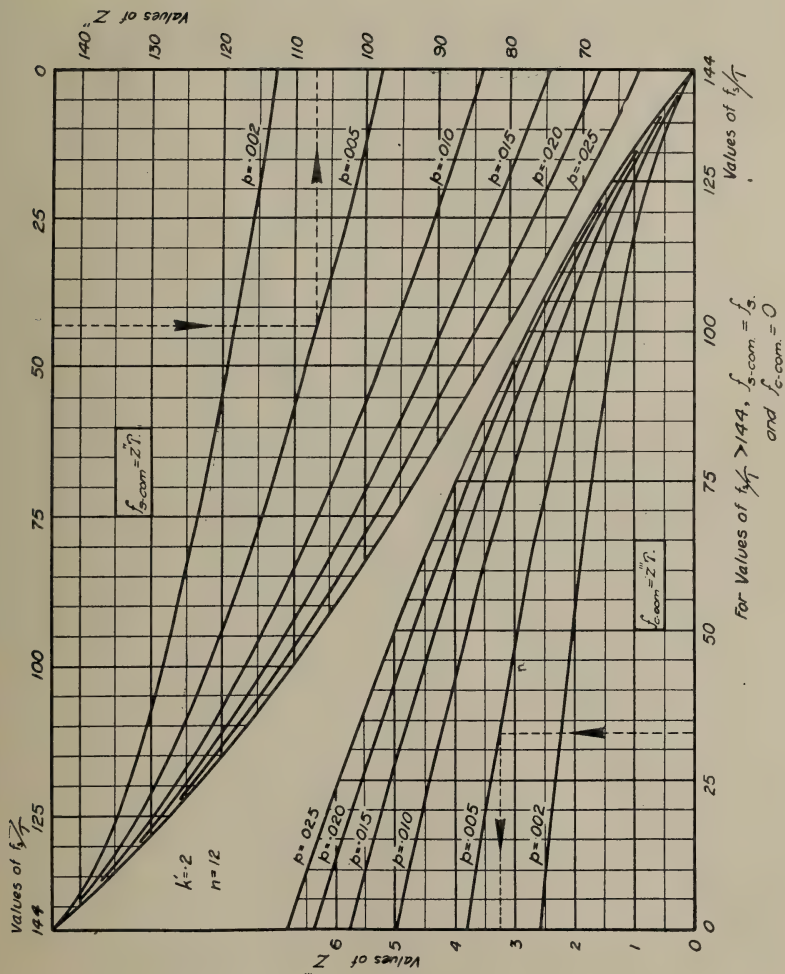


Fig. 15. Use For Combining Stress Due to Temperature Rise T , with Stress f_s Due to Weight and Wind. For Values of c/r_m greater than .5, f_s occurs at Windward; for Values of c/r_m less than .5, f_s does not exist.

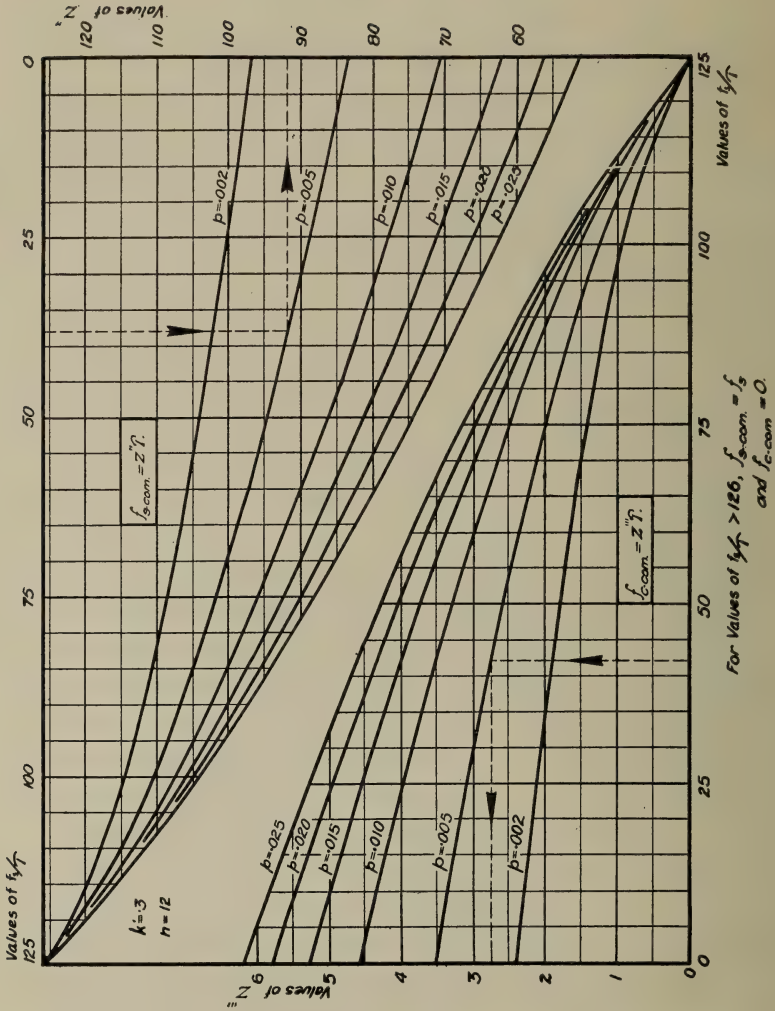


Fig. 16. Use For Combining Stress Due to Temperature Rise T , with Stress f_s Due to Weight and Wind. For Values of c/r_m greater than .5, f_s occurs at Windward; for Values of c/r_m less than .5, f_s does not exist.

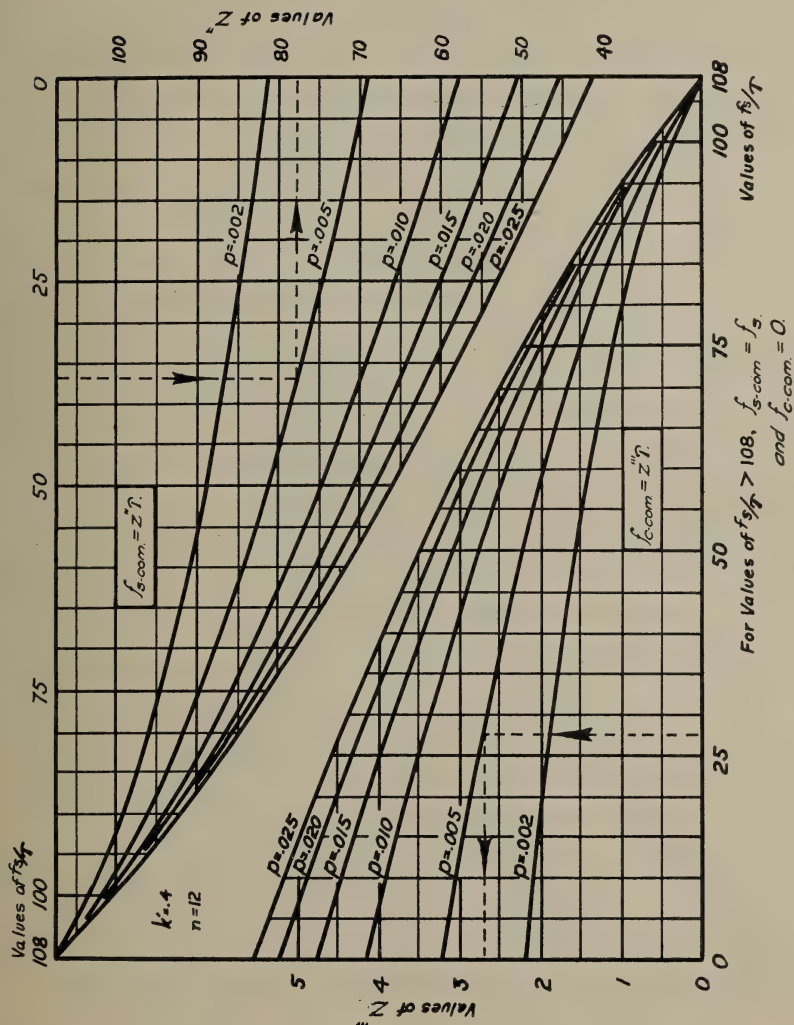


Fig. 17. Use For Combining Stress Due to Temperature Rise T , with Stress f_s Due to Weight and Wind. For Values of c/r_m greater than .5, f_s occurs at Windward; for Values of c/r_m less than .5, f_s does not exist.

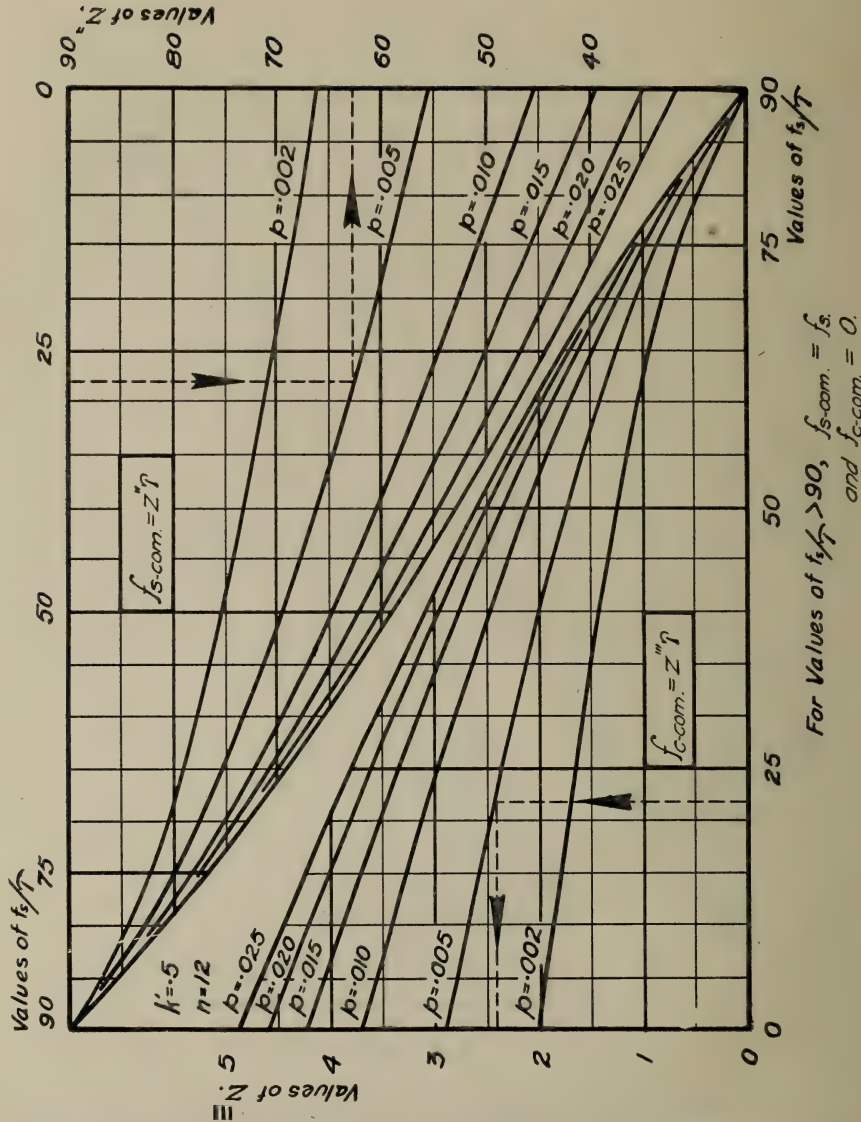


Fig. 18. Use for Combining Stress Due to Temperature Rise T , with Stress f_s Due to Weight and Wind. For Values of c/r_m greater than .5, f_s occurs at Windward; for Values of c/r_m less than .5, f_s does not exist.

limits are those at which the gap in the concrete due to tensile stress f_s in the metal, is closed by the rise in temperature. For example, k' being .2, if f_s be 14,400 lb. per sq. in. and T , 100° , f_s/T will be 144. The gap in the concrete per unit of height due to f_s will be .00048. Due to expansion of the metal an additional .00012 per unit results. It will be obvious that the total will be just closed on the inner face of the shell by the expansion in the concrete due to the 100° rise in temperature, K being .000003. For values of f_s/T greater than 144, the gap will not be closed and no compression in the concrete will occur. The numerical values, 144, 126, 108 and 90, shown on Figs. 15, 16, 17 and 18 were obtained by writing k in eq. 14 equal to zero and solving for f_s/T . Young's modulus for steel has been assumed as 30,000,000 lb. per sq. in.; that for concrete as 2,500,000 lb. per sq. in.

See Fig. 10.

$$KE_c = 15; f_{ct} = 15k''T; f_{st} = \frac{12(t - k''t - k't)}{t} TKE_c = 180T(1 - k'' - k');$$

$$kt = \frac{f_{c-com}}{f_{ct}} k''t.$$

$$\frac{k f_{c-com}}{2} - p t f_{s-com} = t f_c(1 + p n); \quad \frac{f_{ct}}{k''t} = \frac{f_{c-com} + f_{s-com}/n}{t - k't} \quad \text{or}$$

$$f_{ct}(t - k't) = k''t f_{c-com} + k''t f_{s-com}/n.$$

$$\begin{aligned} \frac{k''t f_{s-com}}{n} &= f_{ct}(t - k't) - k''t f_{c-com} \quad \text{and} \quad f_{s-com} = \frac{f_{ct}(t - k't)n}{k't} - \frac{k''t f_{c-com}n}{k''t} \\ &= \frac{15k''T(1 - k')12}{k''} - 12f_{c-com} = 180T(1 - k') - 12f_{c-com} \end{aligned}$$

$$\text{and } f_{s-com}/T = 12[15(1 - k') - f_{c-com}/T] \quad (3)$$

$$\begin{aligned} \text{Now } \frac{f_{c-com}k''t}{2f_{ct}} - p t [180T(1 - k') - 12f_{c-com}] &= t f_c(1 + p n) \quad \text{or } f_{c-com} \left(\frac{k''t}{2f_{ct}} \right) \\ &+ 12p t f_{c-com} - p t T(1 - k')180 - t f_c(1 + p n) = 0. \end{aligned}$$

$$\text{Hence } f_{c-com} = 15T \left[-12p + \sqrt{144p^2 + 24p(1 - k') + \frac{2}{15}(1 + 12p)f_c/T} \right]$$

$$\text{or } f_{c-com}/T = 15 \left[-12p + \sqrt{144p^2 + 24p(1 - k') + \frac{2}{15}(1 + 12p)f_c/T} \right] \quad (4)$$

$$\text{Again } kt = \frac{f_{c-com}}{f_{ct}} k''t; f_{ct} = 15k''T; k = \frac{f_{c-com}k''}{15k''T}.$$

$$\text{Substituting } k = -12p + \sqrt{144p^2 + 24p(1 - k') + \frac{2}{15}(1 + 12p)f_c/T} \quad (5)$$

Equations 3, 4 and 5 are for the case where k is equal to or less than $1-k'$. In a similar manner, equations for f_{s-com}/T , f_{c-com}/T and k may be obtained for the remaining cases that occur. The following summarizes the results:

Case 1, $k \leq 1-k'$.

$$f_{s-com}/T = 12[15(1-k') - f_{c-com}/T] \quad \text{Sp. (3)}$$

$$f_{c-com}/T = 15 \left[-12p + \sqrt{144p^2 + 24p(1-k') + \frac{2}{15}(1+12p)f_c/T} \right] \quad (4)$$

$$k = -12p + \sqrt{144p^2 + 24p(1-k') + \frac{2}{15}(1+12p)f_c/T} \quad (5)$$

Case 2, $k \leq 1$.

$$f_{s-com}/T = 12[f_{c-com}/T - 15(1-k')] \quad (6)$$

$$f_{c-com}/T = \frac{7.5 + 180p(1-k') + \frac{f_c}{T}(1+12p)}{1+12p} \quad (7)$$

$$k = \frac{7.5 + 180p(1-k') + \frac{f_c}{T}(1+12p)}{15(1+12p)} \quad (8)$$

Case 3, $k \leq 1-k'$ but $k \geq 1$.

$$f_{s-com}/T = 12[f_{c-com}/T - 15(1-k')] \quad (9)$$

$$f_{c-com}/T = 15 \left[-12p + \sqrt{144p^2 + 24p(1-k') + \frac{2}{15}(1+12p)f_c/T} \right] \quad (10)$$

$$k = -12p + \sqrt{144p^2 + 24p(1-k') + \frac{2}{15}(1+12p)f_c/T} \quad (11)$$

Case 4, where wind and weight result in tension in the metal (windward side only).

$$f_{s-com}/T = 12[15(1-k') - f_{c-com}/T] \quad (12)$$

$$f_{c-com}/T = 15 \left[-12p + \sqrt{144p^2 + 24p(1-k') - \frac{2}{15}pf_s/T} \right] \quad (13)$$

$$k = -12p + \sqrt{144p^2 + 24p(1-k') - \frac{2}{15}pf_s/T} \quad (14)$$

For cases 1, 2, and 3, the following may be written

$$f_{s-com} = zT; \quad f_{c-com} = z'T.$$

In Case 4, similarly there results

$$f_{s-com} = z''T; \quad f_{c-com} = z'''T.$$

PROBLEMS ILLUSTRATING THE USE OF THE CHARTS

Problem 1

A section of a chimney has an outer diameter of 13'-4" and an inner diameter of 11'-8". Longitudinal reinforcement is equal to .006 of its gross cross section and lies 2.0" from the outer face of the shell. The weight above the section is 250,500 lb. and because of wind, its line of action pierces the section 105 in. from the centre. Determine f_c and f_s .

Here $t = 10$ in.; $r_m = 75$ in.; $\frac{c}{r_m} = 1.4$; $\frac{r_s}{r_m} = 1.04$; $p = .006$.

From Fig. 4, $q = 28.8$ and $X = .808$.

Then $f_c = \frac{.808 \times 250,500}{10 \times 75} = 270$ lb. per sq. in. at leeward

and $f_s = 28.8 f_c = 7,775$ lb. per sq. in. at windward.

Problem 2

A chimney shell is reinforced with .006 of its gross cross-section with circumferential metal lying .4t from the outer face. The difference in temperature between the inner and outer faces being 100° F., determine f_{ct} and f_{st} , the stresses in concrete and steel due to this difference.

From Fig. 8, since $k' = .4$ and $p = .003$, $y = 3.45$.

Then $f_{ct} = 3.45 \times 100 = 345$ lb. per sq. in.

Similarly from Fig. 9, $y' = 67$, and $f_{st} = 6,700$ lb. per sq. in.

Problem 3

Determine at both leeward and windward, the stresses f_{c-com} and f_{s-com} due to the weight and wind stresses computed for the chimney in problem 1, combined with a temperature rise between faces of 90° F.

Leeward

$$\frac{f_c}{T} = \frac{270}{90} = 3; k' = .2.$$

From Fig. 11, for $\frac{f_c}{T} = 3$ and $p = .006$, $z' = 10.0$ and $z = 24$.

Hence $f_{c-com} = 10 \times 90 = 900$ lb. per sq. in. and $f_{s-com} = 24 \times 90 = 2,160$ lb. per sq. in.

Windward

$$\frac{f_s}{T} = \frac{7,775}{90} = 86.4.$$

From Fig. 15, for $\frac{f_s}{T} = 86.4$, $z''' = 2.3$ and $z'' = 116$.

Hence $f_{c-com} = 2.3 \times 90 = 207$ lb. per sq. in. and $f_{s-com} = 116 \times 90 = 10,440$ lb. per sq. in.

Problem 4

Stress in steel on windward side of a chimney shell due to wind and weight only is 3,000 lb. per sq. in. If $k' = .2$ and $p = .005$, determine the combined stresses f_{c-com} and f_{s-com} when $T = 20^\circ$ F.

$$\text{Here } f_s/T = \frac{3,000}{20} = 150.$$

From Fig. 15, since f_s/T exceeds 144,

$$f_{s-com} = f_s = 3,000 \text{ lb. per sq. in. and } f_{c-com} = 0.$$

Problem 5

A weightless chimney has longitudinal reinforcement equal to .015 of its gross cross-section placed .3*t* from the outer face. There being no wind and T being 100° F., determine the resulting stresses in concrete and steel.

$$f_c/T = 0.$$

From Fig. 12, $z' = 5.3$ and $z = 62.5$.

Hence $f_{c-com} = 530$ lb. per sq. in. and $f_{s-com} = 6,250$ lb. per sq. in.

These values agree with those for f_{ct} and f_{st} respectively obtained from Figs. 8 and 9.

Problem 6

In a chimney shell where $k' = .2$, compressive stress in concrete due to weight and wind is 350 lb. per sq. in. If $T = 50^\circ$ F. and $p = 0.010$, determine the combined stress f_{c-com}

$$\text{Here, } f_c/T = 7.$$

From Fig. 11, for $f_c/T = 7$, $z' = 15$ and z is not shown. This latter circumstance is due to the fact that when $f_c/T = 7$, $k = 1$ from equations 8 and 11., and f_{s-com} in compressive.

Hence $f_{c-com} = 15 \times 50 = 750$ lb. per sq. in.

If now f_c be increased to 450 lb. per sq. in., the temperature difference remaining unchanged, $f_c/T = 9$.

From Fig. 11 for $f_c/T = 9$, $z' = 17$.

This shows that when k equals or exceeds unity, increments of any magnitude to f_c/T result in equivalent increments to z' . This of course is to be expected.

ECONOMIC BASE WIDTH OF TRANSMISSION TOWERS

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Incidental to a study of certain phases of the design of steel transmission towers, begun by the authors in 1924, it appeared desirable to determine the least-weight ratio of the width of base at the ground line in square windmill type towers with suspended insulators to the total height above the ground. The base width is the only primary dimension that the structural engineer or designer for the manufacturer may ordinarily vary. The height of the cross arms and spacing of conductors depend upon prescribed clearances and line voltage. In the present enquiry, the height of the tower and the position of conductors has been accepted as fixed, but it has been assumed that neither right-of-way conditions, erection problems, nor any other circumstance debarred the structural engineer from varying the base width or changing the configuration of the trusses between the ground level and the lower cross arm.

Examination of the literature of the subject disclosed the fact that most writers regard the ratio of base width to height for least weight of metal as $\frac{1}{4}$. Current practice appears to be in accordance with that view. A study of 29 different towers of the square windmill type, on 25 different transmission systems, showed that for the standard suspension tower the average ratio of base width to extreme height above ground level was 0.242. The lowest ratio found was 0.173 for the towers of the Virginia Power Co., and the highest 0.321 for the towers of the Niagara, Lockport and Ontario Power Co.

In order to determine the degree of soundness existing in the present practice with regard to the choice of base width, the authors decided to make an original design investigation of the effect on the weight of the tower produced by varying the base width, and at the same time varying the design specifications and the assumption with respect to the compressive value of diagonals. Two different types of suspension square windmill, or pyramidal, towers 79 ft. high were studied, Type "A" tower being as indicated in Fig. 1, and "Type "B" as indicated in Fig. 2. Type "A" tower is of the same configuration as the standard double circuit towers built recently for a power company in Northern Ontario, and of the same dimensions as the prototype tower, except for

the base width, which was varied in order to give seven different towers. The widths investigated were 14 ft., 14 ft. 9 in., 15 ft. 6 in., 16 ft. 3 in., 17 ft., 18 ft. 6 in. and 20 ft. Type "B" tower is the same as Type "A" above the level of the lower cross arm, but below that level it possesses one less main story, and the batter of the legs begins at the level of that arm. Reduction of the number of stories by replacing the four stories of Type "A" tower immediately below the lower cross arm by three stories necessitated the greater use of stay or auxiliary members to support the tower legs. The height of the stories was better graded in Type "B" tower than in Type "A", but the bottom story was the same in the two cases. In selecting Type "B" tower for investigation in comparison with Type "A", it was desired to ascertain the combined effect of reducing the number of stories and of relieving the stress on the diagonals immediately below the lower cross arm through beginning the batter of the legs at that level. Involved in the increase of the story heights is also the influence of the introduction of a greater numbers of stay members. For Type "B" tower five base widths were considered, these being 14 ft., 15 ft 6 in., 17 ft., 18 ft. 6 in. and 20 ft.

Each of the seven towers of Type "A" and of the five towers of Type "B" was designed for two assumptions as to the manner of action of the diagonals in the planes of the faces of the tower. The first assumption was that the diagonals above a certain designated story resist both tension and compression. In the case of Type "A" tower, this condition was assumed to obtain down to and including the story marked 26, 27, 28, 29, and for Type "B" tower it was assumed to obtain down to and including story 22, 23, 24, 25. Below these respective stories the diagonals were assumed by reason of their considerable length to be able to resist tension only. The second assumption was that the diagonals were able to take tension only. In accordance with the first assumption, the horizontal struts below the level of the lower cross arm and above the critical story do not bear stress, while below this story they do. Under the second assumption, all horizontal struts take stress and must be designed therefor.

Each of the 24 towers resulting from the varying of the base widths of both Type "A" and Type "B" and the designing for two different assumptions in respect of diagonals, was designed for three specifications. There thus resulted in all 72 different towers. The three specifications mentioned differed only in respect of the limiting slenderness ratio for compression members. The nature of this variation of the specification is indicated in the text below and also in Figs. 3 to 8.

TABLE I
ANALYSIS OF WEIGHTS OF TYPE "A" TOWERS
See Fig. 1 for Tower Arrangement and Dimensions. All Weights are in Pounds

Type	Spread of Base Ft. In.	Specification	WEIGHTS OF COMPONENT PARTS					Galvanizing, Bolts and Washers	Total Weight		
			Legs	Truss Diagonals	Struts and Cross Arms	Stay Members	Hood Angles, Cap. Plates and Ties			Horizontal Diagonals	Footings, Except Stub Angles
"A" Diagonals Tension and Compression +	14-0	I	2,753	1,382	757	297	149	129	876	682	7,025
	14-9		2,606	1,391	763	305	149	131	876	678	6,899
	15-6		2,411	1,397	768	313	149	139	876	672	6,725
	16-3		2,412	1,404	828	335	149	142	876	675	6,821
	17-0		2,310	1,398	844	344	149	145	876	666	6,732
	18-6		2,232	1,426	860	359	149	157	876	646	6,705
20-0		2,135	1,429	1,040	407	149	171	876	656	6,863	
"A" Diagonals Tension only +	14-0	I	2,802	1,277	903	297	149	119	876	751	7,174
	14-9		2,655	1,284	904	305	149	121	876	746	7,040
	15-6		2,459	1,291	917	313	149	130	876	740	6,875
	16-3		2,460	1,298	976	335	149	133	876	743	6,970
	17-0		2,358	1,268	979	344	149	135	876	734	6,843
	18-6		2,356	1,279	1,002	359	149	148	876	725	6,894
20-0		2,183	1,249	1,176	407	149	161	876	697	7,009	
"A" Diagonals Tension and Compression +	14-0	II	2,753	1,382	741	297	149	129	876	682	7,009
	14-9		2,606	1,391	747	305	149	131	876	678	6,883
	15-6		2,411	1,397	753	313	149	139	876	672	6,710
	16-3		2,412	1,388	776	335	149	142	876	675	6,750
	17-0		2,310	1,398	782	344	149	145	876	664	6,668
	18-6		2,232	1,410	852	359	149	157	876	644	6,679
20-0		2,135	1,412	870	407	149	171	876	649	6,669	
"A" Diagonals Tension only +	14-0	II	2,802	1,277	894	297	149	119	876	751	7,174
	14-9		2,655	1,284	895	305	149	121	876	746	7,031
	15-6		2,459	1,291	907	313	149	130	876	740	6,865
	16-3		2,460	1,298	924	335	149	133	876	741	6,916
	17-0		2,358	1,268	925	344	149	135	876	732	6,787
	18-6		2,356	1,279	996	359	149	148	876	725	6,888
20-0		2,183	1,294	1,015	407	149	161	876	722	6,807	
"A" Diagonals Tension and Compression +	14-0	III	2,753	1,382	741	297	149	129	876	682	7,009
	14-9		2,606	1,391	747	305	149	131	876	678	6,883
	15-6		2,411	1,397	753	313	149	139	876	672	6,710
	16-3		2,412	1,388	759	335	149	142	876	672	6,733
	17-0		2,310	1,398	765	344	149	145	876	663	6,650
	18-6		2,232	1,410	776	359	149	157	876	642	6,601
20-0		2,135	1,412	800	407	149	171	876	646	6,596	
"A" Diagonals Tension only +	14-0	III	2,802	1,277	894	297	149	119	876	751	7,174
	14-9		2,655	1,284	895	305	149	121	876	746	7,031
	15-6		2,459	1,291	907	313	149	130	876	740	6,865
	16-3		2,460	1,298	914	335	149	133	876	741	6,906
	17-0		2,358	1,268	915	344	149	135	876	731	6,776
	18-6		2,356	1,279	927	359	149	148	876	722	6,816
20-0		2,183	1,294	953	407	149	161	876	720	6,743	

DESIGN SPECIFICATION

All towers were designed to resist, in addition to their own weight, the following loads:

- (1) 1,500 lb. vertical at each of seven points *a* to *d*.
- (2) 4,000 lb. horizontal and parallel to the line at any one point *a* to *d*.
- (3) 1,400 lb. horizontal and normal to the line at each of seven points *a* to *d*.

The wind effect on the tower is included in (3). Stresses due to (1) and (2)) and a variable fraction of those due to (3) were combined. The fraction mentioned represented the appropriate part of the transverse wind effect properly considered at a selected level of the tower at the time that either one conductor above or the ground wire was broken. Thus, for a member between the level of the top and middle cross arms, the fraction considered was 5/6; for a member between the level of the middle and the lower cross arm the fraction was 9/10, while for a member below the lower cross arm it was 13/14. In adopting these fractions the relative influences of breaks at different levels above the member in question was neglected. It is obvious that the higher the level at which the break occurs, the greater would be the relief in stress due to the transverse action of the wind.

The working stresses adopted in design were as follows:

- (1) Tension—26,000 lb. per sq. in.
- (2) Compression in lb. per sq. in., for l/r from 0 to 50, 19,000; for l/r from 50 to 150, 24,000—100 l/r ; and for l/r greater than 150, $202,000,000 / (l/r)^2$.
- (3) Shearing on bolts 16,000 lb. per sq. in.
- (4) Bearing on bolts 32,000 lb. per sq. in.
- (5) Maximum permissible values of l/r under the three specifications mentioned were:

Specification I—Legs =	150
Other compression members =	225
Stay members =	300
Specification II—Legs =	150
Other compression members =	250
Stay members =	300
Specification III—Legs =	150
Other compression members =	300
Stay members =	300

(6) Effective length of compression diagonals was assumed to be 3/4 of the length from intersection to intersection.

(7) For single angles used as tension members and connected by one leg only, the effective percentage of the net area was taken as 100—30

TABLE II
ANALYSIS OF WEIGHTS OF TYPE "B" TOWERS

See Fig. 2 for Tower Arrangement and Dimensions. All Weights are in Pounds

WEIGHTS OF COMPONENT PARTS											
Type	Spread of Base Ft. In.	Specification	Legs	Truss Diagonals	Struts and Cross Arms	Stay Members	Hood Angles Cap, Plates and Ties	Horizontal Diagonals	Footings, Except Stub Angles	Galvanizing, Bolts and Washers	Total Weight
"B" Diagonals Tension and Compression +	14-0		2,523	1,562	743	397	158	131	876	697	7,087
	15-6		2,250	1,445	811	422	158	142	876	697	6,801
	17-0	I	2,143	1,363	829	466	158	155	876	690	6,680
	18-6		2,143	1,378	1,013	469	158	169	876	695	6,901
	20-0		2,051	1,394	1,042	532	158	219	876	698	6,970
"B" Diagonals Tension only +	14-0		2,621	1,457	864	397	158	121	876	729	7,223
	15-6		2,349	1,339	938	422	158	132	876	731	6,945
	17-0	I	2,191	1,240	945	466	158	145	876	721	6,742
	18-6		2,192	1,254	1,130	469	158	159	876	731	6,969
	20-0		2,099	1,269	1,166	532	158	210	876	732	7,042
"B" Diagonals Tension and Compression +	14-0		2,523	1,562	737	397	158	131	876	696	7,080
	15-6		2,250	1,445	778	422	158	142	876	696	6,767
	17-0	II	2,143	1,363	794	466	158	155	876	689	6,644
	18-6		2,143	1,378	847	469	158	169	876	689	6,729
	20-0		2,051	1,394	865	532	158	219	876	691	6,786
"B" Diagonals Tension only +	14-0		2,621	1,457	864	397	158	121	876	729	7,223
	15-6		2,349	1,339	913	422	158	132	876	730	6,919
	17-0	II	2,191	1,240	918	466	158	145	876	720	6,714
	18-6		2,192	1,254	972	469	158	159	876	723	6,803
	20-0		2,099	1,269	998	532	158	210	876	725	6,867
"B" Diagonals Tension and Compression +	14-0		2,523	1,562	737	397	158	131	876	696	7,080
	15-6		2,250	1,445	778	422	158	142	876	696	6,767
	17-0	III	2,143	1,363	794	466	158	155	876	689	6,644
	18-6		2,143	1,378	810	469	158	169	876	688	6,691
	20-0		2,051	1,394	826	532	158	219	876	690	6,746
"B" Diagonals Tension only +	14-0		2,621	1,457	864	397	158	121	876	729	7,223
	15-6		2,349	1,339	913	422	158	132	876	730	6,919
	17-0	III	2,191	1,240	918	466	158	145	876	720	6,714
	18-6		2,192	1,254	944	469	158	159	876	722	6,774
	20-0		2,099	1,269	967	532	158	210	876	724	6,835

ug/c^2 , where u is the length of the outstanding leg in inches, g is the gauge in inches of the connected leg and c is the length in inches of the connected leg.

(8) The minimum allowable angle was fixed as $1\frac{1}{2}$ by $1\frac{1}{2}$ by $\frac{1}{8}$ in.

While the two horizontal struts in each face of a tower directly above the critical story 26, 27, 28, 29, in Type "A" tower, and the one horizontal strut in each face above the critical story 22, 23, 24, 25 in Type "B" tower, might theoretically be omitted, they were nevertheless inserted, but were considered merely as stay members with a maximum allowable slenderness ratio of 300.

The ledger angles, that is, the horizontal angles parallel to the line and in the bottom plane of each cross arm, were, in accordance with the results of investigations to be reported later, considered as resisting no stress due to torsion. They, however, were proportioned to resist the amount of tension resulting from the tendency of the two splayed angles in the bottom plane of the cross arm to spread.

In all designs it was assumed that the backs of the angles used for the tower legs were placed on the skeleton lines of the tower. Had the outer gauge line of each leg angle been placed on the appropriate skeleton line, there would of course have been a reduction in the stress in the legs due to the virtual widening of the tower.

In the weights indicated in Figs. 3 to 8 and in Tables I and II, it has been assumed that the footing material, exclusive of the stub angles, is constant and that a standard footing apart from the stub angles is used for all of the towers designed. The effect of varying the footing material in accordance with the variation of base width is discussed under "Footings" below.

WEIGHT CURVES

The computed weights of the towers, including details and the stub angles of the footings, with a constant amount for other footing material, are indicated in Figs. 3 to 8 and in Tables I and 2. Galvanizing has been assumed to have a weight of $3\frac{3}{4}\%$ of the weight of the steel throughout. In the diagrams, the curves marked $+$ refer to towers designed under the assumption that diagonals take only tension, while the curves marked $\pm$ refer to towers in which the diagonals are assumed to take either tension or compression.

Examination of Figs. 3 to 5, pertaining to towers of Type "A", shows that for both assumptions with respect to the action of diagonals, the actual plotted weights do not give smooth regular curves. There is for the tower having a base width of 16 ft. 3 in., a decided "hump" with respect to what might be regarded as the normal curve. This

exists for all three specifications and whether the diagonals are assumed to take tension only or tension and compression. For specifications II and III, Figs. 4 and 5, a "hump" also occurs for the 18 ft. 6 in. base width for towers designed under the assumption that diagonals take tension only.

The departure of these points from a normal curve is due to several causes. The most important of these are: (a) the fixing of an arbitrary slenderness ratio for compression members, (b) the necessity of selecting standard sections which do not make possible the precise assignment of material for a given stress, (c) the arbitrary increase of thickness of members to reduce the number of bolts required in order to meet space limitations, and (d) the strict limitation of the permissible allowance for over-stressing.

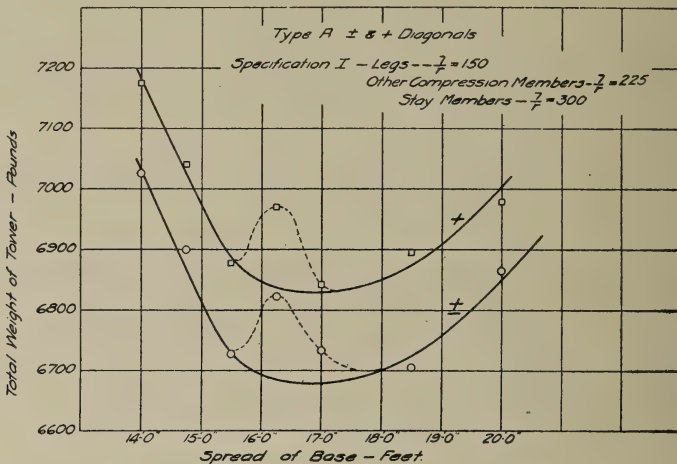


Fig. 3—Weights of Type "A" Tower, Specification I

If such discontinuity factors as the above did not occur in the problem, it would manifestly be possible to draw smooth curves closely representing the actual plotted weights. As a normal guide, then, there have been drawn, in the diagrams considered, full lines representing the normal variation of weights that would arise if the influences making for irregularity did not exist. Dotted lines joining up the actual plotted weights show the nature of the weight variation as actually found.

From Figs. 3 to 5 it is evident that for towers of Type "A" the least weight width of base is about 17 ft. for Specifications I and II, and about 18 ft. for Specification III. Varying the assumption as to the action of diagonals does not appear to alter the least weight width. Raising of the slenderness ratio limit has the effect of increasing the

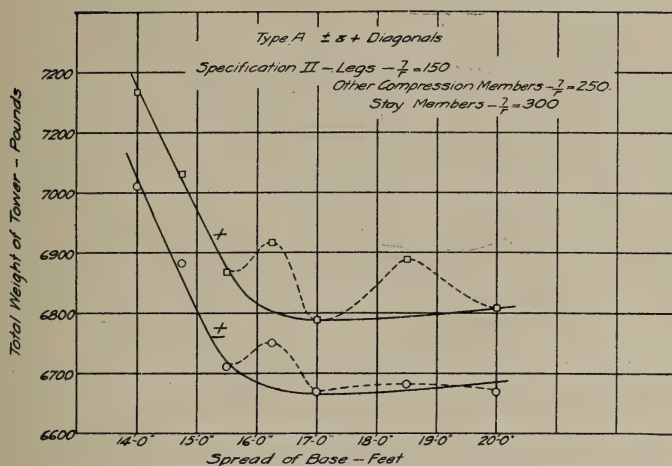


Fig. 4—Weights of Type "A" Tower, Specification II

least weight width and in general of reducing the weight of the tower for the greater base widths.

Computation of the ratio of base width to height for least weight gives 0.215 for Specifications I and II, and 0.228 for Specification III. In practice this ratio has been generally assumed as 0.250.

From Figs. 6 to 8 it is seen that the actual plotted weights of towers of Type "B" lend themselves to representation by regular curves better than towers of Type "A". For Specification I, however, a "hump" is disclosed for both assumptions as to action of diagonals at a base width of 18 ft. 6 in. The causes underlying this irregularity are the same as have been outlined in the case of towers of Type "A". In Figs. 6 to 8 normal curves indicated by full lines have been drawn and the departure from normality in the case of Fig. 6 has been indicated by dotted lines.

In all cases, towers of Type "B" disclose a minimum least weight width of 17 ft. or a ratio of width to height of 0.215. This is appreciably less than the commonly assumed least weight ratio of 0.25, and less than the ratio found for the 29 towers referred to above, for which the average was 0.242.

As in the case of towers of Type "A", the raising of the slenderness ratio causes a reduction in tower weight for the larger base widths, but very little for the smaller widths.

TYPES "A" AND "B" COMPARED

Examination of Figs. 3 to 8 and Tables I and II discloses the fact that towers of Type "B" weigh less than those of Type "A". On the

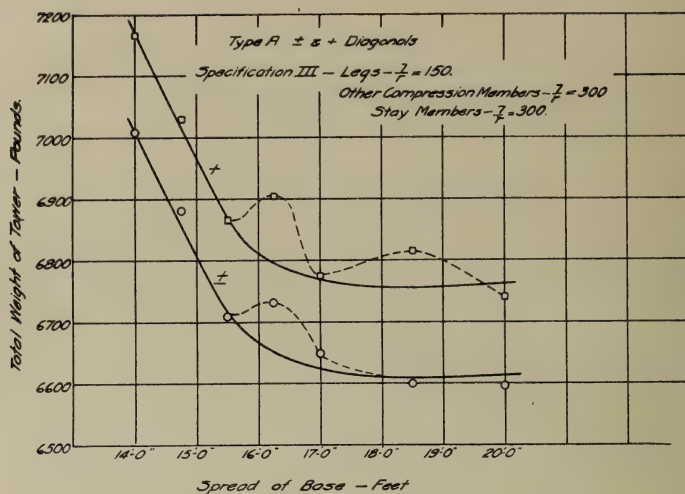


Fig. 5—Weights of Type "A" Tower, Specification III

other hand, there are more members and more bolts for the Type "B" tower than for Type "A", involving a higher manufacturing and erection cost.

A more definite least weight width is evident from a comparison of the curves for the two types of towers, and at the same time there is less spread between the weights obtained for the two opposing assumptions as to the action of the diagonals.

EFFECT OF DIAGONAL ACTION

All of the diagrams show that there is a reduction of weight introduced by considering the diagonals as taking tension or compression and not merely tension only. Designing in accordance with the first assumption actually involves a greater weight of diagonal material, but there is sufficient saving in the legs and struts to offset this and leave a substantial net saving on the whole. Variation of the tower type and base width does not alter the general conclusion as to the value of proportioning diagonals to act under both kinds of stress.

FOOTINGS

In Figs. 3 to 8 there has been included for all base widths a variable weight for the stub angles and a constant weight for the other footing material. If a standard footing be employed in all these cases, with a factor of safety for the least weight width of 2.25, it will be reduced for the 14-ft. width to 1.8, and will be increased for the 20-ft. width to 2.7.

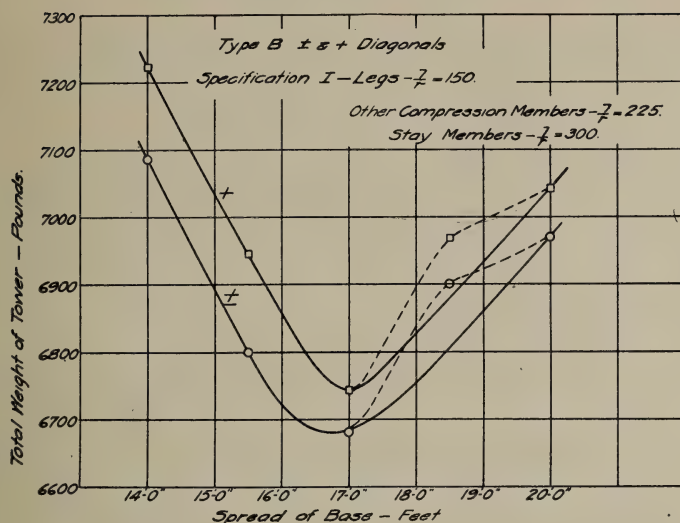


Fig. 6—Weights of Type "B" Tower, Specification I

Bringing the factor of safety to the level applying in the case of the least weight width would involve adding a weight of steel to the footings of about 240 lb. for the 14-ft. width, and reducing the weight of steel for the 20-ft. width by about 200 lb. Extra excavation would also be required for the narrow tower and reduced excavation would arise for the wide tower. Variation of the footing weights would have the effect

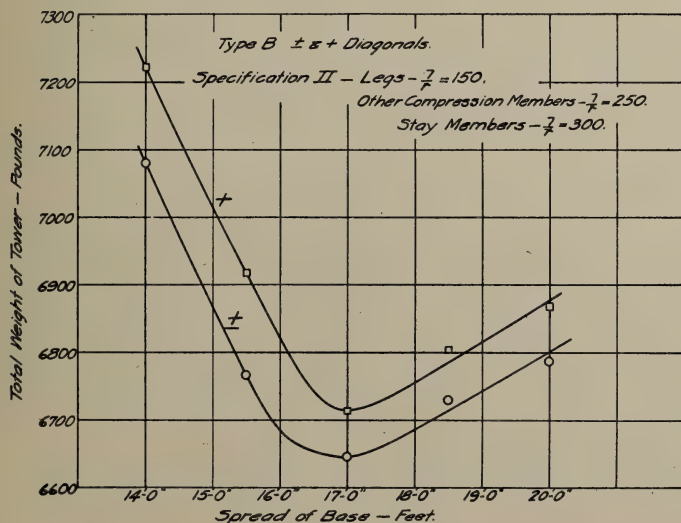


Fig. 7—Weights of Type "B" Tower, Specification II

of raising the curves in Figs. 3 to 8 at the left hand end and dropping them at the right hand end, but no change of their elevation would occur at the least weight width.

From the present investigation it appears that:

(1) The least weight spread of base at the ground line for a typical square windmill suspension tower depends upon the specification to which the design is made and more particularly on the slenderness ratio limits. The ratio of base width to total height above the ground level under usual specifications is 0.215. With a lenient specification in the matter of slenderness ratio this may rise to 0.228.

(2) Reducing the number of stories in a tower, using more stay members and beginning the batter of tower legs at the level of the lower

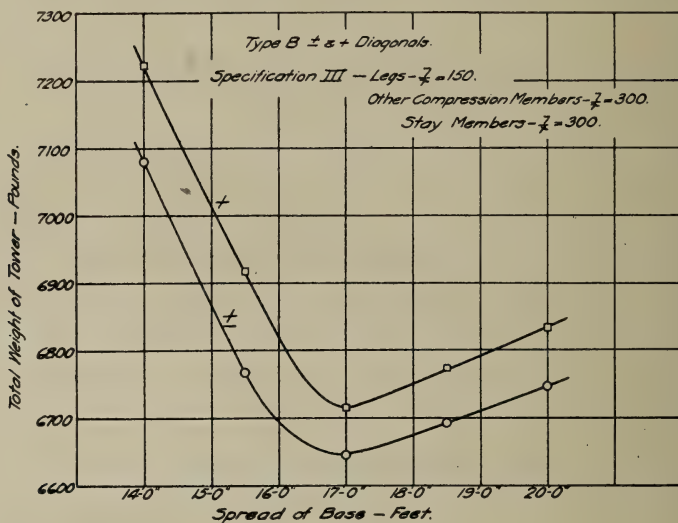


Fig. 8—Weights of Type "B" Tower, Specification III

cross arm, has the combined effect of reducing tower weight, but adds to the pound cost of manufacture and erection. This modification gives a more definite least weight and less difference between the weights of towers designed under the competing assumptions that diagonals take both tension and compression and that they take tension only.

(3) In all towers analysed, weight was saved by proportioning the diagonals to take both tension and compression, rather than resist tension only.

(4) Arbitrary limits with respect to slenderness ratio, area of sections available, number of bolts practicable in end connections, and permissible allowance for over-stressing, tend to make actual weight curves irregular and not in accordance with any continuous mathematical function.

BEHAVIOUR OF MODEL TRANSMISSION TOWERS UNDER LOAD

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In a study of steel transmission towers undertaken by the present authors and Mr. C. E. Lewis, a number of stress uncertainties were encountered upon which it was thought light might be thrown by experimentation with a model tower. This paper is a brief record of some of the results so obtained.

Information of considerable interest on a number of points was obtained from wooden towers about 3 ft. high constructed by Mr. Lewis in 1923. The legs of these towers were of $\frac{3}{4}$ -in. square and the horizontal struts were of $\frac{1}{2}$ -in. square white pine. The diagonal bracing, both in the planes of the side trusses and in the one braced horizontal plane was of iron wire about 0.05 in. diameter. There were two stories in each tower.

MODEL STEEL TOWER

In order to extend the enquiry and obtain, if possible, some quantitative information, the model steel tower shown in Figs. 1 and 2 was constructed. The form selected was the square "windmill" type corresponding to that designed in detail by Mr. S. G. Roebblad, in Hool & Kinne's *Stresses in Framed Structures*, p. 477. For convenience the model tower was constructed to $1/10$ the size of the original, thus giving a height of 6.9 ft. for the model.

Two thicknesses of material were used. Where the material for the actual tower would be greater than $\frac{1}{4}$ in., No. 18 gauge galvanized steel sheeting bent into angle sections was used, and where the thickness would be $\frac{1}{4}$ in. or less, No. 24 gauge sheet metal was adopted. These thicknesses correspond to $1/20$ and to $1/40$ in. respectively. Inability to match exactly the various thicknesses of the original tower made little difference in the relative radii of gyration of the struts, and gave no important discrepancy between the elastic behaviour of the model and that of an actual tower represented by the model. Bolts were of 0.084 in. diameter, or somewhat more than $1/10$ th of the usual $5/8$ -in. size in actual towers.

CENTRE OF TORSION

Prevailing methods of design for towers assume that a horizontal load applied at the end of a cross-arm parallel to the line may be re-

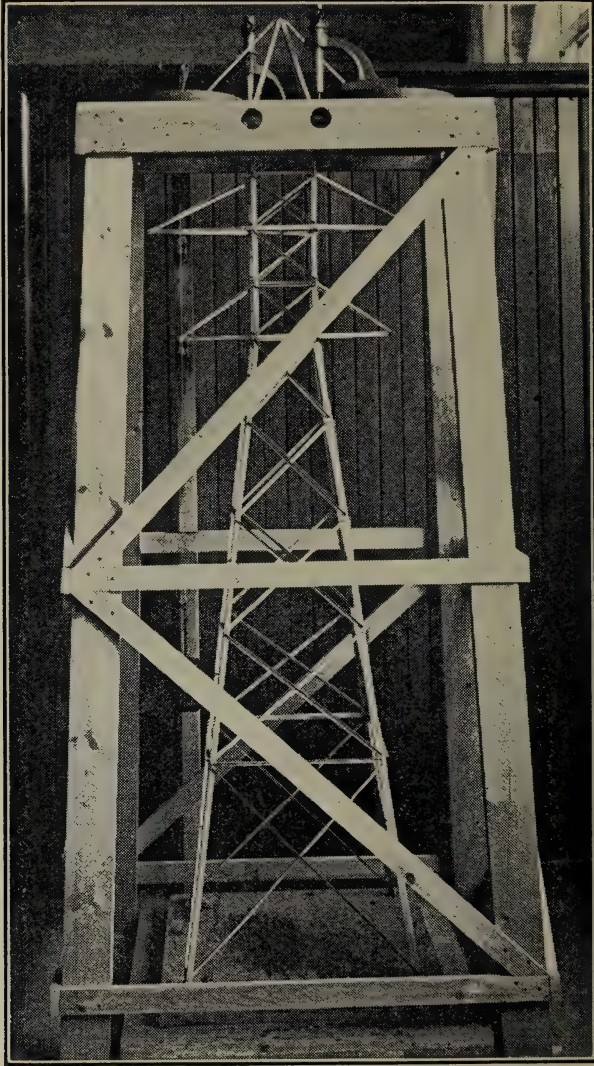


Fig. 1—Model Steel Tower With Observation Platform in Place

placed by an equal and parallel force operating at the tower axis and a couple equal to the force multiplied by the distance of the actual force from the axis. The centre of torsion is thus assumed as at the centre of

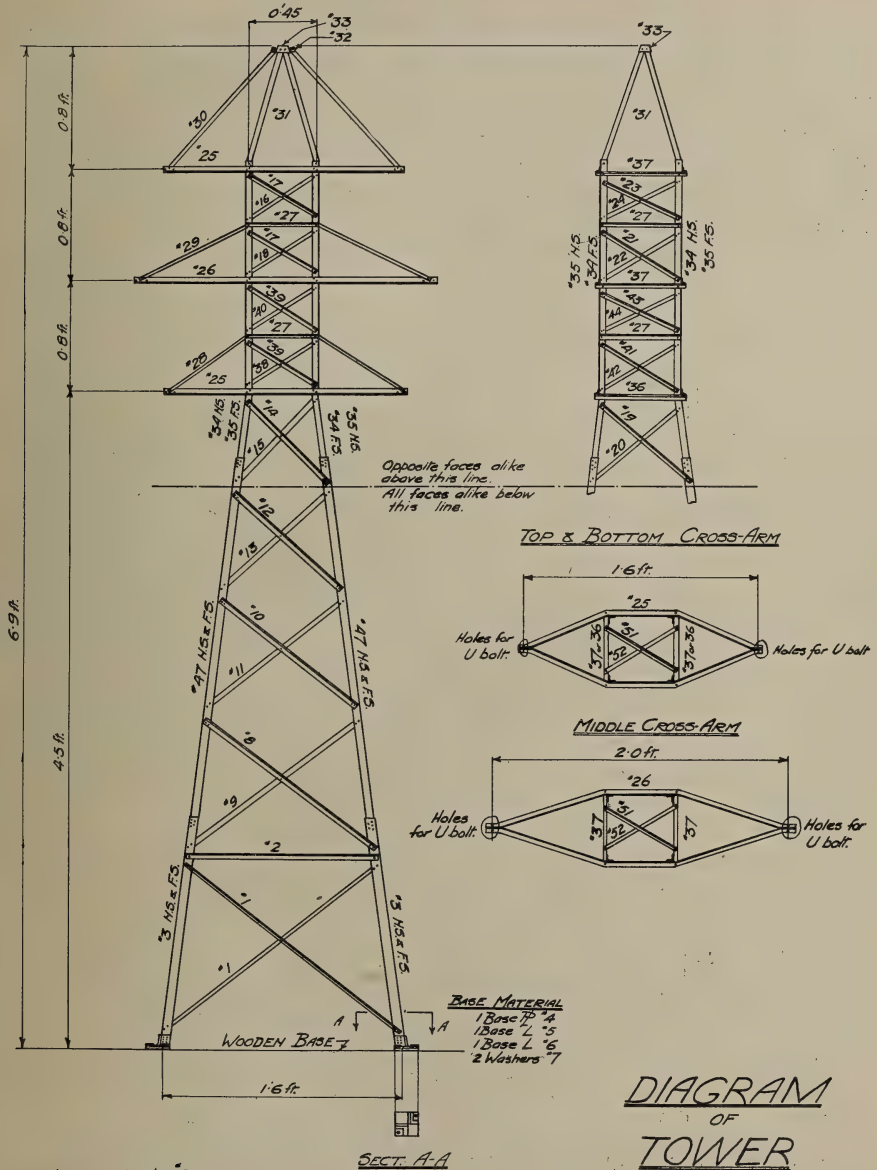


Fig. 2—Make-Up of Model Steel Tower

the tower. To what extent differences in elastic behaviour of the four side trusses of the tower may vary this was one of the matters on which light was sought.

In brief, the procedure for determining the centre of torsion was to

observe the direction and amount of movement of an observation point at the top of each of the four tower legs, and by subtracting vectorially the movement of each point that would arise if the load were considered as centrically applied to the tower, to find the movement of the points due to pure torsion. The centre of rotation, may then be found geometrically by finding the intersection of normals to the torsional paths of the various tower legs.

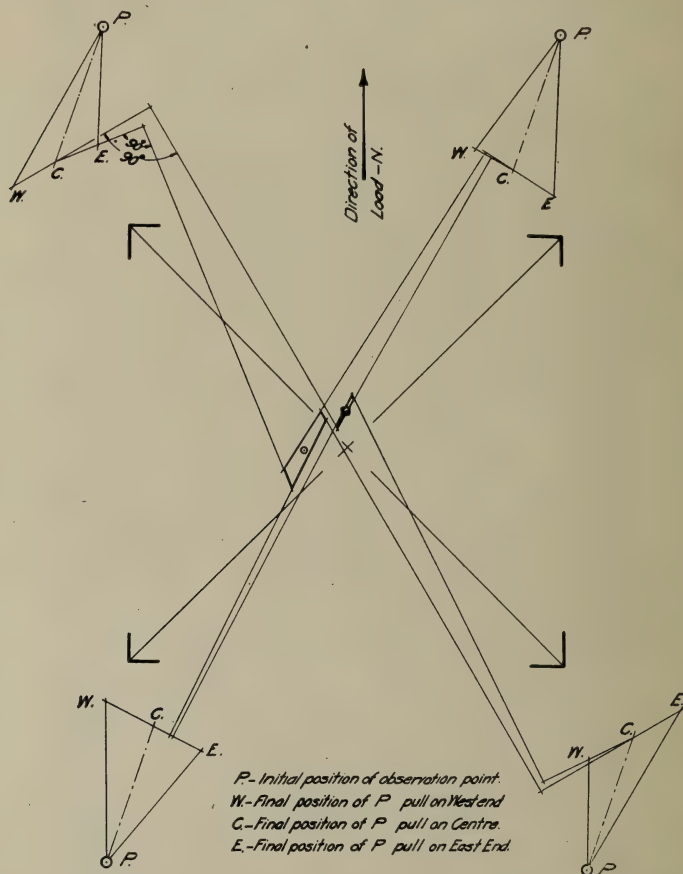


Fig. 3—Vectorial Subtraction of Movements to Obtain Centre of Torsion

Micrometer microscopes were set up on a platform independent of the tower, as shown in Fig. 1, over each of the points and the intersection of the cross lines brought into coincidence with the observed point under the zero load condition. As torsional load was applied, each point moved away from the zero position and the direction of its travel was recorded on a special piece of cardboard directly under-

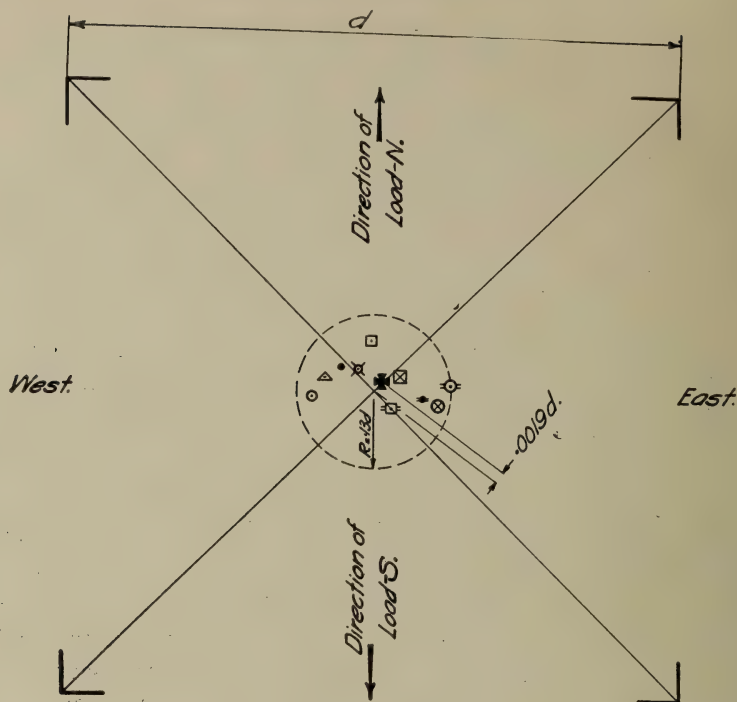
neath the point and in the focus of the microscope. The magnitude of the shift was read by means of the micrometer eyepiece. In order to allow for the movement of the tower legs due to the non-torsional effect of the loads, horizontal loads up to 35 lb. were applied to the centre of the tower and in the direction of the line by means of a specially constructed yoke. The movements, which were substantially parallel to the pull, were measured for each observation point and subtracted vectorially from the movement for the same load applied eccentrically, as indicated in Fig. 3. A perpendicular to the direction of movement of each of the four points determined for the torque above, that is after the vectorial subtraction referred to above had been made, was then drawn through the zero position of each observation point. Theoretically, these perpendiculars should intersect at a point, which should be the centre of torsion. Actually, due to imperfections of workmanship and errors of observation, they formed a four-sided figure near the centre of the tower. The centre of gravity of this figure was then found and assumed as the centre of torsion for the set of readings considered. In Fig. 3, the determination of the torsional centre was made for a pull applied in a northerly direction to the middle cross-arm first at the west end of the cross arm and next for a pull applied at the east end.

On account of the difficulty of access to the upper cross-arm, experiments were confined to the middle and the lower cross-arms. The results obtained are probably characteristic for any cross-arm.

TORSION AT MIDDLE CROSS-ARM

Fig. 4 indicates the results obtained by one observer for the middle cross-arm. Loads were applied at either the east or the west end of the arm and in either the north or the south direction. Distinctive marks on the figure indicate the position of the centre of torsion for each case of loading. The average of the determinations for the southward pull at the two ends of the arm is shown, as also the average for the northward pull at the two ends. These averages give points on opposite sides of the tower axis, but at about the same distance therefrom.

With a view to discovering the difference between the position of the centre of torsion as determined from the movement of the two legs subjected to net compression under an eccentric load and the centre of torsion as determined by the movement of the two legs subjected to net tension, the positions of the torsional centre for the two compression legs and for the two tension legs was plotted for (1) average of north and south applications at east end of arm and for (2) average of north and south applications at west end of arm. Fig. 4 indicates that the



Legend —

- ⊠ — Load applied at West end of X-Arm, Direction S.
- ⊙ — Load applied at East end of X-Arm, Direction S.
- ⊕ — Average of West & East applications, Direction S.
- ⊡ — Load applied at West end of X-Arm, Direction N.
- ⊙ — Load applied at East end of X-Arm, Direction N.
- — Average of West & East applications, Direction N.
- From readings on legs in compression —*
- ⊗ — Load applied at West end of X-Arm, Directions N & S.
- ⊙ — Load applied at East end of X-Arm, Directions N & S.
- From readings on legs in tension —*
- ⊠ — Load applied at West end of X-Arm, Directions N & S.
- ⊙ — Load applied at East end of X-Arm, Directions N & S.
- ⊕ — Mean of all positions.

Fig. 4—Centre of Torsion for Pull on Middle Cross-Arm

deviation of the torsional centre from the tower axis does not differ greatly whether the compression legs or the tension legs are made the basis.

Combining all results for the middle cross-arm, it is seen the mean of

all positions for the centre of torsion is 0.0019 times the width " d " of the tower from the axis, and the extreme deviation, which occurred when the load was applied southward at the east end of the cross-arm was $0.13d$. The mean position of the centre of torsion found on the basis of combining the results for the compression legs and the tension legs coincided with the position of this point for the average of all experiments using four legs as a basis.

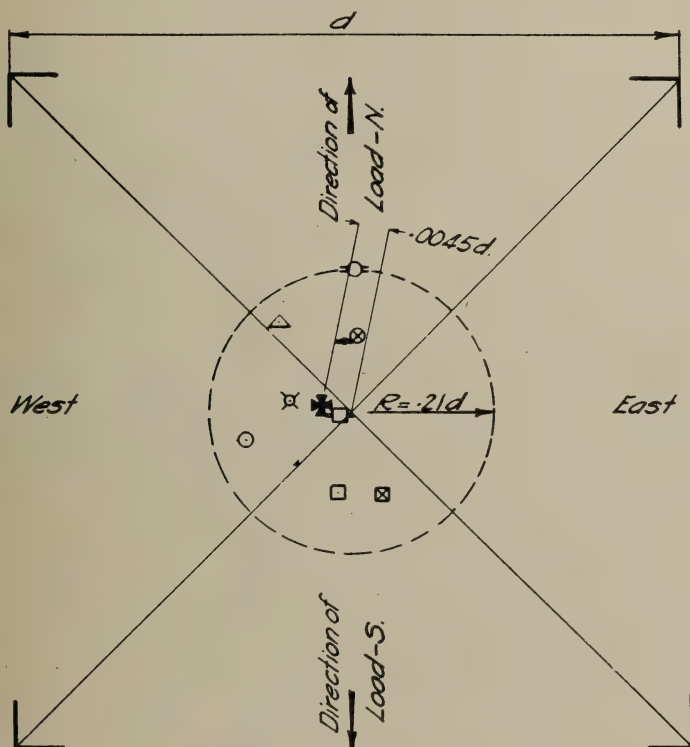


Fig. 5—Centre of Torsion for Pull on Lower Cross-Arm

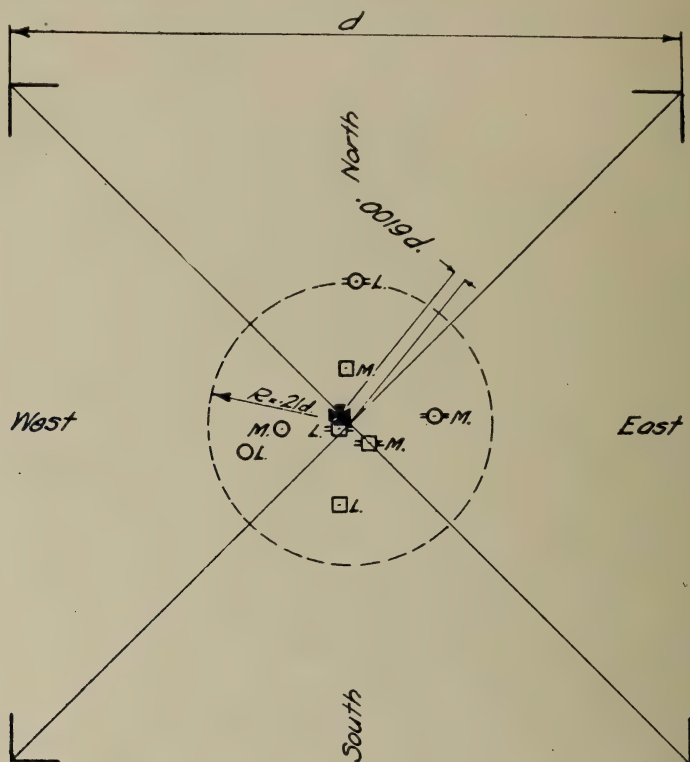
TORSION AT LOWER CROSS-ARM

Tests similar to those reported above were made for the lower cross arm by two observers and the results thereof are shown in Fig. 5. The symbols employed are the same as are employed in Fig. 4.

The averages of the results for the southward pull at the two ends of the arm and of the results of the northward pull at the two ends give points on opposite sides of the tower axis, but at about the same distance therefrom, these distances not differing greatly from those found for the middle cross-arm. The results for compression legs only and for

tension legs only show greater deviations from the tower axis than in the case of the middle arm.

Combined results also show greater deviation for the lower cross-arm. The mean of all positions gives an eccentricity of $0.0045d$ with respect to the tower axis, and the maximum of any one case gives an eccentricity of $0.21d$.



M - Results on middle cross-arm.

L - Results on lower cross-arm.

Fig. 6—Combined Results of Torsion Centre Determination for Middle and Lower Cross-Arms

COMBINATION OF TORSIONAL RESULTS AT TWO ARMS

Averages of the results for the middle and the lower cross-arms on the basis of the behaviour of all four legs are indicated in Fig. 6. The mean deviation from the tower axis for all these observations for the two arms is $0.0019d$, while the maximum deviation is $0.21d$.

While some of the observations indicate a considerable departure from the common assumption that the centre of torsion is at the tower

axis, the average of a large number of readings reduces this deviation to a negligible amount. The larger deviations may possibly be accounted for by the fact that the tower had been successively loaded and unloaded in previous experiments of another character. Possibly some permanent set had been produced in some parts; but in this respect the tower would probably not differ greatly from a transmission tower under actual working conditions.

STRESS GAUGE

With a view to measuring the stress in any member of the model steel tower, a special stress gauge was devised to work in as short a length as 3 in. and have a range of 60 lb. either way from zero. After a great deal of difficulty, and as a result of much preliminary experi-

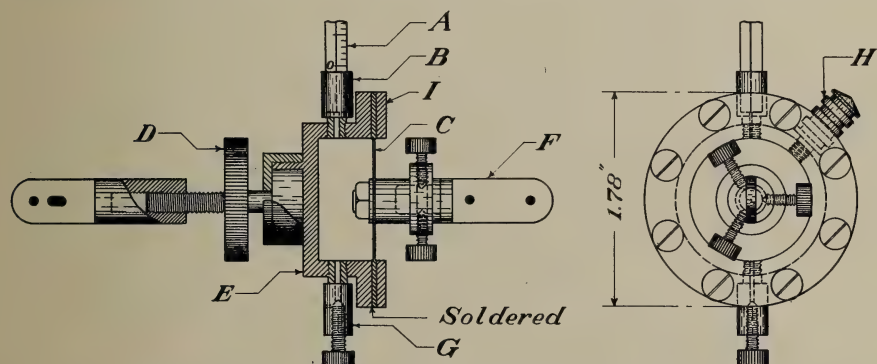


Fig. 7—Gauge for Determining Stresses in Members of Model Tower

menting, the instrument shown in Fig. 7 was devised and found to function satisfactorily.

The gauge consists of a brass flanged cup *E*, over the end of which and soldered thereto is a steel diaphragm *C*, the rigidity of the fixing being still further promoted by the bolting on of a brass ring *I* over the diaphragm. Application of stress, either tensile or compressive, to this diaphragm by the bar *F* changes the volume of the enclosed space.

Changes in this volume were measured by filling the enclosed space with coloured water which would rise or fall upon the application of load, in the capillary tube *A*. The tube was graduated and the calibration indicated in Fig. 8 was made. *B*, in Fig. 7, is a bushing into which the capillary tube was shellaced. *G* is a drain-cock by means of which the height of the liquid in the capillary tube was raised or lowered to some satisfactory level in order to have it at some convenient point on the scale before commencing operations. *H* is the nipple to which

the reservoir consisting of a thistle tube was connected. The supply from the reservoir was controlled by means of a pinch-cock.

Various lengths of bars F were prepared in order to fit the instrument into different locations, and suit different panel sizes. Rotation of the bar is permitted by a three-point set screw attachment. Load is applied by turning the milled screw D in the appropriate direction.

Several diaphragms were tried before one was found that would function properly. Spring brass appeared to fatigue on repeated loadings to 60 lb. Spring steel, either in 2 or 3 leaves, lagged in recovery. Finally, a diaphragm was turned from steel to a thickness of 0.023 in. and upon tempering was found to perform consistently.

A fibre gasket was used originally between the cup flange and the

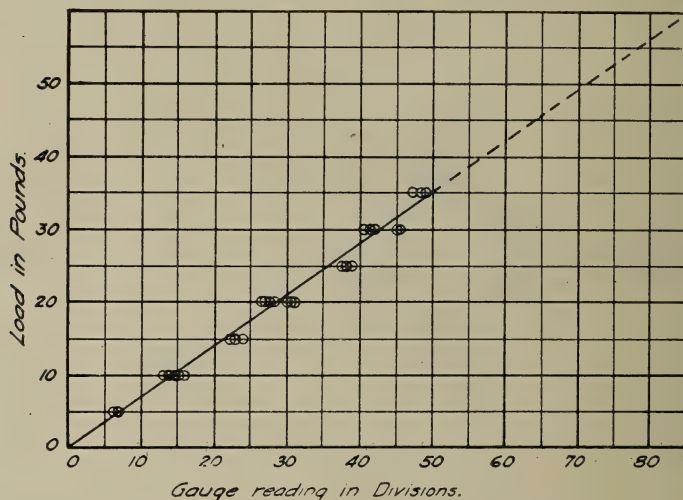


Fig. 8—Calibration Curve for Stress Gauge

diaphragm, but this appeared to introduce a lag in recovery. Finally soldering the diaphragm to the flange was tried and proved satisfactory.

METHOD OF USING STRESS GAUGE

In order to use the gauge to determine the stress in any particular member, four observation points, consisting of needle points, were chosen, one on each leg, just above the plane of the upper cross-arm. Any load applied to the tower produced a movement of these points, and upon the removal of any member of the structure a further change in the position of the points occurred.

The procedure for determining the stress in any member was then to apply the desired load to the tower and note by micrometer micro-

scopes, Fig. 1, the position of the observation points with the applied load acting. The member in which the stress was desired was then removed and a shift of each of the four observation points occurred. Then the stress gauge was inserted in the place vacated by the removed member and such a stress produced in it as would bring the observation points back to their position before the removal of the member. Whatever the stress indicated by the gauge happened to be would then be the stress in the member for the tower loading considered.

Actually, only two microscopes were used, these being placed diagonally opposite each other on a wooden frame separated from both the tower and the observer's platform. The microscopes read to $1/1000$ of a millimeter, or approximately $1/25,000$ of an inch.

Loads were applied to the tower by cords passing over pulleys to suspended weights.

THEORETICAL STRESSES IN LEDGER ANGLES

One of the matters upon which information was desired was the axial stress in the "ledger angles" of a tower, that is in the horizontal longitudinal angles in the two sides of the tower parallel to the direction of the line, and in the plane of a cross-arm, for the case where two diagonal members, equally able to take tension or compression occur in the plane of the cross-arm. These ledger angles are AC and BD in Fig. 9.

Theoretical considerations appeared to indicate zero stress in these members and it was desired either to confirm or to disprove the results of analysis. The theoretical study giving rise to the views held was based on the assumption that the diagonals of the tower, whether they be in the plane of the tower sides or in the horizontal planes of the cross-arms take tension and compression equally. The analysis follows.

Step 1. Consider a square pyramidal tower, of breadth b at the height in question, and, as Step 1 in the analysis, let there be no horizontal bracing at all in the plane of the cross-arm, as shown in Fig. 9 (a).

Let the cross-arm on the loaded side of the tower be cut off and moved to the right into the position $B^1D^1E^1$ in order to illustrate the more clearly the action of the forces involved. The arm of the force P to the centre of torsion O of the tower is still actually e , however. Forces indicated are those attacking the tower, and not reactions of the tower against applied forces.

If no diagonals exist in the square $ABCD$, the forces brought to bear at the tower legs are as shown acting at the four corners of the square. There being no diagonals, no shear can exist in the square.

With plus and minus diagonals in the tower sides that is diagonals equally able to take tension or compression, the forces normal to the line at the four legs must be equal, that is at each,

$$\frac{Pe}{2b} - \frac{P}{4}.$$

Step 2. Consider now that plus and minus diagonals are inserted in the square $ABCD$, as in Fig. 9 (b). Assume that the centre of torsion is at the centre of the tower, which may be shown to be the case by theoretical analysis of the effect of eccentric loading on a series of resistance points equally able to bear load. This has been further sub-

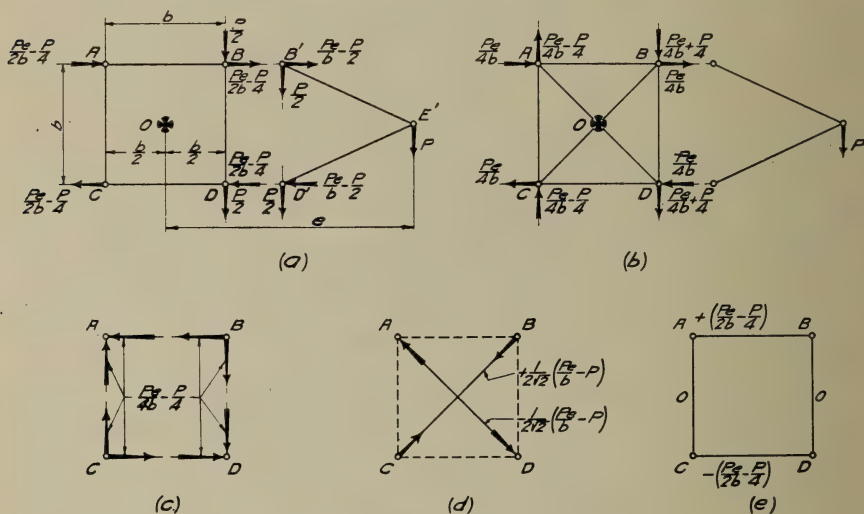


Fig. 9—Theoretical Stresses in Members in Plane of Cross-Arm

stantiated for towers by the experimental evidence presented in the first part of this paper.

The diagonals in the square are able to transfer half of the load P over to the truss AC , and hence each tower leg receives a force of $P/4$, apart from the torsional effect. Equal resistance capacities in the four trusses in the tower sides create two resisting couples each $=Pe/2$. The applied force in each of the four side planes is then $Pe/2b$ and at each leg the torsional force is $Pe/4b$.

The net forces attacking the tower are shown for Step 2 in Fig. 9 (b).

Step 3. Consider now what forces would have to be superimposed on the forces determined in Step 1 at the points A, B, C and D in Fig. 9 (a), in order that the net forces at these points would become as indicated in Step 2, Fig. 9 (b).

At A , the force $\frac{Pe}{2b} - \frac{P}{4}$ normal to the line must then be changed to $\frac{Pe}{4b}$. For all common tower proportions the former is greater than the latter, and hence a force $\left(\frac{Pe}{2b} - \frac{P}{4}\right) - \frac{Pe}{4b} = \frac{Pe}{4b} - \frac{P}{4}$ must be introduced operating to the *left* at A , as shown in Fig. 9 (c). No force parallel to the line existing at A in Fig. 9 (a), a force of $\frac{Pe}{4b} - \frac{P}{4}$ must be introduced at that point acting parallel to the line and opposite in sense to the applied force P .

Similar reasoning shows that forces as shown in Fig. 9 (c) must be introduced at all four corners of the tower to transform the situation depicted by Fig. 9 (a) into that depicted by Fig. 9 (b). In dealing with points B and D it is useful to remember that in any ordinary tower $\frac{Pe}{4b} + \frac{P}{4}$ is greater than $\frac{P}{2}$.

Step 4. Next, it is necessary to ascertain what stress in the diagonal will produce the forces indicated in Fig. 9 (c). From inspection it is evident that a compression in the diagonal AD and a tension in the diagonal BC each amounting to

$$\sqrt{2} \left(\frac{Pe}{4b} - \frac{P}{4} \right) = \frac{1}{2\sqrt{2}} \left(\frac{Pe}{b} - P \right)$$

would give the effects required in Fig. 9 (c). The stresses in the diagonals are then as shown in Fig. 9 (d).

Step 5. As the diagonals serve only to apply forces to the side trusses of the tower by direct application to the joints A , B , C and D , the stresses in the bounding members of the square $ABCD$ remain as in Fig. 9 (a), that is as indicated in Fig. 9 (e). There is, therefore, no stress in the ledger angles, AC and BD , while the stress in the member AB is $+\left(\frac{Pe}{2b} - \frac{P}{4}\right)$ and in CD it is $-\left(\frac{Pe}{2b} - \frac{P}{4}\right)$. It is obvious that the components of the stresses in the two diagonals connecting to the tower at A and C , operating in the same sense could produce no stress in AC . Similarly, for BD .

OBSERVED STRESSES IN LEDGER ANGLES

Three methods were employed to ascertain the stress in the ledger angles.

In accordance with the first method, the tower was first put under zero load, a ledger angle was removed and the stress gauge inserted in its stead, care being taken to ensure that there was no slack in the instrument to be taken up on the application of load. Loads up to 30 lb. were then applied at the end of the cross arm in which the ledger angle occurred with no visible movement of the coloured liquid in the graduated tube of the gauge. Both ledgers, in both the middle and the lower cross-arms, were investigated in this way, the top cross-arm not being accessible for this experiment. The inference was that there is zero stress in these ledgers.

The second method consisted of observing the movement of the legs at the two ends of a given ledger angle. Observation points were selected at the top of the two legs involved and micrometer microscopes set up over these. Loads were applied up to 30 lb. at the end of the cross-arm being investigated, and upon the removal of the ledger angle with the load still on, no apparent movement of the observation points occurred. This operation was repeated several times on both the middle and the lower cross arm with the same result. The loading was now changed to the opposite side of the tower and the experiment repeated with the same result. The outcome of this second series of tests was in conformity with that of the first series.

The third method consisted of measuring the movement of the legs at the two ends of a ledger angle under the imposition of a given load, first, with the ledger angle in place, and, second, with this angle removed.

For the first phase investigated, that is with the ledger angle in place, microscopes were set up over the tops of the two legs involved and increments of load of 10 lb. up to 30 lb. were applied at the end of the cross-arm to which the ledger angle belonged. The load was applied three times with the ledger angle in place. For the ledger angle in the middle cross-arm and on the unloaded side of the tower the movement M_1 of the leg at one end of the ledger ranged from 0.659 to 0.715 mm. for a 10-lb. increment of load, with an average of 0.679 mm. For the leg at the other end the range of movement M_2 was from 0.730 to 0.810 mm., with an average of 0.759 mm.

For the second phase of this investigation the ledger angle on the unloaded side of the middle cross-arm was removed and the loading repeated three times as before. The average of M_1 was 0.680 mm. and the average M_2 was 0.761 mm. It was thus apparent that the presence or absence of the ledger angle had practically no effect on the distortion of the tower and the theory of zero stress in these angles was further supported.

It should be noted, however, that the diagonals in the plane of the cross-arms are frequently attached to the ledger angles, instead of to

the cross-arm angles themselves, and hence bending in the ledger angles will occur due to this type of connection.

OBSERVED STRESSES IN DIAGONALS IN PLANE OF CROSS-ARM

It was thought desirable to check by the use of the stress gauge, Fig. 7, the theoretical stresses in the diagonal in the plane of a cross-arm indicated in Fig. 9 (*d*), that is the correctness of the stress

$$\pm \frac{1}{2\sqrt{2}} \left(\frac{Pe}{b} - P \right).$$

The diagonals in the plane of the top cross-arm were first studied, with the results given in Table I. The gauge readings are the average of three readings in tension and three in compression.

First, two observation points were placed at the top of the legs, at corners of the tower diagonally opposite each other and micrometer microscopes set up over these. Load was applied at the end of the cross-arm being investigated and the position of the observation points noted. Then a diagonal was removed and the points moved away from the position they were in before the removal. The stress gauge was then inserted and stress developed in it to bring the observation points back to their original position. The reading of the graduated tube was taken and translated into pounds by means of the calibration curve. This number of pounds would then be the stress in the diagonal due to the loading.

Study of Table I shows a satisfactory check on the calculated stresses for the diagonals of the upper cross-arm.

TABLE I

CALCULATED AND OBSERVED STRESSES IN DIAGONALS OF UPPER CROSS-ARM

Load at End of Cross-Arm, Lb.	Calculated Stress in a Diagonal, Lb.	Gauge Reading, Divisions	Observed Stress, Lb.
20	5.50	7.97	5.5
30	8.25	11.87	8.3
40	11.00	16.03	11.5
50	13.75	20.00	14.4

Results were not so satisfactory for the middle cross-arm, however. With a 50-lb. load at one end of the arm, with one diagonal in place, and the stress gauge in the place of the other diagonal, the average reading practically coincided with the reading of the gauge when only the latter was in place in the square, that is when there was no diagonal intersecting the line of the gauge. In other words, the stress in either diagonal appeared to be the same whether one or two diagonals existed in the square determined by the four legs.

Observation of the movement of two of the four corners of the square indicated that the removal of either the tension or the compression diagonal in the horizontal bracing gave no appreciable difference in tower distortion as compared with the distortion with both diagonals in place.

It is thus evident that further study is necessary to clear up the behaviour of the horizontal bracing in the planes of the middle and the lower cross-arms. Possibly the horizontal bracing effect of the trusses in the battered faces of the tower, which are nearer to the middle and the lower cross-arms than to the upper cross-arm, is sufficient to obscure the action of the bracing in the planes of these two cross-arms. Such effect does not, however, appear to extend to the upper cross-arm.

EFFECTIVE LENGTH OF BRACING STRUTS IN TRANSMISSION TOWERS

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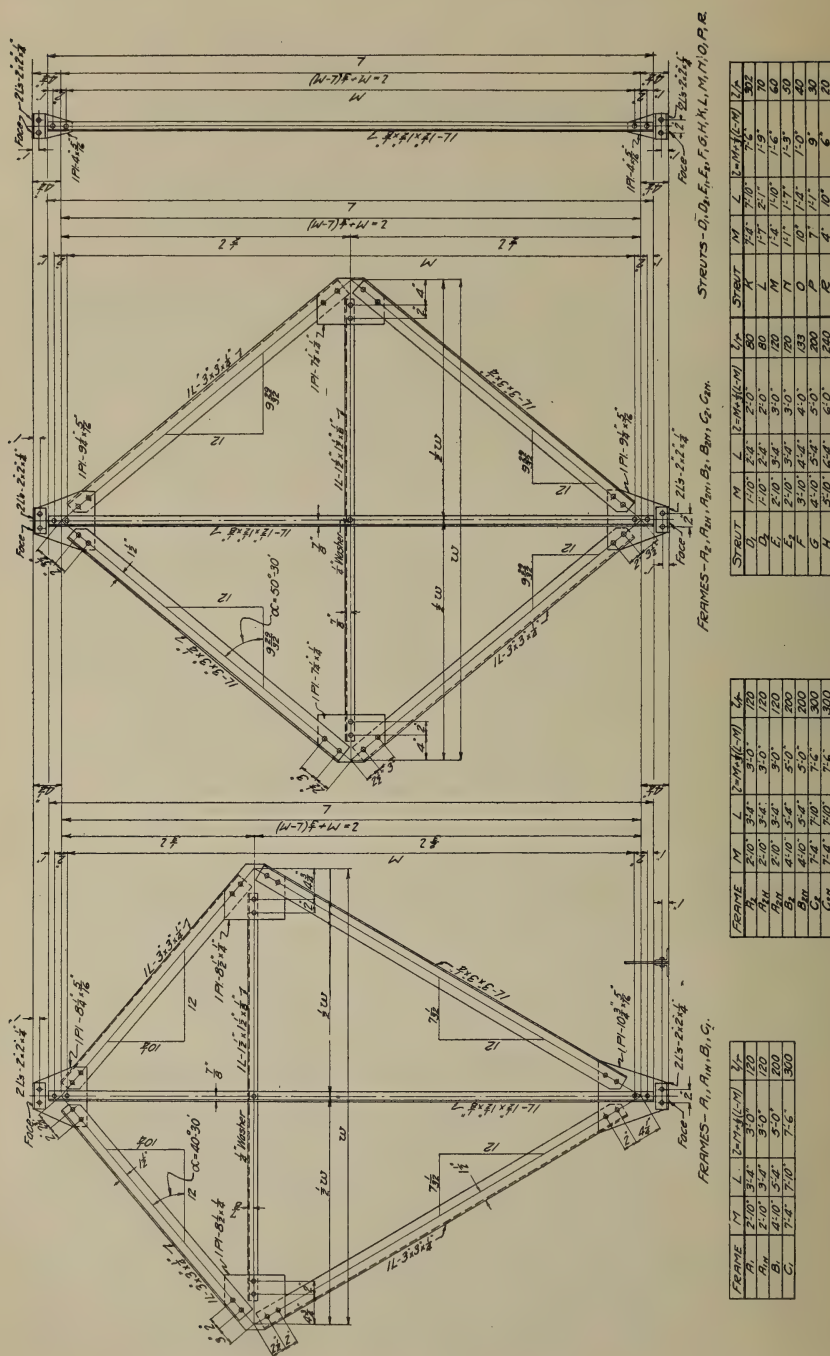
One of the assumptions of design, affecting in an important degree the weight of a transmission line tower, is that relating to the effective length of bracing struts, where these are intersected by corresponding tension members. It has been considered by most designers that the presence of the tension diagonal, which is bolted to the compression strut at the intersection, would materially reduce the effective length of the latter.

Just how great this reduction should be has heretofore been very much a matter of judgment. A rule that has been used fairly widely for such struts is to assume their length as three-quarters of the actual distance from centre to centre of end connections. Some designers consider that this reduction is not sufficiently great, while others assume the whole actual length of the strut as the length to be employed in a compression formula.

UNIVERSITY OF TORONTO TESTS

As one feature of the study that has been made of the design of transmission line towers, it was deemed desirable to throw some experimental light, if possible, on the effective length of bracing struts. To this end a series of specimens were made up and tested with the results that are hereinunder set forth. Fig. 1 shows the details of the specimens. The basic idea observed in the planning of the tests was to subject a vertical single-angle strut supported by a horizontal tension angle carrying approximately the same stress, to a load sufficient to cause failure and then to compare the strength of such a vertical strut with that of a corresponding isolated strut not supported in any way transversely at an intermediate point. This comparison should give some indication of the effective length of the strut enjoying lateral support.

Two types of frames were employed. First, a type giving lateral support to the strut being tested at one of the third points, and, second, a type giving support to the centre of the strut. Specimens of the former series are A_1 , A_1^H , B_1 , and C_1 . These, because of their peculiar shape,



were nicknamed "kite frames". The length of the vertical strut was made for the A frames 3 ft. from centre to centre of end connections of the struts, for the B_1 frame 5 ft., and for the C_1 frame 7 ft. 6 in. The horizontal tie was of the same section as the vertical strut, namely, of a single $1\frac{1}{2}$ by $1\frac{1}{2}$ by $\frac{1}{8}$ in. angle, and intersected the latter at one-third of its length from one end.

The second series of frames were all diamond-shaped and having a nominal length of strut of 3 ft., 5 ft., and 7 ft. 6 in., with the cross tie intersecting the vertical strut at its centre. The specimens of this series are as shown in Fig. 1, namely, A_2 , A_{2H} , A_{2N} , B_2 , B_{2N} , C_2 , and C_{2N} . Specimens A_{2H} and A_{2N} were made up to afford corroboration or otherwise of the results obtained from specimen A_2 , which did not appear to be altogether consistent with other tests. Specimens B_{2N} and C_{2N} were made up similarly to afford a basis of average for the results obtained with B_2 and C_2 respectively.

The holes for the frames were punched 11/16-in. diameter and connections were made by 5/8-in. bolts in order to simulate as much as possible characteristic tower construction.

As a very common assumption made by designers of transmission towers with respect to diagonals is that the tension and compression diagonals in a panel of a side truss are equally stressed, the frames were so devised that the stresses in the vertical strut and in the horizontal intersecting tie would be the same. This was done by applying the theory of frames with redundant members to these specimens and proceeding by the method of trial and error in order to arrive at the proper slope of the outside members of the frame with respect to the vertical. The calculations and detailed design of the frames for test purposes were carried out by H. J. A. Chambers as part of his original work for the Master's degree.

PLAIN STRUTS

In order to form a basis of comparison between a strut enjoying lateral support in a frame and a strut not so supported, a series of plain struts with plate and angle caps and bases were made up as indicated in Fig. 1. These had lengths centre to centre of end connections varying from 6 in. to 7 ft. 6 in., the section used being in all cases a $1\frac{1}{2}$ by $1\frac{1}{2}$ by $\frac{1}{8}$ in. angle.

METHOD OF TESTING

Both the frames and the plain struts were subjected to compressive load in an Olsen wire testing machine. Martens mirror extensometers were attached at three points on the cross-section of the vertical struts

in the frames as indicated in Fig. 2. By observing the strain at each of these points it was possible to determine the stress at the centre of gravity of an angle strut at any loading stage and by multiplying this by the area of the strut the total amount of load on it could be determined. This total stress increased uniformly with the application of load within the ordinary working limits, but was found to increase less rapidly than the applied load after the approximate proportionality limit had been reached.

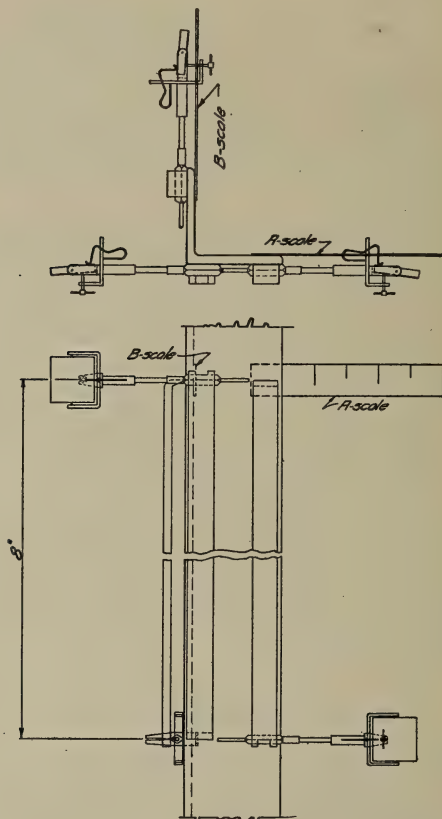


Fig. 2—Attachment of Extensometers to Struts

Scales were attached to the two legs of the vertical strut for the purpose of reading the lateral deflection in two directions at right angles to each other at each loading stage and the results obtained from these observations were made the basis of fixing the proportionality limit.

DEFLECTION DIAGRAMS

In Figs. 3, 4, 5 and 6, are set forth the lateral deflection of the vertical struts in the frames and of the plain struts for loads up to and well

beyond the proportionality limit. These diagrams show the lateral deflection of some convenient selected point, not in all cases the mid-height of the strut, for the various total loads on the specimen in pounds. The deflections are plotted in a continuous scale and hence the actual amounts must be ascertained by taking differences. On these diagrams the proportionality limit is indicated by a heavy horizontal stroke which has been located as a compromise between the curves plotted from the two scales.

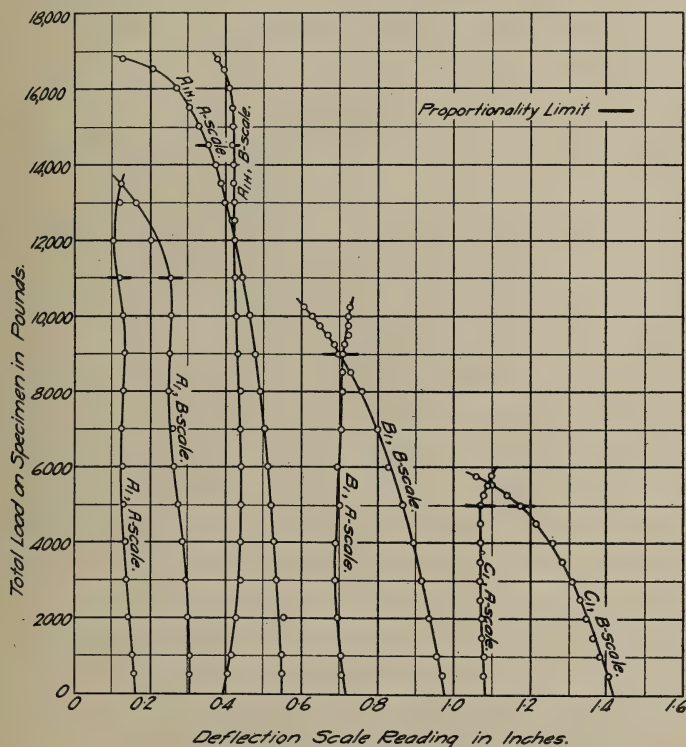


Fig. 3—Lateral Deflections of Struts in Frames A1 A_{1H}, B1 and C1

EFFECTIVE LENGTH DETERMINATION

As a basis for determining the effective length of a strut, laterally supported by a tension member at right angles thereto, and carrying approximately the same total amount of stress, the proportionality limit was found more satisfactory than any other basis. The proportionality limits of the various plain struts were plotted against their slenderness ratios in Fig. 7. In determining the latter ratios for purpose of plotting, the length of the strut was taken as $l = M + 1/3 (L - M)$, where M is the distance between the inner rivet of one end connection

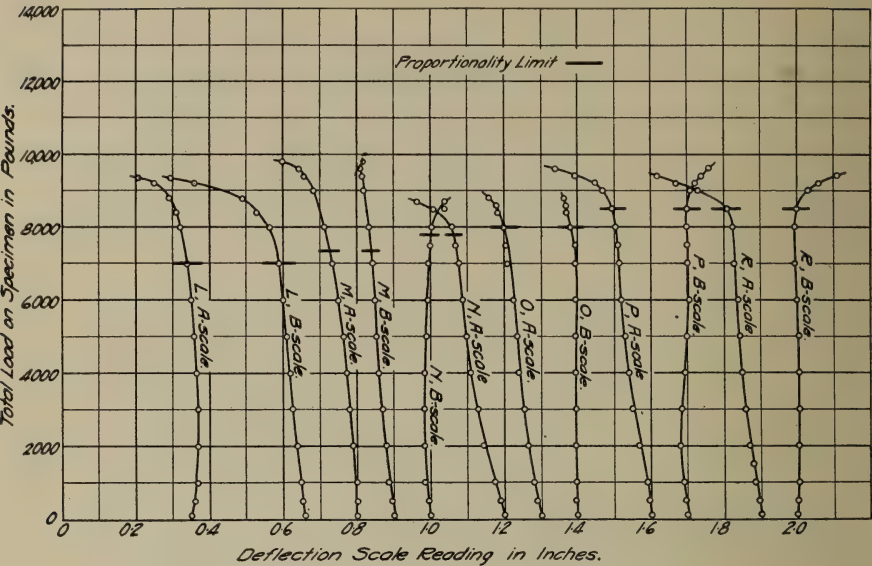


Fig. 6—Lateral Deflections of Struts L, M, N, O, P and R

and the inner rivet of the other connection and L is the total overall length of the angle. This is the rule adopted in Technologic Paper No. 218 of the United States Bureau of Standards entitled "Results of Some Compression Tests of Structural Steel Angles" by A. H. Stang and L. R. Strickenberg. The least radius of gyration was used in all cases for the determination of the slenderness ratio. A smooth curve

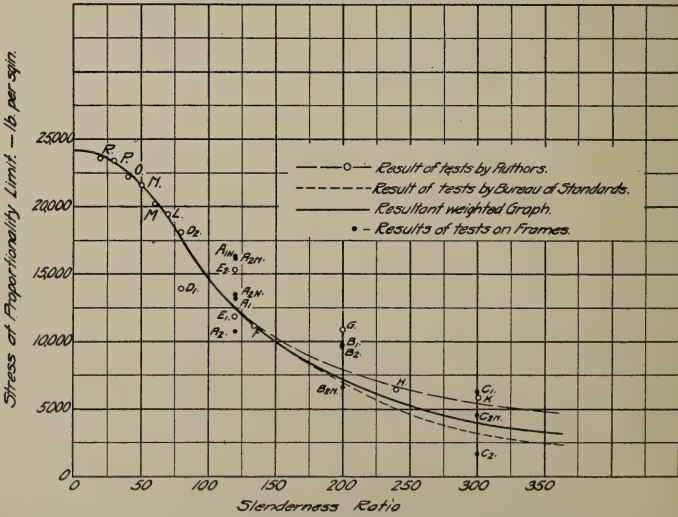


Fig. 7—Relation of Slenderness Ratios of Supported and Unsupported Struts

was then run through the plotted tests giving the variation of the proportionality limit with slenderness ratio.

On the same diagram, Fig. 7, there were plotted the stresses at the proportionality limit in the vertical struts of the various frames tested. By observing the slenderness ratio for which a normal plain strut would give the same proportionality limit as a strut in a frame, it was possible to ascertain the reduction in slenderness ratio brought about by supporting the struts at some intermediate point.

In order to give a broader basis for comparison of the strength of supported struts with ones not supported, there were included in Fig. 7 the tests on structural steel angles made at the United States Bureau of Standards referred to above. These tests were on angles having a variety of end attachments for which the end fixation factors are tabulated in Technologic Paper No. 218. These range from 1.1 to 1.9. It was deemed fair, having regard to the particular end attachments used in the University of Toronto tests, to assume the end fixation factor as 1.2. Consequently the ultimate strengths of the angles in the Bureau of Standards tests were all adjusted to a fixation factor of 1.2. As the proportionality limit of the angles in the Bureau of Standards investigation was not reported, this was derived from the ultimate strength by multiplying the latter for each slenderness ratio by a factor found by dividing the proportionality limit by the ultimate strength found in the University of Toronto tests for the same slenderness ratio. On Fig. 7 these adjusted proportionality limits for the Bureau of Standards results are plotted as a curve with short dashes. The combination of the latter results with the University of Toronto results was made by weighed average and the resultant graph is shown as a full line.

<i>Specimen</i>	<i>Length</i> $L = M + \frac{1}{3}(L-M)$ <i>-inches.</i>	<i>Equivalent</i> <i>unsupported</i> <i>Length-inches</i>	<i>Percentage</i>
<i>A₁</i>	36	33.75	93.7
<i>A_{1H}</i>	36	26.25	72.9
<i>B₁</i>	60	45.75	76.2
<i>C₁</i>	90	66.75	74.2
<i>A₂</i>	36	42.0	116.6
<i>A_{2H}</i>	36	33.0	91.6
<i>A_{2N}</i>	36	27.0	75.0
<i>B₂</i>	60	46.5	77.5
<i>B_{2N}</i>	60	63.75	106.2
<i>C₂</i>	90		
<i>C_{2N}</i>	90	82.5	91.7

Table 1—Effective Unsupported Lengths of Supported Struts

Effective slenderness ratios for the various vertical struts in the frames are determined with respect to this weighed curve and are listed in Table I.

CONCLUSIONS

Examination of Table I shows that the effective or equivalent unsupported length of a strut stayed laterally by an intersecting tension member varies from about 73 to 117% of the distance centre to centre of end connections. There is no clear difference resulting from the shifting of the support from the third-point to the centre. The average percentage, omitting those for A_2 and B_{2N} , which appears to be in error, is 81.6%. This would indicate the advisability of not reducing the centre to centre length by more than 25%, and preferably by not more than 20%.

